Examples of irregular behavior of harmonic mappings of singular domains

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The work is devoted to analysis of harmonic mappings of plane domains with geometrical singularities such as reentrant corners. Behavior of mappings near singularities is investigated. This problem is important for diagnosis of numerical phenomena arising from numerical minimization of mesh functionals based of harmonic maps theory[1]. The most common singular element of domain is rounded corner with small or equal to zero rounding radius. Analytical-numerical method has been developed for this case [2], which allows not only to solve boundary value problems for Laplace (or Poisson) equation but also to do theoretical investigation of solution asymptotics near the rounded corner. Apparatus of theory of functions of complex variable can be applyied to harmonic mappings, since the mapping function $f(z)$ may be introduced in the form $f(z) = h(z) + \overline{g}(z)$, i.e. it can be represented by two analytic functions, and local action of mapping is presented by analytic function $\omega(z) = g'(z)/h'(z)$ called complex dilatation of harmonic mapping. In the work in particular it has been shown that near corner with the rounding radius equal to zero the stability of the angle between coordinate line originating from corner origin and corner side is lost. The finite increment of this angle under infinitesimal region deformation or modification of the metric of harmonic mapping is possible. It is shown also that maximum of dilatation is attained on the boundary of domain which is consistent with numerical experience.

REFERENCES

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