Computation of high temperature plasma dynamics in Z-pinches.

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The mathematical model of the processes in z-pinch includes the following set of electronic magnetohydrodynamic (EMH) equations (in the undimensional form) [1, 2]:

$$\frac{d\rho}{dt} + \rho\left(\frac{1}{r}\frac{\partial(rv_r)}{\partial r} + \frac{\partial v_z}{\partial z}\right) = 0$$

$$\frac{\mathbf{v}}{dt} = -\frac{1}{\rho}\nabla p + \frac{[\ \mathrm{rot}\mathbf{B}\times\mathbf{B}]}{4\pi\rho}$$

$$\frac{\rho}{A}\frac{d}{dt}(z_{eff}\epsilon_e + J(z_{eff})) = -P_e\ \mathrm{div}\ \mathbf{v} + \ \mathrm{div}(k_e\ \mathrm{grad}(T_e)) + Q + Q_{ei}$$

$$\frac{\rho}{A}\frac{d\epsilon_i}{dt} = -P_i\ \mathrm{div}\ \mathbf{v} + \ \mathrm{div}(k_i\ \mathrm{grad}(T_i)) - Q_{ei}$$

$$\frac{\partial\mathbf{B}}{\partial t} = -\ \mathrm{rot}\ \mathbf{E}, \qquad \mathbf{j} = \frac{1}{4\pi}\ \mathrm{rot}\ \mathbf{B}$$

$$\mathbf{E} = -[\mathbf{v}_e\times\mathbf{B}] + \frac{\mathbf{j}}{\sigma}[\mathbf{v}_e\times\mathbf{B}] + \frac{\mathbf{j}}{\sigma} - \frac{1}{n_e}(\nabla P_e - \mathbf{R})$$

$$\mathbf{R} = -0.71 n_e\nabla_{||}T_e - 1.5\frac{n_e}{\omega_e\tau_e}\frac{[\mathbf{B}\times\nabla T_e]}{|\mathbf{B}|}$$

$$Q = \frac{\mathbf{j}^2}{\sigma} + (\mathbf{j}, \frac{\mathbf{R}}{\mathbf{n_e}})$$

$$\mathbf{v}_e = \mathbf{v} - \frac{\mathbf{j}}{en_e}, \qquad p_e = n_e t_e$$

The of electronic current velocity was added to consider the "two-liquid" effects. To solve this non-linear set of partial differential equations numerically, there was implied the splitting method (on physical processes). In the first stage of we neglected dissipative effects in plasma movement. In the second stage the program calculated the magnetic field penetration in plazma material, taking into account the finite conductivity. In the subsequent stages there were considered the dissipative effects (electron and ion termal conductivity) and radiative transport using one-group diffusion approximation.

We use the free Lagrangian grid for the calculational area and built the implicit difference sheme on the base of variation method [1, 2]. The Lagrangian in this model is the following:

$$L = \iint_v \left(\frac{\mathbf{v}^2}{2} - \epsilon_e - \epsilon_i - \frac{\mathbf{B}^2}{8\pi\rho}\right) dm$$

The variation of Hamilton's principal function is equal to zero on the trajectory:

$$S = \int_{t_0}^{t_1} L(t)dt \qquad \delta S = 0$$

The discrete analogue of Hamilton's principal function:

$$S = \int_{t_0}^{t_1}\sum_{ij} m_{ij}\left(\tfrac{u^2+v^2}{2}|_{ij} - (\epsilon_e + \epsilon_)_{ij} - \frac{B_{ij}^2}{8\pi\rho_{ij}}\right) dt$$

The mass conservation law was used as quantity equation while varying the Hamilton's principal function: $\delta(\rho d\Omega) = (dm) = 0$.

As the result of the variation and subsequent transformations, we get the discrete differential equation for the calculation of whole pressure:

$$\hat{\Omega}_{ij}\left(\frac{A_e}{A_p}\right)\hat{p}_e\frac{m}{A}J(\hat{Z}_{eff}) - \frac{m}{A}(Z_{eff}z_e + J(Z_{eff})) + \tau\hat{P}_e\sum_{k=1}^{4}\left(\frac{\partial\Omega_{ij}}{\partial r_k}\hat{U}_k + \frac{\partial\Omega_{ij}}{\partial z_k}\hat{V}_k\right) = 0$$

where Ω - the volume of the cell, η - the artificial viscosity, which is added to the pressure to dicrease the oscilations on shock waves, w - the balance coefficient.

The set of equations for pressure can be solved autonomously and, using the solution, we could find the rest unknown quantities (Fig. 1). This differential scheme takes into

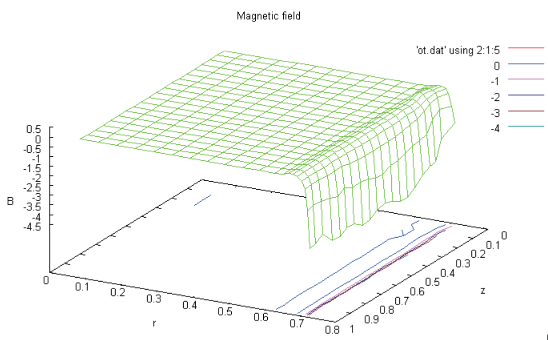

Figure 1: The distribution of magnetic field on the 71th step

account the edge effects and has the second approximation order in the entire calculational area.

During the modelling we can analyse the dynamic of z-pinch: spatially it gripes 10 times, there are shock waves and inhomogeneities (the example – electrode effects). All these effects involve the deformation of the calculating grid, which is rectangular in the beginning and repeatedly deformates during the calculation (Fig. 2). To avoid the reversing

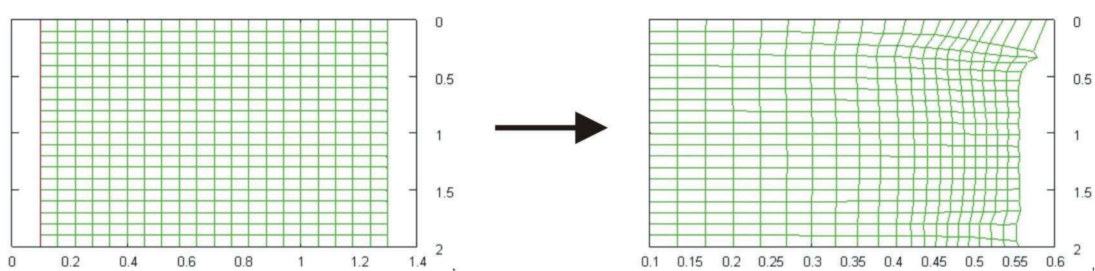

Figure 2: The deformation of the grid during calculation

of the cells and getting physically incorrect results, the limitations on the minimal size of the cells were implied. If spatial size of the cell becomes smaller than the acceptable value or after the definite number of time steps, then all physical parameters were conservatively converted to the new grid [3, 4]. Not only the "traditional" values (such as temperature, impetus and mass) were converted, but also the average charge of ion in cell and losses on ionization.

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References

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