Volume increasing isometric deformation of polyhedral surface

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In this paper, we prove that any closed bounded polyhedron in R^3 can be isometrically deformed so as to enclose greater volume. Deformed surfaces are extrinsically nonconvex, but have the same intrinsic geometry. The regular tetrahedron can be isometrically deformed to enclose over 37.7% more volume (see Figure 1), while the cube, octahedron, dodecahedron, and icosahedron enjoy in rises over 21.3%, 11.5%, 9.3%, and 3.6%), respectively.

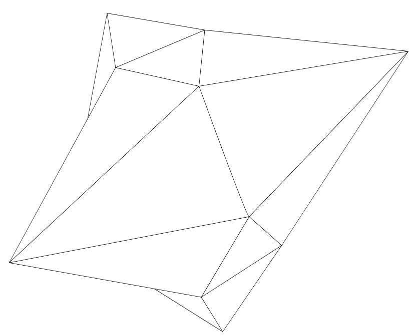

Figure 1

The direct consequence of the Alexandrov's theorem is the fact that a present scan defines not more than one convex realization. Alongside convex realizations there also exist nonconvex realizations. Natural nonconvex realizations obviously reduce volume. For example, we shall separate some vertex of the polyhedron P by plane T and then symmetrically reflect in T the turned over pyramid. The remained part of a polyhedron P plus a pyramid's image at symmetry form a polyhedron of smaller volume, than that of the initial one. Nevertheless, in 1996 D. Bleecker proved that isometric deformation can increase volume of any convex simplicial polyhedron. It is worth noting, that prior to Bleecker's work nonconvex realization of a regular tetrahedron's scan (much exceeding its volume) had been found. Also, a very interesting example of nonconvex realization was given in 2002 by S.M. Mikhalev. Here, we are dealing with polyhedra supposing nonconvex realization increasing volume and keeping all the facets of an initial polyhedron.

The next interesting question is an assumption (stated by N.P. Dolbilin) that any (not only convex and simplicial) closed polyhedral surface can be isometrically deformed with increase in volume. The present work is devoted to the study of the change of volume of a polyhedron which is isometrically deformed. The techniques of solution are inspired by D. Bleecker. More precisely, the method of deformation of a pyramid (see Figure 2) is development of method offered by Bleecker.

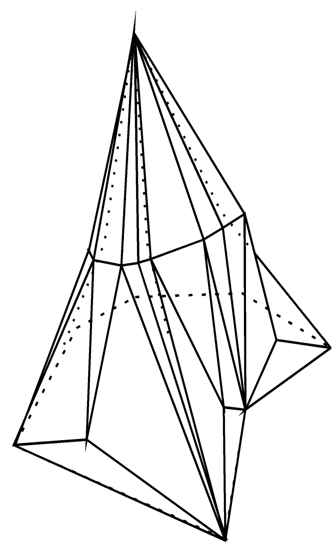

Figure 2

As a result it will be shown, that Bleecker's theorem can be extended to any closed bounded self-not intersected polyhedron. This result can be formulated by the theorem mentioned below:

THEOREM. For any closed bounded and self-not intersected polyhedron in R^3 there exists realization with greater volume.