

DYNAMICS OF A TETRAHEDRAL CONSTELLATION OF SATELLITES-GYROSTATS

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Abstract. *We study orbital motion of a gyrostat with the mass distribution that possesses a tetrahedron symmetry group. We suppose that the body is small compared to the distance between its center of mass and the attracting center, and that gyrostatic moments of rotors are also small. The equations of motion are written in the so-called satellite approximation and their properties are studied.*

1 INTRODUCTION

Problem of motion of symmetric rigid bodies is analogous to the respective problem for regular polyhedra and arises to [1 - 3], where the potential is written as power series with terms depending on the invariants of the appropriate symmetry group. It allows one to describe a complete set of steady motions and to express the sufficient conditions of their stability in terms of these invariants. The sufficient conditions of stability are obtained supposing that the tetrahedron is composed by four equal masses connected by six massless rods. It turned out that the dimension of the element (vertex, edge or face) that points to the attracting center in the steady motion is equal to the degree of instability of this motion.

Dynamics of tetrahedral bodies attracts now a lot of attention due to the NASA program in formation flying that presumes the launch of satellite's constellation in the shape of a regular tetrahedron [4 - 7]. The shape of the constellation is supposed to be maintained by active control tools. In [8 - 12] we began to study the possibility to replace the active control tools by passive elements like tethers. In these publications we identify the constraints that can be implemented by tethers.

2 POSING THE PROBLEM

Consider motion of a rigid body with rotors \mathcal{G} in the Newtonian gravitational field with the center at point N. The mass distribution of \mathcal{G} possesses the tetrahedron group of symmetry. Let O be the center of mass of the body (CM), and A, B, C and S its vertices.

We use three reference frames:

- $Ox_1x_2x_3$ frame connected to the body (CF);
- orbital frame $OX_1X_2X_3$ (OF): its OX_3 -axis is parallel to the vector NO and OX_2 -axis is directed along the vector product of $\mathbf{NO} \times \mathbf{v}$ (here $\mathbf{v} = (v_1, v_2, v_3)^T$ is the velocity of CM);
- an absolute frame $NX_\alpha X_\beta X_\gamma$ (AF).

The motion of the body can be described by the Euler – Lagrange – Poincaré equations (cf. [13])

$$\frac{d}{dt} \frac{\partial L}{\partial \boldsymbol{\omega}} = \frac{\partial L}{\partial \boldsymbol{\omega}} \times \boldsymbol{\omega} + \frac{\partial L}{\partial \mathbf{v}} \times \mathbf{v} + \frac{\partial L}{\partial \mathbf{r}} \times \mathbf{r} \quad (1)$$

$$\frac{d}{dt} \frac{\partial L}{\partial \mathbf{v}} = \frac{\partial L}{\partial \mathbf{v}} \times \boldsymbol{\omega} + \frac{\partial L}{\partial \mathbf{r}} \quad (2)$$

$$L = L(\boldsymbol{\omega}, \mathbf{v}, \mathbf{r}) \quad (3)$$

These equations describe variations of the kinetic moment of the body and its momentum respectively. Here $\boldsymbol{\omega} = (\omega_1, \omega_2, \omega_3)^T$ is the vector of the angular velocity of the body, $\mathbf{k} = (K_1, K_2, K_3)^T$ is the vector of gyrostatic momentum of the rotors, $\mathbf{r} = (r_1, r_2, r_3)^T$ is the position vector of O with respect to the center of attraction N. All the vectors are given by their projections onto the axes of $Ox_1x_2x_3$ frame. The Lagrange function (3) is

$$L = T - U, \quad T = \frac{1}{2} \left(m' \mathbf{v}^2 + (\mathbf{I} \boldsymbol{\omega}, \boldsymbol{\omega}) \right) + (\mathbf{k}, \boldsymbol{\omega}), \quad U = U(\mathbf{r}) \quad (4)$$

Here T is the kinetic energy of the system (we omit an additive constant), U is its potential energy, m' is the mass of the body, $\mathbf{I} = \text{diag} (I_1, I_2, I_3)^T$ is its principal central tensor of inertia.

Equations (1) – (3) should be completed by the Euler kinematic equations

$$\dot{\mathbf{r}} = \mathbf{v} + \mathbf{r} \times \boldsymbol{\omega} \quad (5)$$

and by the Poisson kinematic equations

$$\dot{\boldsymbol{\alpha}} = \boldsymbol{\alpha} \times \boldsymbol{\omega}, \quad \dot{\boldsymbol{\beta}} = \boldsymbol{\beta} \times \boldsymbol{\omega}, \quad \dot{\boldsymbol{\gamma}} = \boldsymbol{\gamma} \times \boldsymbol{\omega} \quad (6)$$

Here

$$\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \alpha_3)^T, \quad \boldsymbol{\beta} = (\beta_1, \beta_2, \beta_3)^T, \quad \boldsymbol{\gamma} = (\gamma_1, \gamma_2, \gamma_3)^T$$

are the AF unit vectors.

Suppose that the gyrostatic moment \mathbf{K} is constant. Then the Lagrange function does not depend on time explicitly, and the equations of motion admit Painlevé - Jacobi first integral

$$\mathcal{J}_0 = \left(\frac{\partial L}{\partial \mathbf{v}}, \mathbf{v} \right) + \left(\frac{\partial L}{\partial \boldsymbol{\omega}}, \boldsymbol{\omega} \right) - L \quad (7)$$

Moreover, by virtue of Eqs. (1), (2) and (5) the equality

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \boldsymbol{\omega}} + \mathbf{r} \times \frac{\partial L}{\partial \mathbf{v}} \right) = \left(\frac{\partial L}{\partial \boldsymbol{\omega}} + \mathbf{r} \times \frac{\partial L}{\partial \mathbf{v}} \right) \times \boldsymbol{\omega} \quad (8)$$

holds true. Equality (8) means that the full kinetic momentum of the system in the AF is constant. So the projection of the vector of the full kinetic momentum onto an arbitrary direction fixed in the absolute space is constant too. Hence there exists a family of first integrals generated by three independent ones. Suppose that these independent integrals are

$$\mathcal{J}_i = \left(\frac{\partial L}{\partial \boldsymbol{\omega}} + \mathbf{r} \times \frac{\partial L}{\partial \mathbf{v}}, \mathbf{i} \right), \quad \mathbf{i} \in \{\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}\} \quad (9)$$

Let us choose the axes of AF so that

$$\mathcal{J}_{\boldsymbol{\alpha}} = 0, \quad \mathcal{J}_{\boldsymbol{\beta}} = p_{\psi}, \quad \mathcal{J}_{\boldsymbol{\gamma}} = 0 \quad (10)$$

Apart from (10), the system admits six geometric integrals

$$\mathcal{J}_{ii} = (\mathbf{i}, \mathbf{i}) - 1 = 0, \quad \mathbf{i} \in \{\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}\} \quad (11)$$

$$\mathcal{J}_{ij} = (\mathbf{i}, \mathbf{j}) = 0, \quad \mathbf{i}, \mathbf{j} \in \{\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}\}, \quad \mathbf{i} \neq \mathbf{j} \quad (12)$$

These integrals together with integrals (9) allow one to reduce system (1) – (3) to the system of Lagrange equations with five degrees of freedom. Since, according to the assumption, the Lagrange function does not depend on time, one need to find four more independent integrals to complete integration of the equations of motion.

3 POTENTIAL OF NEWTONIAN GRAVITATIONAL ATTRACTION AND ITS APPROXIMATIONS

The body mass distribution possesses a discrete symmetry that allow one to describe the principal classes of stationary configurations without an additional hypothesis of smallness of the body with respect to its distance from the attracting center (see [14], pp. 298 – 303). In this

problem the potential turns out to be a function of the polynomial invariants of the appropriate discrete symmetry group, in particular, the tetrahedron symmetry, namely (cf. [3])

$$\mathcal{U} = \mathcal{U}(p_2, p_3, p_4) \quad (13)$$

$$p_2 = r^2 = r_1^2 + r_2^2 + r_3^2, \quad p_3 = r_1 r_2 r_3, \quad p_4 = r_1^2 r_2^2 + r_2^2 r_3^2 + r_3^2 r_1^2 \quad (14)$$

Suppose that the body in question is a regular tetrahedron with equal masses m located in its vertices A, B, C , and S , so the total mass of the body is $m' = 4m$. The expression for the Newtonian potential can be written as

$$U = -fMm \sum \rho^{-1} \quad (15)$$

where f is the gravity constant, M is the mass of the attraction center, ρ is the distance between the vertex and the attraction center and the summation is done for all vertices of the tetrahedron. We suppose that the coordinates of the vertices in the CF are

$$\begin{aligned} \overrightarrow{OA} &= \frac{1}{\sqrt{3}}(a, a, -a)^T, & \overrightarrow{OB} &= \frac{1}{\sqrt{3}}(a, -a, a)^T \\ \overrightarrow{OC} &= \frac{1}{\sqrt{3}}(-a, a, a)^T, & \overrightarrow{OS} &= \frac{1}{\sqrt{3}}(-a, -a, -a)^T \end{aligned}$$

Obviously, the edges of this tetrahedron are the skew diagonals of a cube with the principal diagonal equal to $2a$.

Usually the analysis of such problems is done supposing that the characteristic dimension of the body is much smaller than the radius of its orbit (so-called "satellite approximation" [14]). Using the parameter $\varepsilon = a/r$ and developing the respective series of the gravitational potential, one obtains

$$\begin{aligned} U &= U_0 + U_1 + U_2 + U_3 + U_4 + \dots \quad (16) \\ U_0 &= -\frac{4fMm}{r}, \quad U_1 = 0, \quad U_2 = 0, \quad U_3 = -\varepsilon^3 \frac{20\sqrt{3}fMmp_3}{3r^4} \\ U_4 &= \varepsilon^4 \frac{14\sqrt{3}fMm(r^4 - 5p_4)}{9r^5} \end{aligned}$$

On the other hand, the tetrahedron vertices are equally distant from its center of mass, so one can use the parameters (cf [17])

$$\varepsilon_i = \frac{2(\overrightarrow{OI}, \mathbf{r})}{r_a^2}, \quad i \in \{A, B, C, S\} \quad (17)$$

$$r_a = (r_1^2 + r_2^2 + r_3^2 + a^2)^{1/2} = (r^2 + a^2)^{1/2}$$

to write the gravitation potential as

$$\begin{aligned} U &= U_0 + U_1 + U_2 + U_3 + U_4 + \dots \quad (18) \\ U_0 &= -\frac{4fMm}{r_a}, \quad U_1 = 0, \quad U_2 = -\frac{2fMma^2r^2}{r_a^5} \\ U_3 &= -\frac{20fMma^3p_3}{3r_a^7} \end{aligned}$$

$$U_4 = -\frac{35fMma^4(r^4 + 4p_4)}{18r_a^9}$$

Unlike (16), series (18) is valid for any relation between the tetrahedron dimension and the orbit radius. Moreover, it does not have singularity at $r = 0$. Expressions for U_0 and U_2 depend only on r^2 , so they do not contribute to the gravitational torque with respect to O . The first term of (18) that determine this torque is U_3 .

If the potential is represented as series in a small parameter such as (16):

$$\mathcal{U} = \mathcal{U}_0 + \mathcal{U}_1 + \mathcal{U}_2 + \mathcal{U}_3 + \dots \quad (19)$$

then its terms depend only on variables (r_1, r_2, r_3) through the polynomial invariants of the tetrahedron group p_2, p_3, p_4 .

4 EQUATIONS OF MOTION FOR A SMALL RIGID BODY

Consider now the case when the body is small with respect to the orbit dimension. In this case the gravitational potential can be approximated by formulae (16). Using the small parameter ε , and taking into account (19), one can re-write the initial equations of motion (1), (2) as

$$m'\dot{\mathbf{v}} = m'\mathbf{v} \times \boldsymbol{\omega} - \frac{\partial U_0}{\partial \mathbf{r}} - \varepsilon^3 \frac{\partial U_3}{\partial \mathbf{r}} + \dots \quad (20)$$

$$\begin{aligned} \frac{d}{dt}(\mathbf{I}\boldsymbol{\omega} + \mathbf{k}) &= (\mathbf{I}\boldsymbol{\omega} + \mathbf{k}) \times \boldsymbol{\omega} \\ &+ \mathbf{r} \times \left(\frac{\partial \mathcal{U}_0}{\partial \mathbf{r}} + \varepsilon^3 \left(\frac{\partial \mathcal{U}_3}{\partial r} \frac{\partial r}{\partial \mathbf{r}} + \frac{\partial \mathcal{U}_3}{\partial p_3} \frac{\partial p_3}{\partial \mathbf{r}} \right) + \dots \right) = \\ &= (\mathbf{I}\boldsymbol{\omega} + \mathbf{K}) \times \boldsymbol{\omega} + \varepsilon \mathbf{Q}, \quad \mathbf{Q} = \mathbf{r} \times \frac{\partial \mathcal{U}_3}{\partial p_3} \frac{\partial p_3}{\partial \mathbf{r}} + \dots \end{aligned} \quad (21)$$

One should complete the system by adding the respective kinematic equations of Euler and Poisson. Considering in (20) only the first approximation in ε , one obtains

$$\dot{\mathbf{v}} = \mathbf{v} \times \boldsymbol{\omega} - \frac{\partial U'_0}{\partial \mathbf{r}}, \quad U_0 = m'U'_0 \quad (22)$$

These equations together with (5) describe the motion of the center of mass of the body O . They separate from the rest of the system that correspond to the body's attitude motion. Equations (22), (5) can be integrated resulting in the description of the motion of CM along a Keplerian orbit. We study below only elliptic orbits.

Substituting $\varepsilon = 0$ to (21) and taking into account that the inertia tensor is spherically symmetric and the gyrostatic moment is constant, one gets

$$I\dot{\boldsymbol{\omega}} = \mathbf{k} \times \boldsymbol{\omega} \quad (23)$$

This is a system of three linear differential equations with constant coefficients which can be easily solved. So, in this approximation, the equations of motion of CM and the equations of attitude motion split and can be solved.

To obtain more accurate description of attitude motion of the body, one has to keep the linear with respect to ε terms in the equations of motion. In this approximation the orbital motion affects the body's attitude dynamics:

$$I\dot{\boldsymbol{\omega}} = \mathbf{k} \times \boldsymbol{\omega} + \varepsilon \mathbf{Q}, \quad \mathbf{Q} = \mathbf{r} \times \frac{\partial \mathcal{U}_3}{\partial p_3} \frac{\partial p_3}{\partial \mathbf{r}} \quad (24)$$

However, this influence is small since ε is a small parameter.

5 RELATIVE EQUILIBRIA IN THE "SATELLITE APPROXIMATION"

Consider now the case when the satellite's CM moves along a circular Keplerian orbit, and its gyrostatic moment is constant. Let's determine the equilibrium orientations of the tetrahedron with respect to the orbital reference frame $\mathbf{O}X_1X_2X_3$ that in this case rotates about the normal to the orbit plane with constant (orbital) angular velocity Ω_o . These relative equilibrium configurations can be found by application of the Routh theorem [16,17] to the critical points of the augmented potential

$$U_{aug} = -\frac{1}{2}I\Omega_o^2 + (\mathbf{k}, \boldsymbol{\beta})\Omega_o + \varkappa\gamma_1\gamma_2\gamma_3, \quad \varkappa < 0 \quad (25)$$

In the expression for the gravitational potential we consider the terms up to the third order, U_3 . The first term in (25) is constant and does not affect the existence and stability of the equilibrium configurations. They are found as critical points of the Routh function

$$W = U_{aug} + \frac{1}{2}\lambda(\boldsymbol{\beta}, \boldsymbol{\beta}) + \mu(\boldsymbol{\beta}, \boldsymbol{\gamma}) + \frac{1}{2}\nu(\boldsymbol{\gamma}, \boldsymbol{\gamma}) \quad (26)$$

and can be determined solving the system

$$\frac{\partial W}{\partial \boldsymbol{\beta}} = -\Omega_o\mathbf{k} + \lambda\boldsymbol{\beta} + \mu\boldsymbol{\gamma} = 0 \quad (27)$$

$$\frac{\partial W}{\partial \boldsymbol{\gamma}_1} = \varkappa\gamma_2\gamma_3 + \mu\beta_1 + \nu\gamma_1 = 0, \quad (1\ 2\ 3) \quad (28)$$

together with the geometric integrals

$$(\boldsymbol{\beta}, \boldsymbol{\beta}) = 1, \quad (\boldsymbol{\beta}, \boldsymbol{\gamma}) = 0, \quad (\boldsymbol{\gamma}, \boldsymbol{\gamma}) = 1 \quad (29)$$

Equations (27) – (29) are non-linear. Yet they conserve the symmetries of the initial system and so they admit the solutions when the body points to the attraction center \mathbf{N} by one of the following elements:

- I. one of its vertices;
- II. one of its edges,
- III. the center of one of its faces.

and the vector of the gyrostatic moment is orthogonal to the orbit plane $\mathbf{k} = k\boldsymbol{\beta}$.

These stationary solutions form three one-parameter classes of configurations that can be obtained by rotation of the body about the local vertical that passes by \mathbf{O} . These classes are described as

$$I. \quad \boldsymbol{\beta} = \frac{1}{\sqrt{6}} \left(-2 \sin \varphi, -\sqrt{3} \cos \varphi + \sin \varphi, \sqrt{3} \cos \varphi + \sin \varphi \right)^T, \quad \boldsymbol{\gamma} = \frac{1}{\sqrt{3}} (1, 1, 1)^T \quad (30)$$

$$II. \quad \boldsymbol{\beta} = (\cos \varphi, \sin \varphi, 0)^T, \quad \boldsymbol{\gamma} = (0, 0, 1)^T \quad (31)$$

$$III. \quad \boldsymbol{\beta} = \frac{1}{\sqrt{6}} \left(-2 \sin \varphi, -\sqrt{3} \cos \varphi + \sin \varphi, \sqrt{3} \cos \varphi + \sin \varphi \right)^T, \quad \boldsymbol{\gamma} = -\frac{1}{\sqrt{3}} (1, 1, 1)^T \quad (32)$$

Here

$$\lambda = \Omega_0(\mathbf{k}, \boldsymbol{\beta}), \quad \mu = \Omega_0(\mathbf{k}, \boldsymbol{\gamma}) = -\varkappa \sum_{(123)} \beta_1 \gamma_2 \gamma_3, \quad \nu = -3\varkappa \gamma_1 \gamma_2 \gamma_3 \quad (33)$$

In accordance to the Routh theory, the sufficient conditions of stability can be found analyzing the restriction of the second variation of the Routh function $\delta_r^2 W$ onto the linear manifold defined by geometric integrals (29). The number of negative eigenvalues of the resulting quadratic form matrix is referred to as degree of instability χ . If $\chi = 0$, the system is secularly stable, if χ is odd, the motion is unstable, and with even values of χ the stability in the first approximation is possible.

For the problem in question, the second variation of the Routh function has the form

$$2\delta^2 W = \lambda(\delta\boldsymbol{\beta}, \delta\boldsymbol{\beta}) + 2\nu(\delta\boldsymbol{\beta}, \delta\boldsymbol{\gamma}) + \mu(\delta\boldsymbol{\gamma}, \delta\boldsymbol{\gamma}) + 2\varkappa \sum_{(123)} \gamma_1 \delta\gamma_2 \delta\gamma_3 \quad (34)$$

and the linear manifold is

$$(\boldsymbol{\beta}, \delta\boldsymbol{\beta}) = 0, \quad (\boldsymbol{\beta}, \delta\boldsymbol{\gamma}) + (\boldsymbol{\gamma}, \delta\boldsymbol{\beta}) = 0, \quad (\boldsymbol{\gamma}, \delta\boldsymbol{\gamma}) = 0 \quad (35)$$

To study the restriction of (34) onto (35), one could use the Mann criterion [18, 19] to study the properties of the respective augmented matrix [20]. Here it is more convenient to use the augmented matrix to study eigenvalues of the matrix of the quadratic form $\delta_r^2 W$ that can be found from the equation

$$P_3(\sigma) = 0 \quad (36)$$

where

$$P_3(\sigma) = \det \begin{pmatrix} 0 & 0 & 0 & \beta_1 & \beta_2 & \beta_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & \gamma_1 & \gamma_2 & \gamma_3 & \beta_1 & \beta_2 & \beta_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & \gamma_1 & \gamma_2 & \gamma_3 \\ \beta_1 & \gamma_1 & 0 & \lambda - \sigma & 0 & 0 & \mu & 0 & 0 \\ \beta_2 & \gamma_2 & 0 & 0 & \lambda - \sigma & 0 & 0 & \mu & 0 \\ \beta_3 & \gamma_3 & 0 & 0 & 0 & \lambda - \sigma & 0 & 0 & \mu \\ 0 & \beta_1 & \gamma_1 & \mu & 0 & 0 & \nu - \sigma & 0 & 0 \\ 0 & \beta_2 & \gamma_2 & 0 & \mu & 0 & 0 & \nu - \sigma & 0 \\ 0 & \beta_3 & \gamma_3 & 0 & 0 & \mu & 0 & 0 & \nu - \sigma \end{pmatrix}$$

Solutions of (36) for the configurations of Class I are

$$\sigma_1 = -\frac{2\sqrt{3}}{3}\varkappa, \quad \sigma_2 = \Omega_0 k, \quad \sigma_3 = \frac{\sqrt{3}\Omega_0 k - 2\varkappa}{2\sqrt{3}} \quad (37)$$

Parameter \varkappa is negative, so the solution σ_1 is positive. If $\Omega_0 k > 0$, the motion is secularly stable, if $2\varkappa/\sqrt{3} < \Omega_0 k < 0$, the degree of instability is 1 and the motion is unstable, and for $\Omega_0 k < 2\varkappa/\sqrt{3}$, the degree of instability is 2.

For an equilibrium of Class II $\sigma_1 = \Omega_0 k$, and solutions σ_2 and σ_3 satisfy the equation

$$2\sigma^2 + (\varkappa \sin(2\varphi) - \Omega_0 k)\sigma - \varkappa(\varkappa + \Omega_0 k \sin(2\varphi)) = 0 \quad (38)$$

The respective configuration is secularly stable if $\Omega_0 k > 0$, and both σ_2 and σ_3 are positive, that is, if

$$\varkappa \sin(2\varphi) - \Omega_0 k < 0, \quad \varkappa + \Omega_0 k \sin(2\varphi) > 0 \quad (39)$$

Equilibria of Class III correspond to the following solutions of (36):

$$\sigma_1 = \frac{2\sqrt{3}}{3}\varkappa, \quad \sigma_2 = \Omega_0 k, \quad \sigma_3 = \frac{\sqrt{3}\Omega_0 k + 2\varkappa}{2\sqrt{3}} \quad (40)$$

Parameter \varkappa is negative, so $\sigma_1 < 0$, and the respective configurations cannot be secularly stable. If $\Omega_0 k < 0$, the degree of instability is 3 and the configuration is unstable. If $0 < \Omega_0 k < -2\varkappa/\sqrt{3}$, the degree of instability is 2, and it is necessary to verify the necessary conditions of stability using the equations in variations. If $-2\varkappa/\sqrt{3} < \Omega_0 k$, the degree of instability is 1, and the respective motion of Class III is unstable.

Relations

$$\delta\boldsymbol{\beta} = \boldsymbol{\beta} \times \delta\boldsymbol{\theta}, \quad \delta\boldsymbol{\gamma} = \boldsymbol{\gamma} \times \delta\boldsymbol{\theta} \quad (41)$$

permit one to represent the linear manifold in the parametric form. Here $\delta\boldsymbol{\theta} = (\delta\theta_1, \delta\theta_2, \delta\theta_3)^T$ - three-dimensional parameter vector. Relations (41) permit one to write in explicit form the restriction of the second variation of the Routh function. Obviously, the same result can be obtained by analysis of the eigenvalues for the matrix of the quadratic form.

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