Application of Moving Adaptive Meshes in Hyperbolic Problems of Gas Dynamics *

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Abstract

Coupled algorithm of using the Godunov's type solver and adaptive moving mesh is applied to generate moving adaptive mesh when modeling a 2D explosion with complicated wave structure.

1 Introduction

Coupled algorithm of using the Godunov's type solver and adaptive moving mesh has been offered in [1, 2]. In this approach after every mesh iteration the finite-volume flow solver updates the flow parameters at the new time level directly on the curvilinear moving grid without interpolation from one mesh to another [3, 4]. Thus, we eliminate the errors caused by interpolation procedure which smears the discontinuities. This scheme on one hand utilizes the idea of the Godunov's scheme on the deforming meshes [5] and on the other hand is of the second-order accuracy in time and space.

Method of adaptive grid generation has been suggested in [6]. Method is variational, i.e. we consider the problem of minimizing a finite-difference function approximating the Dirichlet's functional written for surfaces. The discrete functional has an infinite barrier at the boundary of the set of grids with all convex quadrilateral cells and this guarantees unfolded grid generation during computations both in any simply connected, including nonconvex, and multiply connected 2D domains. This folding-resistant property is very important since if any of the cells becomes folded we have to stop calculation of the flow problem and use special procedures to continue modeling. The barrier property is also of particularly importance in the vicinity

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of shock waves where the cells are very narrow and maximal aspect ratio achieves 50 and larger [7].

When modeling 2D hyperbolic problems with discontinuous solution on the moving adaptive mesh it is possible to reduce the errors, caused by shocks smearing over the cells, by many factors of ten and, therefore, to decrease significantly, by several times [7], the overall error.

2 Modeling of explosion

We consider a spherical explosion between two parallel walls at $z{=}0$ and $z{=}1$ [8]. Initially the gas is at rest with parameters $(p,\rho)_{out}{=}(1,1)$ evewhere except in a sphere centered at $(0,0,0.4)$ with radius 0.2. Inside the sphere $(p,\rho)_{in}{=}(5,1)$, the ratio of specific heats $\gamma{=}1.4$. The initial jump in pressure results in an outward moving shock and a contact discontinuity and an inward moving rarefaction wave. There occurs interactions between these waves and between waves and the walls. The flow by $t{=}0.7$ consists of several shocks and strong tangential discontinuity surrounding the low density region near the center.

First, until the initial outward shock reaches the wall $z{=}0$, the solution is spherically symmetric. Further, it remains cylindrically symmetric and we use the 2D axisymmetric formulation of the problem. In half-plane $\phi{=}const.$ (where ϕ is an angle in cylindrical coordinates) we compute the domain $(x,z){\in}[0,1.4]{\times}[0,1]$ and on the axis $x{=}0$ define the symmetric boundary conditions. On the walls $z{=}0,1$ we use the condition of reflection and at line $x{=}1.4$ undisturbed values $(u,v,p,\rho)_{out}$ are defined. The fixed mesh is rectangular. The pressure contours at $t{=}0.7$ computed on the fixed mesh $100{\times}140$ are presented in Fig.1.

We perform adaptation by minimizing the Diriclet's functional [1, 2]. On the walls and axis of symmetry we apply constrained minimization to provide consistent redistribution of the grid nodes inside the domain and on its boundary [7]. Up to $t{=}0.45$ we calculate on the rectangular mesh and then the adaptive procedure is switched on. Since strong grid lines compression results in decreasing the admissible time step (value of $\triangle t$ falls proportionally to the increase of the cell aspect ratio) we use the following strategy. Main part of the time interval we compute on not very condensed mesh and at some moment $t'{=}0.7{-}\delta t$ we attain a maximum compression of grid lines in the vicinity of discontinuities. Following this methodology within the time interval $(0.45,0.65)$ we set rather a small coefficient of adaptation [1, 2] $c_a{=}0.07$ to 0.1. Here it is sufficient to perform 3 mesh iterations at every time step. The admissible time step $\triangle t$ is decreased by 5 to 10 times in comparison with $\triangle t$ on the rectangular mesh.

Further, within the final interval $(0.65, 0.7)$, the coefficient c_a is increased up to 0.18 to 0.4 and then time step falls down to 0.05 to 0.005 of $\triangle t$ on the rectangular mesh. It is not a constant value during computation because the mesh constantly "breaths" and accordingly the value of $\triangle t$ periodically first decreases then increases, etc.. At this stage it is sufficient to perform 1 to 2 mesh iterations at every time step.

In the first series of computations with adaptation the pressure p is used as a control function f [1, 2]. In Figs.2,3 the pressure contours and adapted mesh are shown, respectively. We see that the shocks thickness is decreased significantly in comparison with the fixed mesh solution presented in Fig.1. However the grid lines can be strongly condensed to the outer shock and two more ones adjoining to it. This is due to insufficient number of grid points. In the next calculations we use the mesh 200×250, see Fig.4. Here both the intensive shocks and compression waves are depicted by the condensed grid lines rather fine. Pressure contours, computed on the rectangular and adapted meshes, are presented in Figs.5,6.

Since p is continuous across the tangential discontinuity the mesh does not feel it. In the next calculations the density ρ it is used as a control function f. Here the mesh 100×140 depicts the tangential discontinuity very clear, see Fig.7. Density contours computed on the quasiuniform and adaptive meshes are presented in Figs.8,9. Comparing the grids with the control function $f{=}p$, Fig.3, and $f{=}\rho$, Fig.7, one can note that in spite of the fact that in the second case c_a is twice larger, in the first case the shocks are indicated by the grid lines a bit more clear. This occurs due to influence of the contact discontinuity on the mesh in the second case.

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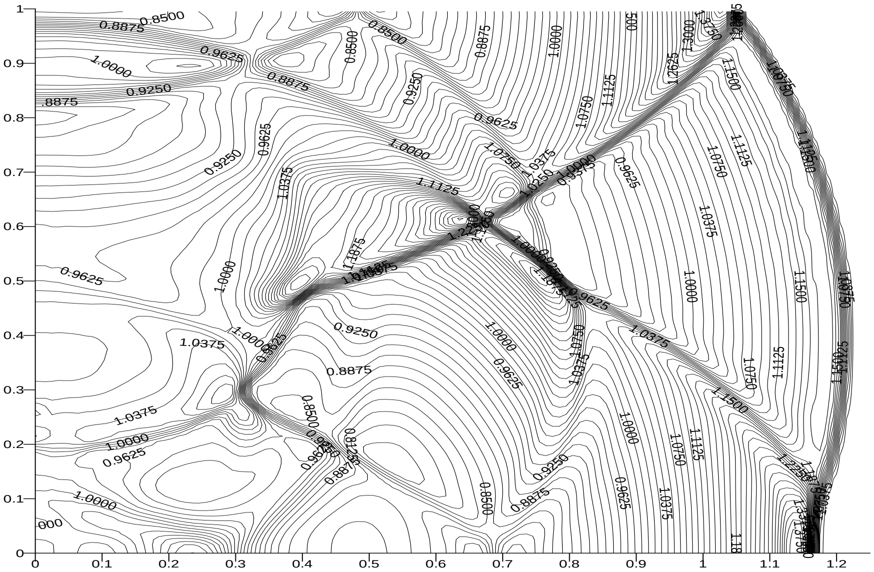

Figure 1: Pressure contours from 0.775 to 1.5 with increment 0.0125 in the $x-z$ plane at t=0.7 computed on the rectangular mesh 100×140.

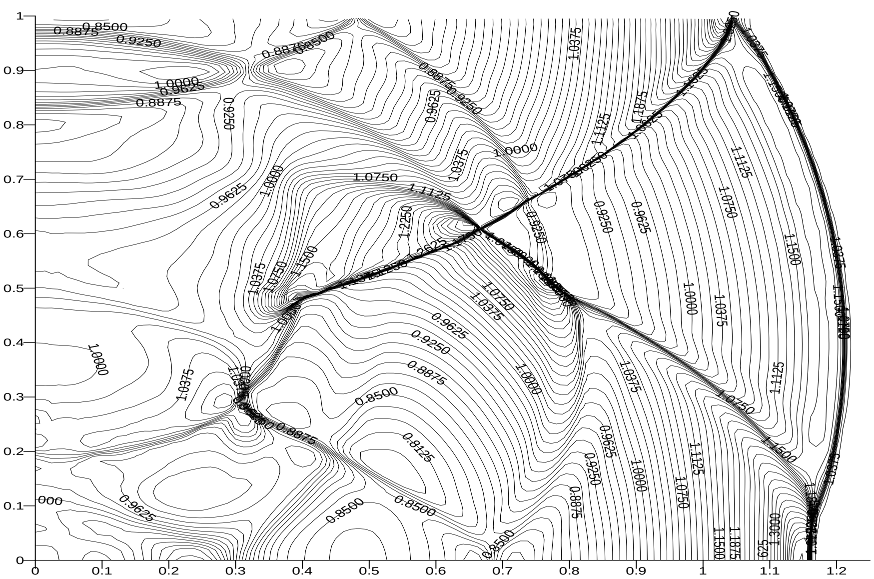

Figure 2: Pressure contours computed on the adapted mesh 100×140.

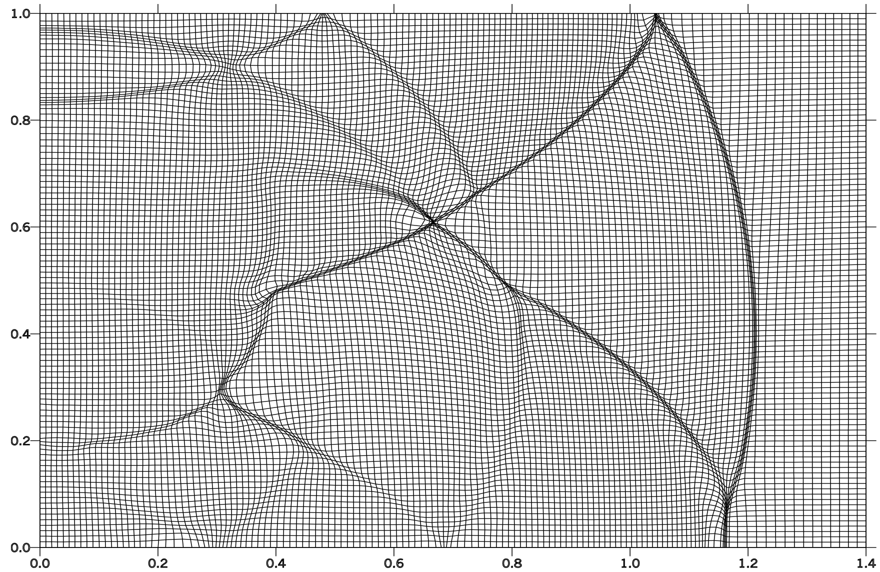

Figure 3: Adapted mesh 100×140, p is a control function, c_a=0.21.

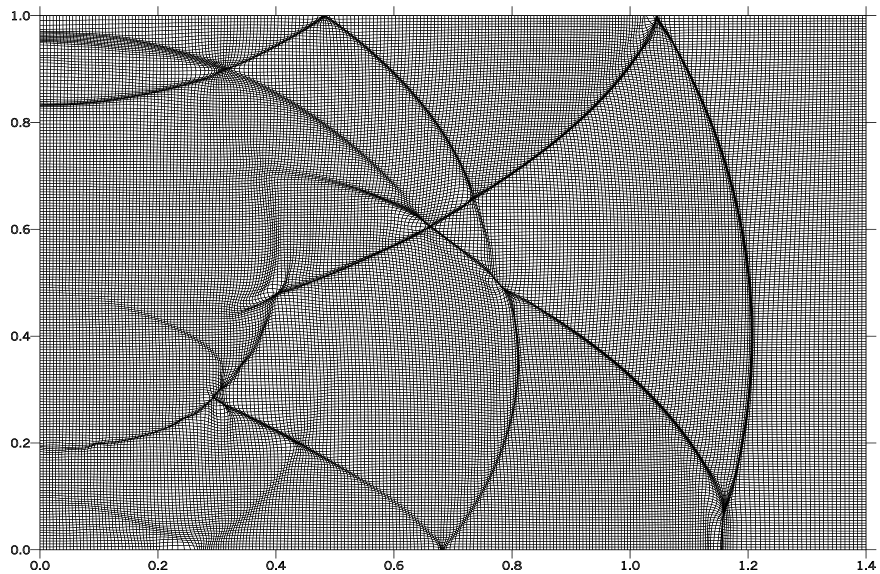

Figure 4: Adapted mesh 200×250, p is a control function, c_a=0.18.

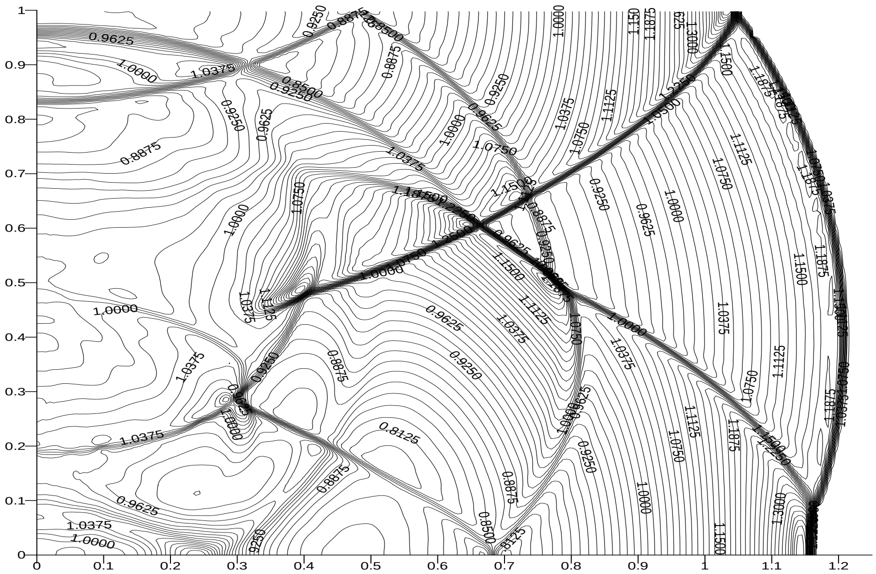

Figure 5: Pressure contours computed on the rectangular mesh 200×250.

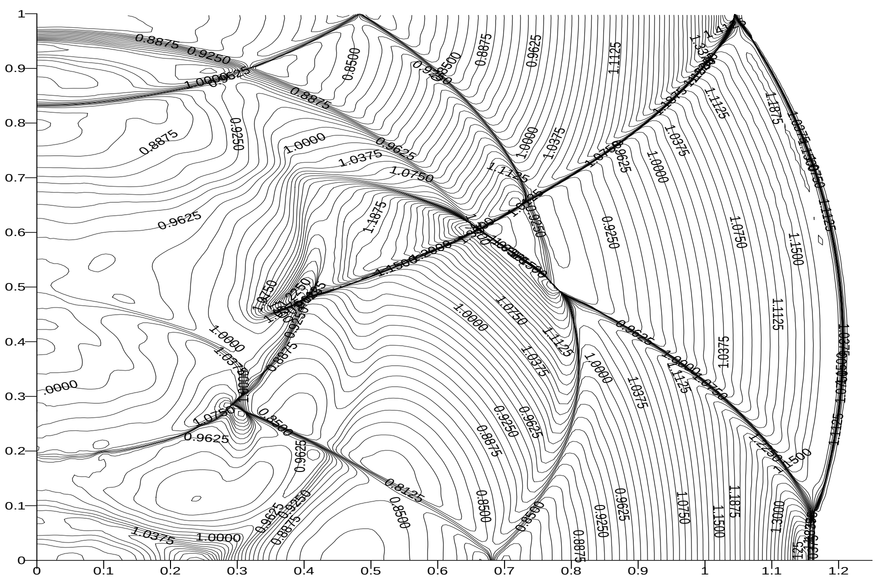

Figure 6: Pressure contours computed on the adapted mesh 200×250.

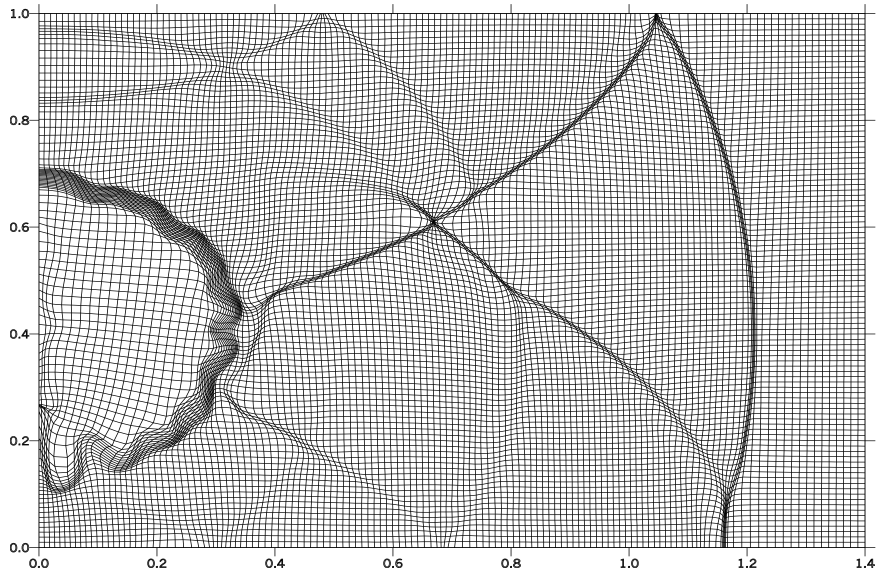

Figure 7: Adapted mesh 100×140, ρ is a control function, c_a=0.4.

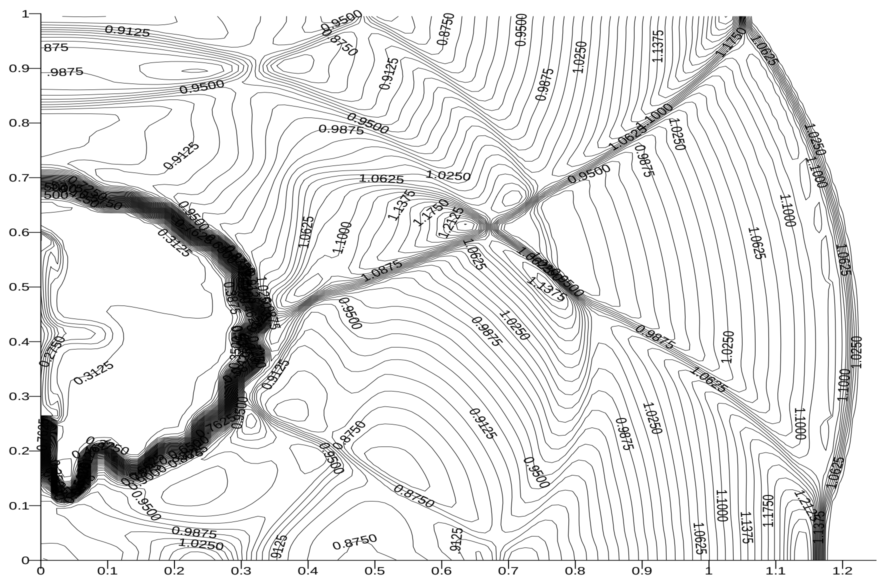

Figure 8: Density contours from 0.2 to 1.3 with increment 0.0125 computed on the rectangular mesh 100×140.

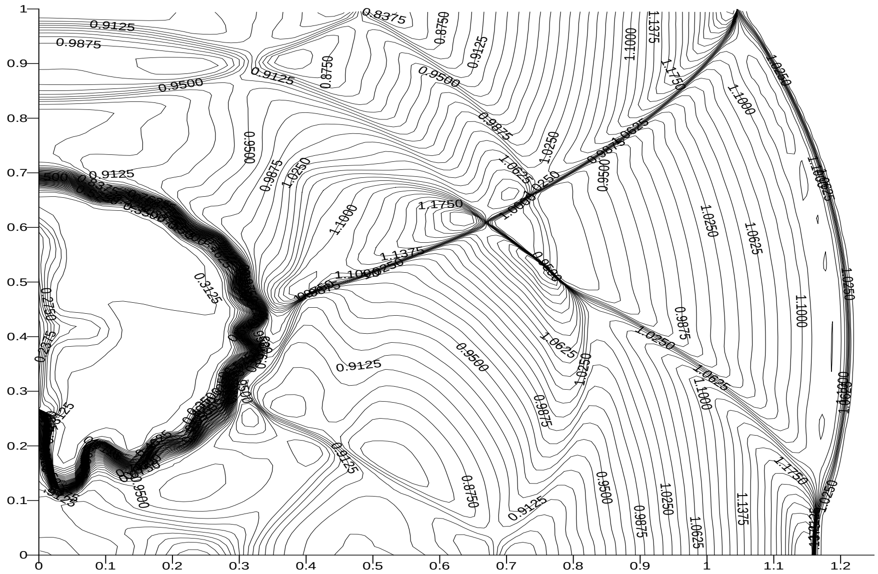

Figure 9: Density contours computed on the adapted mesh 100×140.

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