Improved monitoring of flow phenomena in moving mesh methods

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1 Introduction

Numerical solutions of flow-related problems form a long-standing field of research. This is mainly due to the wide range of flow features that can occur and—more importantly—the effort needed to accurately capture these features in numerical solutions.

Our main focus lies on a moving mesh method that is able to capture not only large shocks, but also smaller and more subtle flow phenomena automatically. The Winslow-based structured moving mesh is steered by a monitor function. We use a monitor that is not only smoothly normalised in space and time, but also includes additional physical quantities that predict flow phenomena more accurately. The better a monitor function is, the more applicable a moving mesh strategy becomes, as the user does not need many expert knowledge to get excellent results.

2 Moving meshes and compressible flow

The mesh movement and solution monitoring are essentially independent of the governing partial differential equations (PDEs). Here we will show some examples of hyperbolic PDEs, more specifically the Euler equations of compressible gas dynamics. The PDEs are solved by a finite volume solver on a two-dimensional structured mesh.

The mesh movement is based on the elliptic 'equidistribution' system

$$\nabla_\xi \cdot (G \nabla_\xi \mathbf{x}) = 0, \tag{1}$$

where $G = \mathrm{diag}(\omega_1, \omega_2)$, i.e., anisotropic mesh movement. The quality of the mesh is mainly influenced by the choice of monitor functions ω_i. It is easy to obtain results that look good at first sight: include gradients of the solution q, for example as in the arc length-type monitor

$$\omega_i(q) = \sqrt{1 + \alpha \left(\frac{\partial q}{\partial x_i} \right)^2} \quad (i = 1, 2).$$

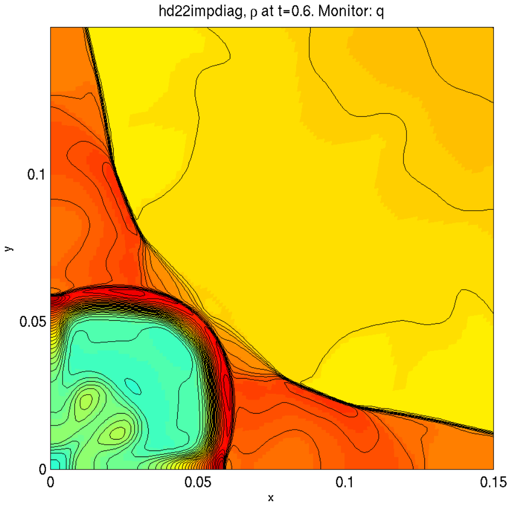
This approach has several disadvantages. For each problem an appropriate value for α has to be chosen, which may be difficult without prior knowledge about the solution. Besides, the solution will often change through time, requiring a time-dependent α. A monitor that solves this produces high quality meshes and solutions as we have found for 1D (magneto-)hydrodynamics applications too [2]. Finally, for systems of PDEs, simply including all components of $\mathbf{q}$, e.g., the conservative flow variables, may not have the desired effect. Derived physical quantities such as vorticity of the velocity field, or the entropy may be stronger indicators of the birth of new flow phenomena. Including these will move the mesh in time in order to obtain the needed resolution.

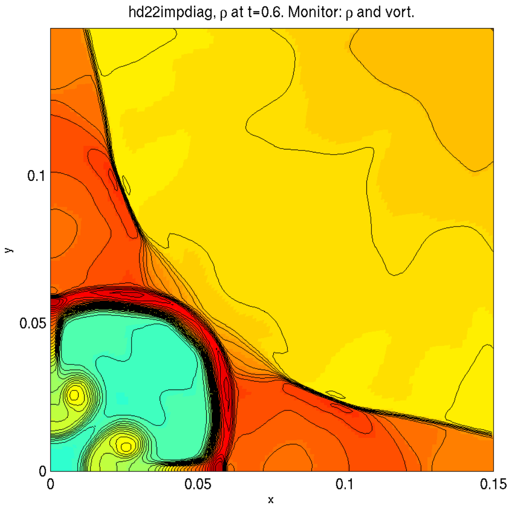
3 Improved monitor functions capture more phenomena

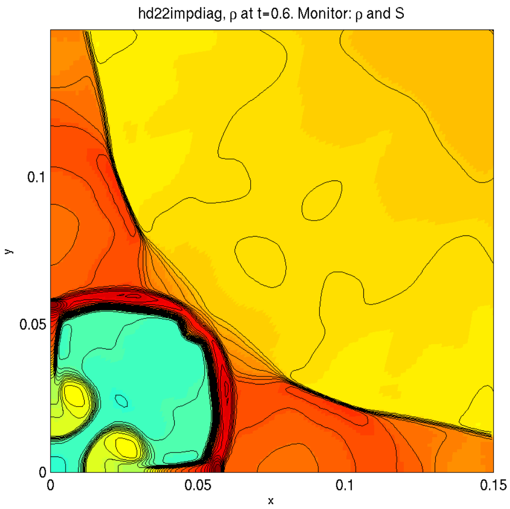
As an example, we solve the implosion problem by Hui et al. [1]. This symmetric problem features many different shocks that continue reflecting against the walls. The diagrams at the right show contour plots of the density. The first diagram shows the result with all solution components included in the monitor. The shocks are properly captured but near the origin there is little detail. The jets that should appear, might be detected by including the vorticity of the velocity field in the monitor. The second diagram indeed shows some improvement. Finally, we combine density with entropy, since high entropy fluctuations indicate that potentially new phenomena can occur in the flow. The third diagram shows that this gives the best result.

This example is not exceptional. Many other problems feature small structures, such as the double Mach reflection problem by Woodward and Colella, which contains a jet, and the various problems showing Richtmyer–Meshkov instabilities on contact discontinuities. Extremely fine discretisations are needed, or higher-order PDE solvers in order to capture these phenomena. Alternatively, we want the moving mesh to pick up even these subtle structures to obtain high accuracy.

Besides entropy, other derived flow quantities may provide high quality monitoring. We found the combination of this 'expert knowledge' and the automatically balanced monitor function a powerful technique for steering mesh movement.







References

[1] W.H. Hui, P.Y. Li, and Z.W. Li. A Unified Coordinate System for Solving the Two-Dimensional Euler Equations. *J. Comput. Phys.*, 153:596–637, Aug 1999.

[2] A. van Dam and P.A. Zegeling. A Robust Moving Mesh Finite Volume Method applied to 1D Hyperbolic Conservation Laws from Magnetohydrodynamics. *J. Comput. Phys.*, 216:526–546, 2006.