ON HOMEOMORPHIC PIECEWISE-SMOOTH MAPPINGS AND THEIR APPLICATION IN THE THEORY OF GRID GENERATION

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Piecewise-smooth mappings of homeomorphic domains in N - dimensional space are considered. Sufficient condition under which the mappings are homeomorphisms of corresponding domains is presented. The main result is the following.

Let Ω_0, Ω be homeomorphic bounded domains in R^N, and $\overline{\Omega}_0$, $\overline{\Omega}$ be their closures, and $\partial\Omega_0$, $\partial\Omega$ be their boundaries. Let a domain Ω_0 be subdivided to nonintersected subdomains Ω_i $(i = 1, \ldots, m)$ such, as

$$\overline{\Omega}_0 = \bigcup_{i=1}^{m} \overline{\Omega}_i.$$

Let $f : \overline{\Omega}_0 \to \overline{\Omega}$ be a continuous mapping which is C^1 - smooth on $\overline{\Omega}_i$. Restriction of the mapping f to $\overline{\Omega}_i$ is denoted by f_i $(i = 1, \ldots, m)$. For every poing $x \in \overline{\Omega}_0$ let

$$J(x) = \min_{\substack{\alpha_1 \geq 0, \ldots, \alpha_{i(x)} \geq 0 \\ \alpha_1 + \ldots + \alpha_{i(x)} = 1}} \det \left(\sum_{i \in i(x)} \alpha_i f_i'(x) \right),$$

where $i(x)$ is the set of all such indices i, that $x \in \overline{\Omega}_i$.

Theorem. *Let f maps the boundary $\partial\Omega_0$ homeomorphically onto the boundary $\partial\Omega$. If $J(x) > 0$ for every $x \in \overline{\Omega}_0$, then f is a homeomorphism from $\overline{\Omega}_0$ onto $\overline{\Omega}$.*