Postfiltration of IC factors for load balancing in parallel PCG linear solvers

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Solution of elliptic problems is an important component of grid generation for industrial applications. We consider the solution of large SPD systems on parallel computers with the use of additive preconditionings. The latter are based on nearly equal partitioning of the given coefficient matrix into overlapping submatrices, which are distributed between the processors.

The number of nonzero elements in Incomplete Cholesky (IC) factors on each processor may become essentially unbalanced due to different submatrix properties. To provide a reasonable load balancing the postfiltration of IC factors may be used.

When the first order IC factorization (i.e. factorization with the only drop tolerance threshold) is exploited one can perform the postfiltration of the factors having the largest number of nonzero entries by some other postfiltration threshold which is locally chosen at each processor to make the numbers of nonzeros in the factors to be equal. Usually it may deteriorate the preconditioner quality and the number of PCG iterations increases. In the factorizations of the second order of accuracy (IC2, i.e. factorization with two different thresholds for factors and error matrices, respectively) the same effect may also be observed but influence to the convergence behavior is more smooth.

In the case of IC2 factorization, another possibility is to increase the number of nonzeros in the factors using the elements from the already computed main term of the error matrix. It may reduce the number of PCG iterations and provide the same time per iteration as compared to the unbalanced algorithm.

The numerical experiments for 2D and 3D test cases with anisotropic coefficients are performed. The most difficult to solve problems from the University of Florida Sparse Matrix Collection are also considered.

References

[1] I. Kaporin. High quality preconditioning of a general symmetric positive definite matrix based on its $U^TU + U^TR + R^TU$-decomposition. *J. Numer. Lin. Alg. Appl.*, 1998, Vol. 5, No. 5, 483–509.

[2] I. E. Kaporin and I. N. Konshin. A parallel block overlap preconditioning with inexact submatrix inversion for linear elasticity problems. *J. Numer. Lin. Alg. Appl.*, 2002, Vol. 9, No. 2, 141–162.