Marching noniterative generation of orthogonal grids

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A noniterative algorithm for the marching generation of orthogonal grids in exterior of contours in a plane and on surfaces is constructed. The stable asymptotic behaviour of grids is specified by a special law of change of the grid cell volume which relates the curvature of the current grid line to its behaviour at infinity. A necessary criterion for the possibility of fitting an orthogonal grid to a given contour with a prescribed asymptotic form is derived. A new version of the definition of a star-shaped (convex) contour relative to a prescribed asymptotic family of contours is proposed. To demonstrate the efficiency of the algorithm some examples of grids are presented.

Introduction

A new algorithm for generating orthogonal grids is constructed and investigated. This algorithm is characterized by the feature that it is noniterative and marching. Hence, it can generate the grid sequentially, or layer by layer, starting from a given initial contour.

The well-known principles and algorithms for constructing orthogonal grids, which are reviewed in Thompson, Warsi, and Mastin (1985), Godunov et al. (1979), are based on the solution of elliptic equations in a finite domain. This is necessarily done by iterative and nonmarching methods. The work in this area includes also the paper Tomamidis and Assanis (1991), in which a family of constructions of orthogonal grids, including the construction of a conformal mapping of the given domain into a rectangle, are successfully combined.

Marching methods (iterative and noniterative) are used, in particular, to generate quasi-orthogonal grids (Steger and Chaussee 1980; Steger 1991), see also the review by Chan (1999). Special mention should be made of the paper by Steger and Chaussee (1980), in which an original marching algorithm for

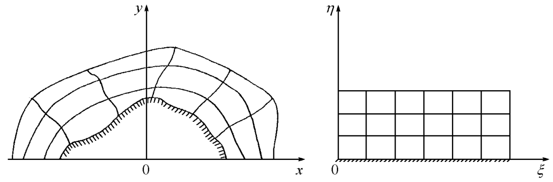

Figure 1:

constructing quasi-orthogonal grids was proposed for the first time. In the examples of Steger and Chaussee (1980), the degree of orthogonality of grid construction, the cosine of the angle between the directrices, varied between 0.04 and 0.2.

The present paper was directly motivated by the results obtained in Steger and Chaussee (1980), and is a positive answer to the possibility of constructing a marching algorithm for generating rigorously orthogonal grids. Individual cases of constructing an algorithm of this kind have been considered by Semenov (1990, 1991, 1195a, 1995b, 1996), and Lipchinsky and Semenov (1997)

Apart from its theoretical interest, the problem arose from a consideration of certain practical problems. The first was to construct a fast algorithm for fitting grids around the hulls of ships in order to calculate the flow past them. Orthogonal grids have to be constructed for each cross-section of the hull. Another problem was to construct grids for investigating laminar and turbulent boundary layers near smooth contours and bodies.

An algorithm for generating orthogonal grids in a plane is constructed in Section 1 below. It is generalized in Section 2 to contours which lie on surfaces, and to three-dimensional grids. The results obtained in generating various grids are given in Section 3.

1 Construction of plane orthogonal grids

In this section we arrive a numerical algorithm for constructing an orthogonal grid in exterior of a single simply connected contour given in the (x, y) plane, see Fig. 1. It is a marching algorithm, that is, it will construct the grids successively, beginning from the original contour. The condition is imposed that at some distance from the original contour the grid shall be a simple configuration, composed of concentric circles (Steger and Chaussee 1980), ellipses, parabolas, etc.

1.1 The correctness of marching generation of orthogonal grids

The orthogonal grids will be constructed on the basis of the system of the following equations (Steger and Chaussee 1980; Thompson, Warsi, and Mastin 1985):

$$x_\xi x_\eta + y_\xi y_\eta = 0, \qquad f_\beta = \frac{\partial f}{\partial \beta}, \quad f = \{x, y\}, \quad \beta = \{\xi, \eta\}; \tag{1.1}$$

$$x_\xi y_\eta - x_\eta y_\xi = V(\xi, \eta) = \frac{1}{J(\xi, \eta)}. \tag{1.2}$$

Equations (1.1)–(1.2) define a one-to-one mapping of (x, y) onto (ξ, η), see Fig. 1. The (x, y)-grid is the grid in the physical plane, the (ξ, η)-grid is a uniform orthogonal grid in the parametric plane. Equation (1.1) is the condition for an orthogonal grid, Eq. (1.2) gives the cell volumes of the grid $V(\xi, \eta)$, and $J(\xi, \eta)$ is the Jacobian of the transformation.

It was shown by Steger and Chaussee (1980) that after local linearization system (1.1)–(1.2) becomes hyperbolic, after which smooth quasi-orthogonal grids can be constructed by the marching method. In order to construct the rigorously orthogonal grids, it is necessary to investigate the system in its original form. To do so, we first rewrite it in the equivalent form:

$$x_\eta = -y_\xi \, G, \quad y_\eta = +x_\xi \, G, \quad G = \frac{V(\xi, \eta)}{x_\xi^2 + y_\xi^2}. \tag{1.3}$$

System (1.3) is of no definite type. Instead of investigating (1.3) directly, we consider its extended Courant–Lax system (Courant and Lax 1949). We must differentiate Eqs. (1.1)–(1.2) with respect to ξ and introduce the new variables $X = \partial x/\partial \xi = x_\xi$, $Y = \partial y/\partial \xi = y_\xi$, where ξ is the longitudinal variable and η is the marching variable. The extended system has the form

$$\frac{\partial \mathbf{W}}{\partial \eta} = A \frac{\partial \mathbf{W}}{\partial \xi} + \mathbf{d}, \quad \mathbf{W} = [X, Y]^{\mathrm{T}}, \quad \mathbf{d} = \frac{V_\xi}{R}[-Y, +X]^{\mathrm{T}}, \tag{1.4}$$

$$A = \frac{V}{R^2} \begin{bmatrix} 2XY & Y^2 - X^2 \\ Y^2 - X^2 & -2XY \end{bmatrix}, \quad R = X^2 + Y^2,$$

System (1.4) is hyperbolic (Rozhdestvenskii and Yanenko 1983; Kulikovskii, Pogorelov, and Semenov 2001) if $R \neq 0$. Its eigenvalues, in particular, are equal to $+V/R = +G$ and $-V/R = -G$, and system has a complete system of eigenvectors. Also, the Riemann invariants can be written out for this system. Thus, Eqs. (1.4) is a hyperbolic system, to solve which it is not only proper, but quite natural, to use a marching method. As a further investigation, we can assume that the marching method is only proper for a separate simply connected contour. The marching method gives satisfactory results only in the case of a multiply connected curve until the grids generated independently from each unconnected part of the curve meet.

As a result of the change to the extended system, the formulation of the problem must be stated as follows: it is required to find $V(\xi,\eta)$ such that the resulting hyperbolic system of Eqs. (1.4): (i) is non-degenerate, that is, the grid does not have self-intersections ($R \neq 0$), and (ii) does not have singularities as a result of the characteristics approaching or intersecting one another (Rozhdestvenskii and Yanenko 1983). These conditions are quite important, because they govern the practical application and efficiency of the marching method. It can be said that the function $V(\xi,\eta)$ must somehow depend on local curvature of the contour and provide the asymptotic grid as the distance along the marching variable η increases (Steger and Chaussee 1980).

1.2 Specifying the asymptotic form and selecting the function $V(\xi,\eta)$

To generate the orthogonal grid we must specify its asymptotic form in some way. It will be convenient to specify it in two forms. First, implicit, form is the form of the one-parameter family $F(x,y) = c$, where c is the parameter. In particular, the family $F(x,y) = x^2 + y^2 = c$, $c > 0$, is a family of concentric circles with center at the point $(0,0)$, and $F(x,y) = y = c$ is a family of lines parallel to the x-axis, etc. Second, explicit, expression has the form $x = x_c(\xi)$, $y = y_c(\xi)$, where x_c and y_c should satisfy the first form identically: $F(x_c(\xi)), y_c(\xi)) \equiv c$. In particular, for the family of circles $x = x_c(\xi) = \sqrt{c}\,\sin\xi$ and $y = y_c(\xi) = \sqrt{c}\,\cos\xi$.

The condition that the grids shall finally attain the asymptotic form given by F involves estimation how much they deviate from that form. The function $\frac{1}{2}F_\xi^2 \geq 0$ will be used as a criterion. In fact, the value of this criterion is small when the grid is close to the limiting form, because the value of F is nearly constant.

We must now specify the law by which the grid attains its asymptotic form. Thus, as the marching variable η increases, we must have

$$\tfrac{1}{2}F_\xi^2|_{\eta=\eta_2} \leq \tfrac{1}{2}F_\xi^2|_{\eta=\eta_1} \quad \text{if} \quad \eta_2 \geq \eta_2, \quad \text{or} \quad \frac{\partial}{\partial\eta}\left(\tfrac{1}{2}F_\xi^2\right) \leq 0. \tag{1.5}$$

In this case the grid will tend gradually to the given one in the limit. Condition (1.5) is unsuitable for use in specific applications, and so we will express it in a partial form as follows. Instead of Eq. (1.5), we take the rigorous equality

$$\frac{\partial}{\partial \eta}\left(\tfrac{1}{2}F_\xi^2\right) = -f\left(\tfrac{1}{2}F_\xi^2, V, x_\xi, y_\xi, \ldots\right), \quad f \geq 0. \tag{1.6}$$

We make the condition that the unknown non-negative function f satisfy

$$f(0, V, x_\xi, y_\xi, \ldots) \equiv 0.$$

The differential equation (1.6) with respect to the function V is nonlinear, and determined by the choice of f. The next step follows Semenov (1991, 1995b). We choose f so that: (i) the equation for V is linear, (ii) its solution can be written out in explicit form, and (iii) the function f has the simplest possible form. So that a marching algorithm can be constructed, the result obtained for V should not depend on derivatives with respect to η. A function f of this kind has been constructed (whether or not it is unique remains an open question at present), and has the form $f = gGF_\xi^2$, $G = V/(x_\xi^2 + y_\xi^2)$, see Semenov (1991, 1995a, 1995b), where g is a positive constant, governing the rate at which the asymptotic form is attained. With this choice of f, the relation (1.6) takes the form

$$\frac{\partial}{\partial \eta}\left(\tfrac{1}{2}F_\xi^2\right) = -gGF_\xi^2.$$

We then obtain

$$F_{\xi\eta} = -gGF_\xi \quad \text{or} \quad (F_x x_\eta + F_y y_\eta)_\xi = -gGF_\xi, \qquad F_x = \frac{\partial F}{\partial x}, \quad F_y = \frac{\partial F}{\partial y};$$

from which, using expressions for x_η and y_η from Eq. (1.3), we obtain an equation for G, and therefore also for V:

$$(-F_x y_\xi G + F_y x_\xi G)_\xi = -gGF_\xi, \tag{1.7}$$

or

$$(-F_x y_\xi + F_y x_\xi)_\xi\, G + (-F_x y_\xi + F_y x_\xi)\, G_\xi = -gGF_\xi.$$

Solving Eq. (1.7), we find an expression for G, and then for V:

$$V = Q(\eta)\frac{x_\xi^2 + y_\xi^2}{|K|}\exp\left(-g\int_0^\xi \frac{F_\xi}{K}\, d\xi\right), \quad K = -F_x y_\xi + F_y x_\xi, \tag{1.8}$$

where K is some characteristic of the curvature of the contour which does not contain derivatives with respect to η and for which a marching algorithm is therefore possible. The function $Q(\eta) > 0$ specifies the law of distribution of nodes along the η-axis. The integrand in Eq. (1.8) can be given the following geometric interpretation:

$$\frac{F_\xi}{K} = \frac{F_x x_\xi + F_y y_\xi}{-F_x y_\xi + F_y x_\xi} = \frac{1}{\tan\alpha},$$

where α is the angle between the vectors $[F_x, F_y]^{\mathrm{T}}$ and $[x_\xi, y_\xi]^{\mathrm{T}}$.

The properties of V follow at once from expression (1.8):

- $V > 0$;
- this choice of V ensures that there is no local degeneration of the grid.

In fact, it follows from Eq. (1.4) that

$$\frac{\partial R}{\partial \eta} = (x_{\xi\xi} y_\xi - y_{\xi\xi} x_\xi)\frac{2V}{R}, \quad R = x_\xi^2 + y_\xi^2,$$

and therefore, if $R > 0$ on the initial contour, this inequality holds in a finite, possibly small, interval of η.

1.3 A criterion for the applicability of the marching algorithm

A necessary criterion for the applicability of the marching algorithm follows from the form of $V(\xi, \eta)$. We will fix the initial contour from which grid generation starts in the form $x = x_0(\xi)$ and $y = y_0(\xi)$. We fix the asymptotic grid in the form $x = x_c(\xi)$ and $y = y_c(\xi)$. We then consider two functions: $K_0(\xi) = K(x_0(\xi), y_0(\xi))$ and $K_c(\xi) = K(x_c(\xi), y_c(\xi))$, where K is defined in (1.8).

Then a necessary condition for the problem of constructing a marching algorithm for an orthogonal grid about a given contour with given asymptotic form to be solvable is:

- the functions K_0 and K_c must be sign-constant with respect to ξ;
- the values of the functions K_0 and K_c must be finite, non-zero, and of the same sign.

We will call this the criterion for the given contour x_0, y_0 to be star-shaped relative to the asymptotic form x_c, y_c. This suggests a new definition of the convexity of a contour (Cassels 1959), or of a star-shaped body (Malyshev 1979). In particular, if the asymptotic grid is taken as a system of concentric circles with center at O, will include the well-known definition of a star-shaped contour relative to some point O as a contour for which any straight ray that begins at O intersects it only once outside O.

The proof of necessity of the criterion follows from the condition for the possibility of a continuous limiting transformation of the initial contour into an asymptotic contour as $\eta \to +\infty$, non-degeneracy of the grid that realizes that transformation being observed, and also $V \neq 0$ and $V \neq +\infty$. Otherwise, if K_0 and K_c change sign, the value of K of (1.8) must past through zero. The grid will then start to degenerate.

Thus, when the criterion is satisfied, it is *a priori* possible to construct a grid, at least in a small neighborhood of the given contour. As far as the practical global generation of a grid is concerned, it has been found by numerical calculation that the algorithm (and thus the criterion) can be used for problems of different kinds, see Section 3 below.

2 Grids on surfaces and in 3D space

We will give formulae for V in the case where the grids are constructed on surfaces and in three-dimensional space.

In the case where orthogonal grids are generated on a surface given by the metric tensor $g_{ij}(\xi, \eta)$, where $g_{ij} = g_{ji}$ and $i, j = 1, 2$; Eqs. (1.1)–(1.2) acquire the form

$$g_{11}x_\xi x_\eta + g_{12}x_\xi y_\eta + g_{21}x_\eta y_\xi + g_{22}y_\xi y_\eta = 0, \quad x_\xi y_\eta - x_\eta y_\xi = V(\xi, \eta).$$

System (1.3) acquires the form

$$x_\eta = -(g_{12}x_\xi + g_{22}y_\xi)\, G, \quad y_\eta = +(g_{11}x_\xi + g_{21}y_\xi)\, G, \tag{2.9}$$

$$G = \frac{V}{|x_\xi, y_\xi|^2}, \quad |x_\xi, y_\xi|^2 = g_{11}x_\xi^2 + 2g_{12}x_\xi y_\xi + g_{22}y_\xi^2.$$

Function V can be constructed similarly, as in Section 1:

$$V = Q(\eta)\frac{|x_\xi, y_\xi|^2}{|K|}\exp\left(-g\int_0^\xi \frac{F_\xi}{K}\, d\xi\right),$$

$$K = (-g_{12}x_\xi - g_{22}y_\xi)F_x + (g_{11}x_\xi + g_{12}y_\xi)F_y$$

The function Q and the constant g are the same as in Eq. (1.8). If the metric tensor $g_{11} = g_{22} = 1$, $g_{12} = g_{21} = 0$ is chosen, we obtain formula (1.8).

Now we will construct grids in three-dimensional space x, y, z. The initial data is a surface on which a grid (may be non-orthogonal) has already been constructed. Then, starting from that surface (or grid), we generate a new surface, the tangent plane to which is orthogonal to the normal of the previous surface. Equations similar to (1.1)–(1.2) now acquire the form

$$x_\xi x_\eta + y_\xi y_\eta + z_\xi z_\eta = 0, \quad x_\eta x_\theta + y_\eta y_\theta + z_\eta z_\theta = 0, \tag{2.10}$$

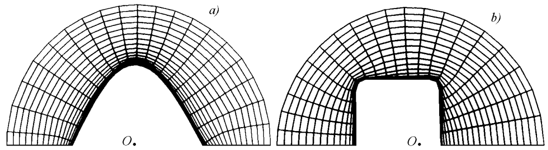

Figure 2:

$$V(\xi,\theta,\eta) = \det \begin{bmatrix} x_\xi & x_\eta & x_\theta \\ y_\xi & y_\eta & y_\theta \\ z_\xi & z_\eta & z_\theta \end{bmatrix}. \tag{2.11}$$

Equation (2.11) can be rewritten as

$$x_\eta \Delta_x + y_\eta \Delta_y + z_\eta \Delta_z = V(\xi,\theta,\eta),$$

$$\Delta_x = \det \begin{bmatrix} y_\theta & y_\xi \\ z_\theta & z_\xi \end{bmatrix}, \quad \Delta_y = \det \begin{bmatrix} x_\xi & x_\theta \\ z_\xi & z_\theta \end{bmatrix}, \quad \Delta_z = \det \begin{bmatrix} x_\theta & x_\xi \\ y_\theta & y_\xi \end{bmatrix}.$$

Equations (2.10) are two orthogonality conditions and (2.11) specifies the grid cell volumes. The variable η is a marching variable, and ξ and θ are longitudinal variables in the parametric space ξ, θ, η. Equations (1.3) now take the form

$$x_\eta = G\Delta_x, \quad y_\eta = G\Delta_y, \quad z_\eta = G\Delta_z, \quad G = \frac{V(\xi,\theta,\eta)}{\Delta_x^2 + \Delta_y^2 + \Delta_z^2}.$$

We can also construct $V(\xi,\theta,\eta)$:

$$V = Q(\eta)\frac{\Delta_x^2 + \Delta_y^2 + \Delta_z^2}{|K|} \exp\left(-g_1 \int_0^\xi \frac{F_\xi}{K}\, d\xi - g_2 \int_0^\theta \frac{F_\theta}{K}\, d\theta\right), \tag{2.12}$$

$$K = F_x\Delta_x + F_y\Delta_y + F_z\Delta_z, \qquad g_1 \geq 0, \quad g_2 \geq 0.$$

In Eq. (2.12) the constants g_1 and g_2 set the rate at which the asymptotic form is attained in the different directions, where as before, the asymptotic form is defined implicitly in terms of a one-parameter family of the form $F(x,y,z) = c$, and c is the parameter.

3 Construction of orthogonal grids

Numerical grid generation was performed using system (1.3) and (2.9), which was approximated by finite differences on a six-point pattern, two-layers in η

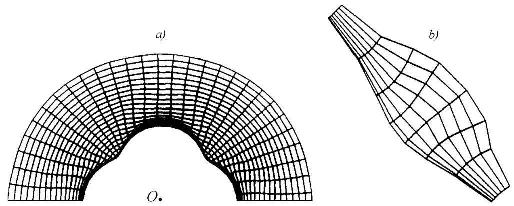

Figure 3:

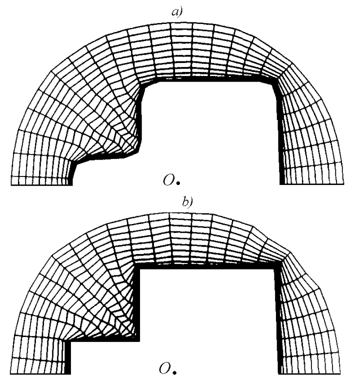

Figure 4:

and three-points in ξ,

$$\frac{\hat{x}_k - x_k}{\Delta\eta} = -(\hat{y}_{k+1} - \hat{y}_{k-1} + y_{k+1} - y_{k-1})\frac{G_k}{4\Delta\xi}, \tag{3.13}$$

$$\frac{\hat{y}_k - y_k}{\Delta\eta} = +(\hat{x}_{k+1} - \hat{x}_{k-1} + x_{k+1} - x_{k-1})\frac{G_k}{4\Delta\xi}, \tag{3.14}$$

$$G_k = \frac{4V_k\Delta\xi^2}{(x_{k+1} - x_{k-1})^2 + (y_{k+1} - y_{k-1})^2}.$$

The subscript is the index of the grid node in the ξ-direction, the symbol $\hat{f}$ denotes f for the upper layer in η, and the absence of this sign indicates the known lower layer in η, and $\Delta\xi$ and $\Delta\eta$ are the steps of the difference grid along the ξ- and η-axes, respectively. It is easy to see that the orthogonality condition (1.1) is satisfied identically with this approximation and, hence, for points with coordinates $\tilde{x}_k = \frac{1}{2}(\hat{x}_k + x_k)$, $\tilde{y}_k = \frac{1}{2}(\hat{y}_k + y_k)$ it has th second-order accuracy. After the coordinates $(\hat{x}_k, \hat{y}_k)$ were obtained, the final grid was constructed over nodes $(\tilde{x}_k, \tilde{y}_k)$.

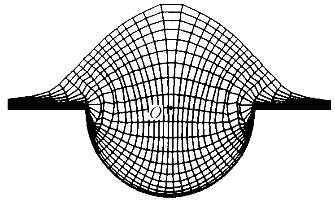

Figure 5:

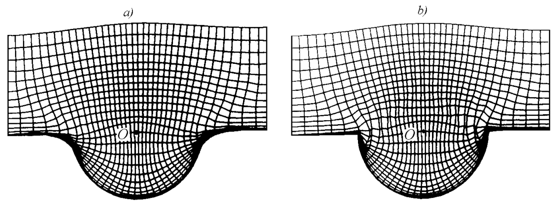

Figure 6:

The resulting system of equations is linear and can be solved by matrix sweep method (Godunov and Ryabenkii 1987; Samarskii and Nikolaev 1989) or by reducing it to the inversion of a five-diagonal matrix by the Gauss elimination, choosing the principal element in the row, the so-called scalar monotone sweep method (Samarskii and Nikolaev 1989).

Note, that in calculations it is better to use system (1.3) than (1.4), which was only used for analysis. The grids were constructed for $\Delta\xi = \Delta\eta = 1$. The initial contours had a characteristic length of unity. The values of the constant g were chosen in the range 0.3–0.66. The function Q was also taken to be constant: $Q = 10^{-2}$–10^{-4}. The concentration of the grid nodes along η was controlled by specifying a rule for choosing those grid layer indices which are to remain in the final result. We chose this to be an arithmetic progression. In each stage of grid construction, a check had to be made to see whether there were any self-intersections. If there were, grid generation was interrupted. The quality of the grid was also monitored by means of the ratio of the maximum to the minimum cell volume along the grid layer.

Figure 2a shows an example of a grid constructed around an original contour in the shape of a symmetric sinusoidal segment. The asymptotic grid was chosen as concentric circles with center at O. Symmetry conditions for x: $x_\xi = 0$ and the condition $y = 0$, were specified for closure on the x-axis when generating the grid according to (3.13)–(3.14). Figure 2b shows the grid

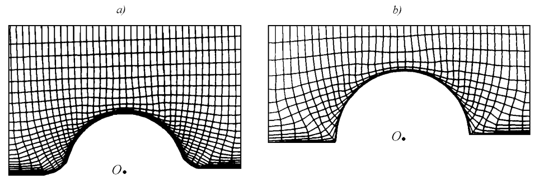

Figure 7:

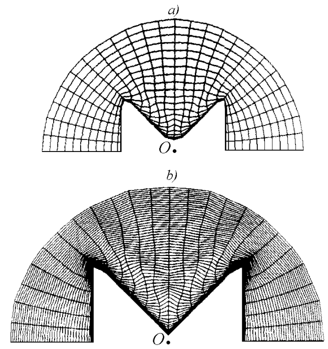

Figure 8:

for a smoothed rectangle, the center of the asymptotic grid here having been shifted with respect to the axis of symmetry of the initial contour. Figure 3a shows the general form of the grid around a semi-circle with imposed sinusoidal perturbation, Fig. 3b shows a fragment of this grid. Figure 4 shows the grids around rectangles with a cut. The asymptotic form consisted of concentric circles, with $F = x^2 + y^2$ for all the grids in Figs. 2–4.

Figure 5 shows the grid which fits a contour lying mainly along the x-axis, with a semi-circular shaped concavity with center at the point O. A "witches of Agnesi" asymptotic form was used:

$$F = (0.5 + x^2)y, \quad y_c = \frac{c}{0.5 + \xi^2}, \quad x_c = \xi.$$

Figure 6 illustrates the relation between the grids for the smoothed contour (a) and the nonsmoothed contour (b) of the type shown in Fig. 5. But now we used a combined asymptotic form: at the initial stage, an asymptotic form as in Fig. 5, until the grid left O, when we used straight lines parallel to the x-axis: $F = y$, $x_c = 0$, $y_c = \xi$. This example demonstrates the flexibility of the method, in which the asymptotic form F can be changed while the grid is being constructed.

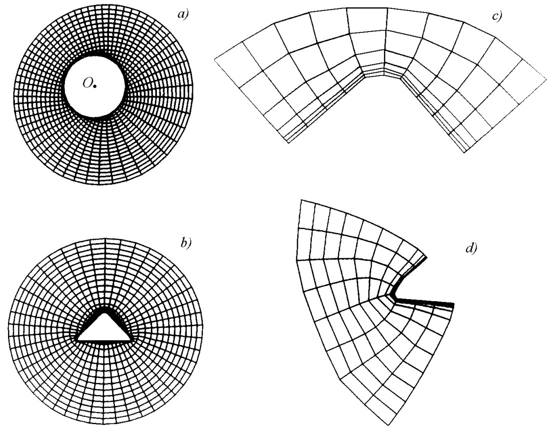

Figure 9:

Figure 7a–b illustrates the behaviour of the grid around a contour with a convexity in the form of a semi-circle. Here $F = y$.

Note, that this grid generation method can be used for any initial contour, even an nonsmooth one, as in Fig. 4b and 7b. In the latter case, the quality obtained might be low, even though the orthogonality condition is, formally, satisfied identically at the discrete level. The reason for this is the large error of approximation of the initial orthogonality condition (1.1), owing to the nonsmooth behaviour of the contours. In this case one must either smooth the initial contour, or concentrate the grid near large gradients in the ξ direction in order to satisfy the approximation conditions.

Figure 8a–b gives an example of a grid constructed around quite a "difficult" initial convex-concave contour: the less detailed grid is at the top (a), the more detailed grid is below (b). Here $F = x^2 + y^2$.

Figure 9a shows a closed grid constructed around a circle when the asymptotic form also consists of circles, but with their centers shifted relative to the center of the original contour. Figure 9b–d shows the grid and its fragments around a triangle. For closed contours, system (1.3) was solved by cycle sweep method (Samarskii and Nikolaev 1989). For a smooth join, the boundary conditions at the place where the first and last nodes of the grid lines met were given: $x_{\xi\xi\xi\eta} = y_{\xi\xi\xi\eta} = 0$.

Figure 10 shows examples of grids constructed on surfaces (a hyperboloid, an ellipsoid, and a cylinder) for initial contours of the type shown in Fig. 7.

We note, in conclusion, that the method can be used either independently, or in combination with other iterative methods of grid generation (Thompson, Warsi, and Mastin 1985; Godunov et al. 1979; Tomamidis and Assanis 1991;

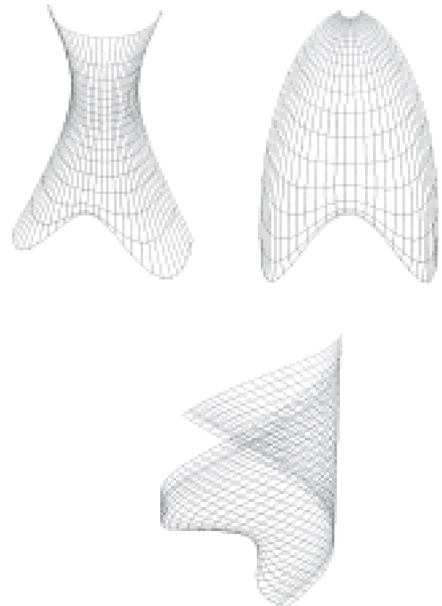

Figure 10:

Ivanenko 1999). In particular, it can be used to generate the initial distribution of grid nodes. An unsuccessful arrangement of grid nodes on the boundary could reduce the degree of orthogonality of the resulting grid, which only increases after the number of nodes is increased (Steger and Chaussee 1980). Otherwise, the method could be used to generate grids in an infinite band (channel), taking one of the sides of the channel as the initial contour, with the shape of the other side given (or approximated) in the form of a one-parameter family F. Then, by varying the rate of convergence g, it is possible to achieve a prescribed degree of orthogonality of the grid on the other side of the channel, see Fig. 11. And, finally, the formulae obtained for V, see Eq. (1.8) and (2.12), can be used directly for the methods by Steger and Chaussee (1980), Steger (1991), Chan (1999).

We have constructed and investigated a marching noniterative algorithm for the numerical generation of rigorously orthogonal grids around simply connected curves in a plane and on surfaces. The method has been shown to be effective and reliable for initial conditions of different kinds and can be recommended for practical use.

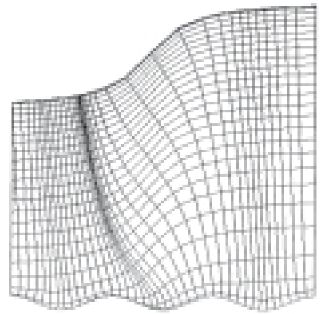

Figure 11:

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