Computation of discrete extrinsic curvatures using the duality principle

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Approximation of surfaces by polyhedra is challenging problem of modern discrete differential geometry. Computation of curvature of polyhedral surfaces is important problem in animation, pattern recognition, geometric modeling, reverse engineering, structural mechanics, computational biology, as well as in interface-related problems in physics, chemistry and biology.

Intrinsic curvature computation is based on length measured along surface and integral Gauss-Bonnet relations, while extrinsic curvature is based on the concept of spherical Gauss map. One can compute discrete extrinsic curvatures using relation between mean curvature and surface Beltrami operator, or relation between variation of surface area and swept volume.

Duality principle for computation of boundary length and area of convex domains was introduced by ancient greek geometer and physicist Archimede. In fact he introduced numerical method for computation of of arclength and area of a circle via sequence of inscribed and circumscribed polygons. This idea allowed to prove convergence of numerical approximations to arclength and area and to obtain two-sided error estimates.

In this work it is shown that duality principle allows to obtain convergent piecewise-affine approximation to spherical Gauss map when regular surface is approximated by a sequence of pairs of locally polar polyhedra. As a result on can obtain convergent discrete pointwise approximations to mean and Gaussian curvature and to curvature tensor as well as convergent discrete approximations to integral curvature measures.