Development of numerical methods for Navier-Stokes equations in primitive variables

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Abstract

In the approach reduction of computational efforts is based on mass conservation equations and original pressure splitting. Computations are performed in predictor-corrector manner. The paper represents the computational algorithm and results of benchmark and applied problems. Numerical experiments demonstrate efficiency of the approach. Maximum efficiency is observed for directed fluid flows.

Introduction

Finite-differenced 2D Navier-Stokes equations can be written in the following matrix form

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & 0 \end{pmatrix} \begin{pmatrix} \boldsymbol{u} \\ \boldsymbol{v} \\ \boldsymbol{p} \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ 0 \end{pmatrix} . \tag{1}$$

Absence of pressure in the continuity equation leads to zero block on main diagonal of the coefficient matrix ($a_{33} = 0$). As a results, the system (1) cannot be solved by standard methods.

Many algorithms for solving (1) have been proposed and developed. However accurate computation of pressure requires impressive computational efforts [3].

Very efficient algorithms have been proposed for solution of the Navier-Stokes equations in boundary layer approximation [1]. In this case pressure is changed in single spatial direction.

Proposed solver uses simplified Navier-Stokes equations in boundary layer approximation as predictor for full Navier-Stokes equations.

1 Pressure splitting

In general, pressure can be represented as

$$p(t, x, y, z) = p^x(t, x) + p^y(t, y) + p^z(t, z) \\ + p^{xyz}(t, x, y, z) .$$

Basic idea of proposed solver consists of use of very efficient methods proposed for simplified Navier-Stokes equations for computation of “one-dimensional” components $p^x(t, x)$, $p^y(t, y)$ and $p^z(t, z)$ [1]. “Multidimensional component” $p^{xyz}(t, x, y, z)$ is computed by artificial compressibility method to avoid the boundary conditions for pressure [2].

The algorithm consists of corrector (auxiliary problem for computation “one-dimensional” components) and corrector (full Navier-Stokes equations). General formulation of the auxiliary problem is discussed in details.

2 Benchmark and applied problems

Proposed algorithm is tested on the following problems:
1) incompressible 2D flow in driven cavity
2) incompressible 2D flow over a backward-facing step
3) incompressible 2D flow in catalyst of mirco jet propulsion
4) compressible 2D flow in Laval nozzle of mirco jet propulsion

Numerical test show that one of the “one-dimensional” components of pressure will be dominant for directed fluid flows. It results in impressive reduction of total amount of computations.

References

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