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UNTANGLING AND OPTIMIZATION OF UNSTRUCTURED HEXAHEDRAL MESHES

by

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An effective method for untangling and optimization of hexahedral unstructured non-conformal meshes is presented. The method has been developed as a part of the Numeca new mesh generator (HEXPRESS™). It is able to untangle invalid (concave) cells and optimize valid but poorly shaped cells resulting from grid generation process. The goal is to obtain a mesh with all concave cells.

The HEXPRESS™ grid generation process is based on a top-down approach and includes several stages. First, an initial non-boundary-conforming mesh is created and refined based on geometry particularities. The cells that fall outside or intersecting the domain are trimmed out of the volume mesh. Next, the surface of the resulting staircase mesh is projected on the domain boundary and layers of buffer cells are inserted between the volume mesh and the corresponding surface mesh in order to obtain a nicely body-conforming mesh. Concave and poorly shaped cells may occur during the projection step and usually are concentrated near the boundary. The final stage is thus to apply the new untangling and optimization tools to transform these cells in convex ones and recover the mesh compatible with simulation. Optionally, layers of high aspect ratio cells for viscous flow computations may be inserted.

The grid generator is coupled with a new flow solver, which aggressively adapts the mesh based on local solution error estimation in order to obtain a mesh optimized for the particular flow solution. Subdividing a concave cell may result in new cells with

negative volumes. The latter are very undesirable for the robustness and accuracy of the flow solver. That is why the automatic optimization procedure is necessary.

The untangling algorithm is based on the successive analysis and correction of concave cells, via local untangling of sets of cells referencing a mesh node. The algorithm employs the discrete analogy of Jacobian positiveness of a tri-linear mapping of the unit cube onto a hexahedral cell [1]. This analogy decomposes the problem into untangling of the tetrahedra that constitute one hexahedral cell. This subdivision is however not unique and must be appropriately chosen to ensure the efficiency of the algorithm. As a tetrahedron is a simplicial element, its untangling can be performed simply by translating any of its vertices to a position that provides positive volume. The condition of the Jacobian positiveness represents a linear constraint. An efficient way to solve the untangling problem for the set of tetrahedra sharing one common vertex is obtained by applying the simplex algorithm [2].

The optimization algorithm is also based on the quality analysis and optimization of the sets of cells attached to a mesh node. Successive optimization of such sets containing valid cells results in an overall mesh quality improvement. The local optimization of hexahedral cells is performed via optimizing a variational functional for the set of tetrahedra representing the cell. It is important that identical decompositions of a cell by tetrahedra are used in both untangling and optimization. Indeed, in the scope of the method this consistency requirement is necessary since only tetrahedra with positive volumes are the only ones subject to effective optimization.

The efficiency of this new untangling-optimization approach will be demonstrated in the paper by various example meshes representative of complex geometries of interest for the industry.

[1] Sergey A. Ivanenko, Harmonic mappings, Handbook of grid generation. Ed. By Joe F. Thompson, Bharat K. Soni, Nigel P. Weatherill, CRC Press 1998, pp. 8-1 – 8-43.

[2] Numerical Recipes in FORTRAN. The Art of Scientific Computing. Second Edition, W. H. Press, S. A. Teukolsky, W. T. Vetterling, B. P. Flannery, Camridge University Press, 1992, pp. 423-435.