PL-analogue of Nash—Kuiper theorem

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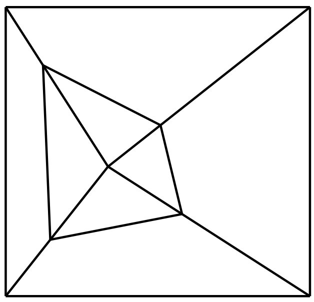
This talk will be devoted to PL-isometry — a piecewise linear mapping, which preserves lengths of curves. Origami is an example of 2-dimensional PL-isometry.



A PL-isometry is defined (up to isometry) by its plane partition into convex polygons. So it is more convenient to research this plane partition.

A plane partition corresponding to a PL-isometry has the following properties: the number of polygons meeting in every vertex of this partition is even; the alternating sum of the angles for any vertex equals 0.

Similar properties can be formulated for greater dimensions.

We will show that any short mapping of the finite set can be extended to the PL-isometry.

Definition. *Let $\mathcal{X}$ be one of the three spaces $\mathbb{E}^d$, $\mathbb{S}^d$ or $\mathbb{H}^d$.*

Definition. *A PL-isometry for the space $\mathcal{X}$ is a mapping $f : \mathcal{X} \to \mathcal{X}$ which for given triangulation maps every simplex isometrically.*

Main results:

Theorem 1. *Let $\mathcal{A}$ be a finite subset of $\mathcal{X}$. Then every short map $\varphi_{\mathcal{A}} : \mathcal{A} \to \mathcal{X}$ admits an extension to a PL-isometry $\varphi : \mathcal{X} \to \mathcal{X}$.*

The proof of this theorem is constructive and is realized as an algorithm.

The concept of PL-isometry and this theorem can be applied to some problems of the computational geometry.