VORONOI ADJUSTMENT MODEL ON 2-SPHERE FOR SMALL NUMBER OF POINTS

MASAHARU TANEMURA

We consider here to get a "good" (an evenly distributed) configuration of points on 2-sphere (two-dimensional unit sphere) by using a method introduced by the author [1]. Roughly speaking, the method is based on the idea that a good configuration would be obtained by a certain repelling mechanism among points.

Let $\{x_1(t), x_2(t), \ldots, x_N(t)\}$ be the configuration of N points on 2-sphere $\mathcal{S}$ at discrete time $t (t = 0, 1, 2, \ldots)$. In our method, we move the position position $x_i(t)$ of a point i at time t to the "center" of the spherical Voronoi cell of the point as its position at the next time $t+1$, namely, $x_i(t+1)$. By applying the similar procedure for all points, we get the set of points $\{x_1(t+1), x_2(t+1), \ldots, x_N(t+1)\}$ for the next time $t+1$. Then, we get a spherical Voronoi tiling for the time $t+1$. By repeating the above movement, it is expected that a "good" configuration of points will be attained from a certain random initial configuration. We call the procedure here the Voronoi adjustment model.

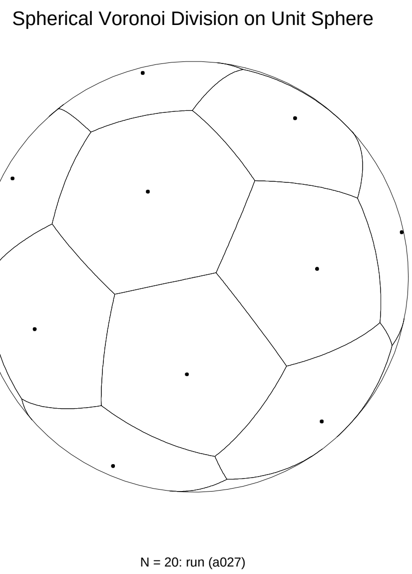

FIGURE 1. An example of "good" configuration of points and its spherical Voronoi tiling for $N = 20$.

For small number of points, it is hoped that stable equilibrium configurations would be obtained. In this paper, we present "good" configurations of points obtained by the Voronoi adjustment model in the range $12 \leq N \leq 50$. In order to characterize the structure of configuration, we introduce the index $(\prod_{f\geq 3}\prod_i f_i^{\alpha})$, where f_i represents the f-sided spherical polygon whose area size is i-th among existing f-sided polygons, and where α indicates the number of the polygons of index f_i with same size. The area of the polygon of index f_i is denoted by $s(f_i)$. Figure 1 shows an example of configuration attained for $N = 20$. In this case, the index is $(5_1^{12}6_1^6 6_2^2)$, and $s(5_1) = 0.607444, s(6_1) = 0.654636$ and $s(6_2) = 0.674612$, where numerical values are given at a precision of six decimals.

In the talk, comparison of configuraions obtained in the present method with those obtained under the other principle [2] will be performed.

References

[1] M. Tanemura, *Random packing and tessellation network on the sphere*, Forma, **13** (1998), 99–121.

[2] T. Erber and G.M. Hockney, *Equilibrium configurations of N equal charges on a sphere*, J.Phys. A, **24** (1991), L1369–L1377.

Department of Statistical Modelling,, The Institute of Statistical Mathematics, 4-6-7 Minami-Azabu, Minato-ku, Tokyo 106-8569, Japan

E-mail address: tanemura@ism.ac.jp