A Portfolio Management Approach Based on Continuous VaR-criterion

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This work continues the development of the author's considerations about using multistage (or continuous) version of VaR-criterion in portfolio management. In the market of quite arbitrary nature, principles for constructing the optimal portfolio of an investor with own view on market properties and own risk preferences are established. These principles allow transgressing the bounds of both theory of second order of Marcowitz in portfolio management and the standard VaR-criterion often used in problems of portfolio hedging. In the work, a scenario approach is applied. In the space of all combinations of some factors which operate in the market and generate the random return of all market securities, a finite subset is chosen that can be considered as a representative sample of the original factor space. The cardinality of this subset will determine the precision of the investor's problem solution and should be coordinated with the multiplicity of market instruments. Some examples are considered, which demonstrate the efficiency of the developed method. It is shown how uniformly the problems of managing an arbitrary securities portfolio and hedging can be described.

This work continues author's investigations presented in [1, 2]. An arbitrary one-period market with N elementary securities $\hat{s}_i, i \in I = \{1, \dots, N\}$, is considered. We treat a security as *elementary* if it can not be represented as a linear combination of some others. Below we use vector and matrix notifications, so we denote a set of all $\hat{s}_i, i \in I$, as a vector $\hat{\mathbf{s}}$. Their prices are given as $m_i = |\hat{s}_i|, i \in I$. In the market, there operate some factors, which affect the actual random return on securities. All their combinations form a space Ω_0. We choose in this space a finite subset $\Omega \subset \Omega_0$, which can be considered as a representative sample of the original set Ω_0. All these sampling points will be referred to as scenarios. We call for $\Omega = I$. A single long position in ith instrument $\hat{s}_i$ generates the random return y_i. We denote the return on instruments $\hat{s}_i, i \in I$, under scenarios $j \in \Omega$, as $\hat{s}_i(j) = y_{ij}$; they form a matrix $\mathbf{Y}$. Let's introduce new instruments. We determine an instrument $u_i, i \in I$, as the *basic* instrument if $\hat{u}_i(j) = \delta_{ij}, i \in I$, where δ_{ij} is the Kroneker symbol. If we replicate $\hat{\mathbf{u}}$ by $\hat{\mathbf{s}}$ in the form $\hat{u}_i = \sum_{j \in I} z_{ij}\hat{s}_j$, or $\hat{\mathbf{u}} = \mathbf{Z}\hat{\mathbf{s}}$, we readily derive that $\mathbf{Z} = \mathbf{Y}^{-1}$ (we assume that the matrix $\mathbf{Y}^{-1}$ exists), and so $\hat{\mathbf{u}} = \mathbf{Y}^{-1}\hat{\mathbf{s}}$. If we neglect all transaction costs we get $\mathbf{c}^T = \mathbf{Y}^{-1}\mathbf{m}^T$, where $c_i = |\hat{u}_i|, i \in I$, are prices of basic instruments, and superscript T denotes the operation of transposition. We can attach to these prices a probability sense by multiplying them all by normalizing factor $r_{rf} = 1/\sum_i c_i$. The point is that the sum of all basic instruments forms a single risk-free instrument. So the factor r_{rf} determines the risk-free return relative that can be received in the market (typically $r_{rf} > 1$) and $\mathbf{c}^0 = r_{rf}\mathbf{c}$ can be interpreted as implied scenario probability vector.

Next we describe an arbitrary investor, which has own market property forecast and own risk preferences. Investor's forecast is given by a vector $\mathbf{d} = (d_1, d_2, \ldots, d_n)$ of scenario probabilities, which does not need to coincide with vector $\mathbf{c}^0$. Investor's risk preferences are given by a continuous monotone non-decreasing critical return function $B_{cr}(\varepsilon), \varepsilon \in [0,1]$. We require $P_t\{R < B_{cr}(\varepsilon)\} \leq \varepsilon$ for all $\varepsilon \in [0,1]$, where $P_t\{E\}$ is a probability measure of the event E as forecasted by our investor, and R is the random return on the investment. To solve this problem we use Neuman-Pearson method [3]. A likelihood ratio vector $\mathbf{l} = (l_1, l_2, \ldots, l_n)$, where $l_i = c_i/d_i$, is introduced and all scenarios from I are being reordered in diminishing order of these ratios. We denote by $\boldsymbol{\xi} = (\xi_1, \xi_2, \ldots, \xi_n)$ a one-to-one mapping of I onto itself corresponding to the above order. This mapping can be also given by a substitution matrix $\boldsymbol{\Xi} = ||\xi_{ij}||$, for which $\xi_{ij} = 1$ if $j = \xi(i)$ and $\xi_{ij} = 0$ otherwise, $i, j \in I$. Now, with the help of this matrix, we can find the reordered vector $\mathbf{d}_{ro}^T = \boldsymbol{\Xi}\mathbf{d}^T$. Then we determine vector $\boldsymbol{\varepsilon}$ by the rule: $\varepsilon_k = P_t\{X_k\} = \sum_{i \leq k} d_{\xi(i)}$, where $X_k = (\xi_1, \xi_2, \ldots, \xi_k)$. Also we find investor's risk preference vector $\mathbf{b} = (b_1, b_2, \ldots, b_n)$, where $b_k = B_{cr}(\varepsilon_k)$.

This information suffices to derive investor's optimal portfolio $\hat{g}$ and its properties. It holds

$$\hat{g} = (\sum)_i b_i \hat{u}_{\xi(i)} = \mathbf{b}\boldsymbol{\Xi}\hat{\mathbf{u}}^T = \mathbf{g}\hat{\mathbf{s}}^T,$$

where $\mathbf{g} = \mathbf{b}\boldsymbol{\Xi}\mathbf{Y}^{-1}$.

The value of this optimal portfolio (i.e. the investment amount) is

$$|\hat{g}| = \mathbf{b}\Xi\mathbf{Y}^{-1}\mathbf{m}^T = \mathbf{g}\mathbf{m}^T.$$

Optimal average return of the investor is

$$R_{opt} = \mathbf{b}\Xi\mathbf{d}^T$$

and optimal average return relative is

$$r_{opt} = R_{opt}/|\hat{g}| = (\mathbf{b}\Xi\mathbf{d}^T)/(\mathbf{g}\mathbf{m}^T),$$

which must be greater than r_{rf}.

Example 1. We wish to apply the above approach to a simplified version of the roulette game. We assume that the roulette has only 36 cells and does not have a "zero" cell. The player has own view on roulette properties. It is readily seen that elementary instruments can in this example be identified with basic ones. As such, we can choose the stakes of our player on arbitrary one cell of all 36 cells.

Let's suppose that, in contrast to casino, the player thinks the probability of roulette halt is the largest for 13^{th} cell and equals $1.09/36$, and is the lowest for opposite 31^{st} cell and equals $0.91/36$. In intermediate cells, the probability varies linearly. Also, we take the risk preference function of the player in the form $B_{cr}(\varepsilon) = \varepsilon^\gamma, \varepsilon \in [0,1]$. It is obvious that $r_{rf} = 1.00$ for all $\gamma > 0$. If $\gamma = 4$ (i.e. the player is very much inclined to the risk) the use of Neuman-Pearson procedure provides the optimal portfolio (i.e. combined stake), for which the largest weight falls on 13^{th} cell and the lowest weight falls on 31^{st} cell. Also, the scattering of weights is significant. The financial properties of this portfolio are as follows:

$$|g| = 0.20, R_{opt} = 0.21, r_{opt} = 1.06.$$

If $\gamma = 0.25$ (i.e. the player is averse to the risk to great extent) then, analogously, the largest weight of the optimal portfolio falls on 13^{th} cell and the lowest weight falls on 31^{st} cell. But this time, the scattering of weights is very small. Therefore financial results of this portfolio are much more modest. This is testified by comparison of average returns relative in both cases:

$$|g| = 0.80, R_{opt} = 0.81, r_{opt} = 1.01.$$

Example 2. Let's consider a one-period option market and an investor, which has own view on market behavior in the future and own risk preferences in the form of function $B_{cr}(\varepsilon) = \varepsilon^{\gamma}, \varepsilon \in [0,1]$. It is reasonable to take the underlier's price as the only factor affecting the prices of all options. For simplicity, we assume that, in the market, 10 call options trade with the successive strikes $a{+}0.1, a{+}0.2, \dots, a{+}1.0$, where a is some reference price. As it was agreed, we have to choose a sample of exactly 10 values — scenarios — of underlier's prices. As such we take the same sequence but shifted to the right of 0.1. Remembering the payoff profile of the call option we obtain that the first line of the matrix $\mathbf{Y}$ is the vector (0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0). By analogy, the second line of this matrix is derived from this line by shifting it of one position to the right, throwing away the rightmost number and putting a zero in the first position. And so forth, until the last line. Let's suppose that

$$\mathbf{m} = (0.52, 0.43, 0.34, 0.27, 0.20, 0.14, 0.09, 0.06, 0.03, 0.01).$$

Consider the first case when the investor estimates the market as less volatile than it is evidenced by call prices. Therefore we can set, for example,

$$\mathbf{d} = (0.08, 0.09, 0.10, 0.11, 0.12, 0.11, 0.10, 0.09, 0.08, 0.07)/0.95\,.$$

If investor's critical return function is $B_{cr}(\varepsilon) = \varepsilon^4, \varepsilon \in [0,1]$, we derive by applying standard Neuman-Pearson procedure the optimal portfolio in the form (here $r_{rf} = 1.05$)

$$\mathbf{g} = (0.006, 0.12, 0.58, 1.77, 4.23, -10.87, 0.05, 2.77, 1.04, 0.28).$$

Its parameters are

$$|g| = 0.21, R_{opt} = 0.26, r_{opt} = 1.25.$$

If, however, the investor with the same risk preferences supposes that the market volatility will increase and so, for example,

$$\mathbf{d} = (0.11, 0.10, 0.09, 0.08, 0.07, 0.08, 0.09, 0.10, 0.11, 0.12)/0.95$$

(i.e. the market and the investor simply exchange their roles in comparison with the first case), the vectors $\boldsymbol{\varepsilon}$ and $\mathbf{b}$ remain as previously and all financial characteristics of the optimal portfolio are the same too (here again $r_{rf} = 1.05$):

$$|g| = 0.21, R_{opt} = 0.26, r_{opt} = 1.25\,.$$

However, the optimal portfolio itself, which is given by the vector

$$\mathbf{g} = (5.83, -9.95, 2.77, 1.04, 0.28, 0.04, 0.12, 0.58, 1.77, 4.23),$$

undergoes a very thorough change.

As supplement to this example, let us now address to the *hedge problem*. Formally, the distinction of this problem from one just considered becomes apparent when investor choose a special type of the critical return function. If, for example, the investor wishes to eliminate the downside risk of the underlier's price reduction, he or she can add to the original function, say $B_{cr}(\varepsilon) = \varepsilon^\gamma$, the positive constant h, which determine the minimal return for investor. Letting that the market and investor's forecast are given as formerly and investor's risk preferences are given by the function $B_{cr}(\varepsilon) = \varepsilon^4 + h, \varepsilon \in [0, 1]$, we could find optimal portfolios for various h by the same way as we did it above for the case $h = 0$. The hedge problem, however, is being usually stated slightly otherwise. One needs to find such a value h that it holds $|\hat{g}| = \alpha h$ for some $\alpha < 1$ prescribed by the investor. To solve this problem we need to organize α search of desired h, at which this relation is valid. We carry out a series of computations for prescribed α and for a set of values of h, as we did it above. Let, for example, $\alpha = 0.8$, i.e. our investor's desire is not to lose more than 20% of investment amount. Setting 0.5 and 1.0 as start values for h and using Newton approach together with dichotomy considerations we can quickly seek out an appropriate value of h with the precision of, say, 0.01. As it can be ascertained, our investor's hedge problem is solved by the value $h = 0.71$. At that $|g| = 0.887, r_{opt} = 1.097$. Comparing this average return relative (1.10) to the average return relative without hedge (1.25) shows that hedging comes for investor to substantial decline in average yield. In fact, such hedging with the parameter h means that the investor does not state own risk preferences in advance. Rather they are being constructed as the investor receives results of test computations. To some extent, it contradicts to philosophy, which is advocated in this work. However, it also testifies that such dialog procedure used by investor to work out the risk preferences is meaningful and can entirely be realized in the analyzed model.

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A Comparison of Effectiveness of Circular and Radial Reserve in Survivability of Symmetrical Hierarchical Networks

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A multicommodity flow network [1] is called symmetrical hierarchical (SHN) if its logical graph has the structure of a star (i.e., the source-sink pairs are given in the form $(v_0, v_i), i \in M \stackrel{def}{=} \{1, \dots, m\}$, with the common source v_0) and all demands are equal to d.

It is known that the physical structure of a star possesses poor properties of survivability. There are two ways to increase the survivability of SHN: to create an additional circular structure connecting all the sinks and to extend the capacity of the radial edge. Let c be an initial capacity vector of SHN. Then survivability of SHN is defined by $\theta_\gamma^g(c)$ as follows:

$$\theta_\gamma^g(c) = \min_{y \in Y_\gamma(c)} \max_{z \in Z(y)} \min_{i \in M} \frac{z_i}{d},$$

where $Y_\gamma(c) = \{y \geq 0 | \sum_{i=1}^{2m} y_i = (1-\gamma) \sum_{i=1}^{2m} c_i, \quad y \leq c\}$ and $Z(y)$ is the set of all feasible multiflows $z = (z_1, \dots, z_m)$ in the network with capacity vector y. Here $\gamma \in (0,1)$ is a parameter which characterizes the power of network destruction.

In this paper, we consider survivability of SHN taking into account destruction of the main radial edge, the circular and radial reserve, and compare the efficiency of circular and radial reserve.

Lemma 1. *Let $c = (d, \dots, d, t, \dots, t)$ be the capacity vectors of SHN with circular reserve, and reserve value $t = md/8$. Then*

$$\theta_\gamma^g(c) = \begin{cases} 1 - \gamma(1 + \frac{m}{8}), & if \quad \gamma \le \gamma^*, \\ 1 + (\frac{1}{4} - \gamma)m - \frac{1}{8}\gamma m^2, & if \quad \gamma^* < \gamma < \bar{\gamma}, \\ 0, & if \quad \gamma \ge \bar{\gamma}, \end{cases}$$

where $\gamma^* = \frac{2m}{(m-1)(m+8)}$, $\bar{\gamma} = \frac{2m+8}{m(m+8)}$.

Lemma 2 *Let $c = (d+t, \dots, d+t, 0, \dots, 0)$ be the capacity vectors of SHN with radial reserve, then*

$$\theta_\gamma^g(c) = \begin{cases} \frac{(d+t)(1-m\gamma)}{d}, & if \quad \gamma < \frac{1}{m}, \\ 0, & if \quad \gamma \ge \frac{1}{m}. \end{cases}$$

Lemma 3. *Let $c' = (d, \dots, d, t, \dots, t)$, $c'' = (d + t, \dots, d + t, 0, \dots, 0)$ be the capacity vectors of SHN with circular and radial reserve correspondingly, with reserve value $t = md/8$. Then we have*

$$\begin{cases} \theta_\gamma^g(c') < \theta_\gamma^g(c'') & if \quad \gamma < \hat{\gamma}, \\ \theta_\gamma^g(c') = \theta_\gamma^g(c'') & if \quad \gamma = \hat{\gamma}, \\ \theta_\gamma^g(c') > \theta_\gamma^g(c'') & if \quad \hat{\gamma} < \gamma < \bar{\gamma}, \end{cases}$$

where $\hat{\gamma} = \frac{m}{(m-1)(m+8)}$, $\bar{\gamma} = \frac{2m+8}{m(m+8)}$.

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Resources Allocation in Industry with Deficit of Circulating Assets*

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This paper is devoted to modeling of functioning of industry. We consider a generalization of Houthakker-Johansen model. It takes into account peculiar properties of Russian economy such as different price structure on Russian and world markets, peculiarities of Russian monetary system, producers' deficiency of floating assets, ineffective resources allocation, and incomplete production capacity.

Patterns of Party Competition in British and Russian General and Finnish Municipal Elections

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key words: *elections, constituencies, over-time stability*

A classification of the electoral outcomes is given in the parliamentary elections in the constituencies of Great Britain and Russia in terms of the relative support of the main parties. Then the over-time stability of the competitive settings in various constituencies is analyzed. Next the same problem is solved for seven most recent Finnish municipal elections. It turns out that only very few classes are needed to characterize the average patterns of support distribution. Our main finding is that out of thousands of conceivable over-time trends only few are needed to characterize the support development in all three countries.

Party competition belongs to the traditional foci of political research. Like in any competition, the successful strategies of parties depend on the strategies of the competitors and on the rules of the game. In party competition the latter consists primarily of the electoral system. What works in a system of single-member constituencies, may not work in multi-member ones. In fact, two well-known

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principles of electoral behavior, viz. Duverger's Law and Duverger's Hypothesis, state that certain types of electoral systems give rise to specific types of party systems (see Duverger, 1954; Riker, 1976; Riker, 1982). There may be interesting principles also within a given electoral system. Our focus is on such principles. More specifically, we shall study the patterns of electoral competition prevailing in a given system over time, i.e. over a period of several elections. We introduce a method for grouping election outcomes into classes according to the nature of competition prevailing in various constituencies. Once we have determined the clusters ("system states") of nearly identical electoral support our next task is to look at the "movements" from one cluster to another that have occurred in districts over the span of several recent elections. We thus present a method of finding out the natural states and state transitions viewed from the angle of electoral support. We present the distribution of constituencies in Great Britain (separately for England, Scotland and Wales) general elections of 1992, 1995 and 1999 over various types including the dynamics, i.e. the over-time movements of constituencies from one type to another as well as the most common time-paths in the observed three elections. We solve the same problem for the constituencies in Russia during last three general elections. Although national elections are generally viewed as most significant, local elections are also important in political systems where the political authority is decentralized to geographically defined units. We give a short exposition of Finnish party system and then we proceed with the same type of analysis in a sequence of seven most recent municipal elections. What we are interested in is whether it is possible to classify the Finnish municipalities into classes with similar support distribution patterns. It is of course trivial to argue that no two municipalities are identical, but then again it is obvious that small changes in support do not necessarily mean essential changes in the competition setting. What we suggest is that there is a way of finding out clusters of similar municipalities so that the distinctive features of the party competition are nearly identical within each cluster.

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Numerical Methods of Abstract Convex Analysis

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In 1999 We proposed an efficient method of global optimization called a cutting angle algorithm. It has been proved to be more efficient with respect to simulated annealing and simple B & B.

The method is applicable to problems with star-shaped and arbitrary Lipschitz functions and solves problems with up to 100 variables.

The extensions of method are able to optimize non-convex increasing functions. This possibility is very important for mathematical economics. The approach is based on the use of min-plus algebra and disjunctive programming, so we have developed some methods of relaxation and reduction to such sub-problems.

The results of numerical experiments are very interesting.

Unimodality of the Simpliest Gaussian Mixture with More Than Two Components

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key words: *Gaussian mixture, contracting operator, unimodality*

Popularity of final Gaussian mixtures as universal approximater causes the solution of such mathematical problems as definition of the modes and the preliminary estimation of their number connected with property of their unimodality. Generally the problem of a unimodality of a Gaussian mixture is not solved. In the known papers only two-component mixtures, $k = 2$, are investigated. In this paper, on the basis of a principle of contracting maps and classical theorems of the mathematical analysis some sufficient conditions for unimodality of mixture of k normal distributions with equal variances and with various expectation values μ_i, $i = 1, \ldots, k$, $3 \leqslant k < \infty$ taken with the a priori probabilities π_i are obtained.

A Nondegenerate Maximum Principle for the Impulse Control Problem with State Constraints*

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key words: *impulse control, state constraints, Maximum Principle, non-fixed time, time transversality conditions, nondegeneracy*

In this article, a non-fixed time impulsive control problem with state constraints and equality and inequality constraints on the trajectory endpoints is considered. For smooth problem a nondegenerate Maximum Principle is derived by using a penalty function method. In addition to the main result for nonsmooth (measurable) in t problem weakened Maximum Principle is obtained where time transversality conditions are deduced with the help of some extra convexity supposition on state constraints.

Statement of the problem. We shall address the following impulse control optimization problem:

$$J(p,u,\mu) = e_0(p) \to \min, \tag{1}$$

$$dx = f(x,u,t)dt + g(x,t)d\mu, \ t \in [t_0,t_1], \tag{2}$$

$$e_1(p) \le 0, \ e_2(p) = 0, \varphi(x,t) \le 0,$$

$$u(t) \in U \subset \mathbb{R}^m, \quad \mu \ge 0,$$

$$p = (x_0, x_1, t_0, t_1), \quad x_0 = x(t_0), \ x_1 = x(t_1).$$

Here e_1, e_2, φ are given vector-functions with values in $\Re^{k_j}$, $j = 1,2,3$, respectively, $t \in \Re^1$ is the time variable, μ is a nonnegative scalar valued Borel measure on time interval $[t_0,t_1]$, referred to by

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impulse control, and x is the state variable with values in $\Re^n$. The notation "a.e." stands for almost all $t \in [t_0, t_1]$ with respect to Lebesgue measure. The vector u with values in $\Re^m$ is called control. An admissible control is an essentially bounded measurable function $u(t)$ such that $u(t) \in U$ a.e.. The vector $p \in \Re^{2n+2}$ is called endpoint. Suppose that all functions are continuously differentiable in all arguments, the vector function f is linear in u, the set U is convex and closed.

The aim of this article is to derive necessary conditions for optimality in the form of a nondegenerate maximum principle (MP) for problem (1). Say that MP degenerates if condition (3) doesn't hold. Accordingly the main our result is nondegenerative MP, which consists in theorem 1. Note here, that MP for impulsive systems with state constraints was researched by many authors [2, 6, 8, 10]. However no one of pointed authors was not interested in nondegeneracy problem in impulse control. The nondegeneracy problem is important problem in optimal control and for convenient optimal control problem with state constraints was researched by Ferreira and Vinter [4], Dubovickies [3] and others.

Hypotheses. Let us formulate basic definitions and assumptions we use below. Definition of regularity of endpoint and state constraints and definition of compatibility of state constraints with endpoint constraints are given in [1].

Definition. The admissible trajectory $x(t)$, $t \in [t_0, t_1]$, is called *controllable* at the end points (with regard to state constraints), if there exist $u_k \in U$ and $m_k \in [0, +\infty)$, $k = 0, 1$, such that

$$(-1)^k \left[\langle f(x_k, u_k, t_k) + g(x_k, t_k)m_k, \varphi_x^j(x_k, t_k)\rangle + \varphi_t^j(x_k, t_k)\right] < 0$$

$\forall j$ s.t. $\varphi^j(x_k, t_k) = 0$. Here $x_k = x(t_k)$, $k = 0, 1$.

We adopt the concept of solution to (2), following the one firstly given in [5].

The main result. Let $\lambda = (\lambda_0, \lambda_1, \lambda_2)$ and consider the following scalar functions

$$H(x, u, \psi, t) = \langle f(x, u, t), \psi\rangle, \qquad Q(x, \psi, t) = \langle g(x, t), \psi\rangle,$$

$$l(p, \lambda) = \sum_{j=0}^{2}\langle e_j(p), \lambda_j\rangle.$$

Theorem 1. Let (p^*, u^*, μ^*) be a solution to problem (1). Suppose that state constraints are compatible with endpoint constraints, the state and endpoint constraints are regular and the optimal trajectory is controllable. Then, there exist

number $\lambda_0 \geq 0$, vectors $\lambda_1 \in \Re^{k_1}$, $\lambda_1 \geq 0$, $\lambda_2 \in \Re^{k_2}$,

vector function $\psi \in V^n(T^*)$, scalar function $\varphi \in V(T^*)$,

vector measure $\eta = (\eta^1, .., \eta^{k_3})$, $\eta^j \in C_+^*(T^*)$, s.t. $\mathrm{Ds}(\mu^*) \cap \mathrm{Ds}(\eta^j) = \emptyset \ \forall j$, and

for every atom $r \in \mathrm{Ds}(\mu^*)$, there exist its own vector function $\sigma_r \in V^n([0,1])$, scalar function $\theta_r \in V([0,1])$, and vector measure $\eta_r = (\eta_r^1, \dots, \eta_r^{k_3})$, $\eta_r^j \in C_+^*([0,1])$, $j = 1, .., k_3$, such that $\psi(t) = \psi_0 - \int\limits_{t_0^*}^{t} H_x(s)ds - \int\limits_{[t_0^*,t]} Q_x(s)d\mu_c^* + \int\limits_{[t_0^*,t]} \varphi_x^\top(x^*, s)d\eta + \Sigma(\psi, t), t \in (t_0^*, t_1^*]$,

$$\Sigma(\psi, t) = \sum_{r \in \mathrm{Ds}(\mu^*),\ r \leq t} [\sigma_r(1) - \psi(r^-)], \Theta(\varphi, t) = \sum_{r \in \mathrm{Ds}(\mu^*),\ r \leq t} [\theta_r(1) - \varphi(r^-)],$$

$\varphi(t) = \varphi_0 + \int_{t_0^*}^{t} H_t(s)ds + \int_{[t_0^*,t]} Q_t(s)d\mu_c^* - \int_{[t_0^*,t]} \varphi_t^\top(x^*, s)d\eta + \Theta(\varphi, t), t \in (t_0^*, t_1^*]$,

$$\left\{ \begin{array}{lcl} d\alpha_r^*(s) & = & g(\alpha_r^*(s), r)\Delta_r^* ds, s \in [0,1], \\ d\sigma_r(s) & = & -g_x^\top(\alpha_r^*(s), r)\sigma_r(s)\Delta_r^* ds + \varphi_x^\top(\alpha_r^*(s), r)d\eta_r, s \in [0,1], \\ d\theta_r(s) & = & \langle g_t(\alpha_r^*(s), r), \sigma_r(s)\rangle \Delta_r^* ds - \varphi_t^\top(\alpha_r^*(s), r)d\eta_r, s \in [0,1], \\ \alpha_r^*(0) & = & x^*(r^-), \sigma_r(0) = \psi(r^-),\ \theta_r(0) = \varphi(r^-),\ \Delta_r^* = \mu^*(\{r\}), \end{array} \right.$$

$$\psi_0 = \frac{\partial l}{\partial x_0}(p^*, \lambda), \quad \psi_1 = -\frac{\partial l}{\partial x_1}(p^*, \lambda),$$

$$\varphi_0 = -\frac{\partial l}{\partial t_0}(p^*, \lambda), \quad \varphi_1 = \frac{\partial l}{\partial t_1}(p^*, \lambda),$$

$$\langle g(\alpha_r^*(s), r), \sigma_r(s)\rangle = 0 \quad \forall s \in [0,1] \quad \forall r \in \mathrm{Ds}(\mu^*),$$

$$\mathrm{supp}(\eta_r^j) \subseteq \{s \in [0,1] : \ \varphi^j(\alpha_r^*(s), r) = W^j(\alpha_r^*(s), r) = 0\} \quad \forall j,$$

$$\langle \lambda_1, e_1(p^*)\rangle = 0,$$

$$\varphi^j(x^*(t), t) = 0 \ \ \eta^j\text{-a.e.} \quad \forall j,$$

$$\max_{u \in U} H(u,t) = H(t) \ \text{ a.e.}, \qquad \max_{u \in U} H(u,t) = \varphi(t) \quad \forall t \in (t_0^*, t_1^*),$$

$$Q(t) \leq 0 \quad \forall t, \qquad Q(t) = 0 \quad \mu^*\text{-a.e.},$$

$$\lambda_0 + \mathcal{L}\Big(\{t : |\psi(t)| > 0\}\Big) + \sum_{r \in \mathrm{Ds}(\mu^*)} \mathcal{L}\Big(\{s : |\sigma_r(s)| > 0\}\Big)\Delta_r^* = 1. \ (3)$$

Here we adopt the following short notation:
$T^* = [t_0^*, t_1^*]$, $\psi_k = \psi(t_k^*)$, $\varphi_k = \varphi(t_k^*)$, $\Delta_k^* = \mu^*(\{t_k^*\})$, $k = 0, 1$,
$H(t, u) = H(x^*(t), u, \psi(t), t)$, $\quad H(t) = H(t, u^*(t))$,
$Q(t) = Q(x^*(t), \psi(t), t)$, $H_x(t) = H_x(x^*(t), u^*(t), \psi(t), t)$, $\ldots$, etc. In other words, if H, Q or their partial derivatives miss some of arguments x, ψ, u, then it is understood that the values $x^*(t)$, $\psi(t)$, and $u^*(t)$ are considered in their place.

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Research of the Soros Equation for the Currency Exchange Market

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In [1], there are described basic macroeconomic processes influencing the exchange rate of national currency and thoughts of the importance of some the reasonings parameters is shown. Designations, used in reasonings:
e — exchange rate of national currency; i_{int} — nominal internal interest rate; i_{ext} — nominal external interest rate; N — not speculative movement of the capital; S — speculative movement of the capital; T — trade balance.

For convenience, we shall consider that all processes go "out" of the country, i.e. positive S means import of the capital (instead of export, as in [1]), and N is nonspeculative export of the capital (instead of import, as in [1]), which defines, basically, service of the external loans. In general reasonings, the exchange rate is defined by a supply and demand of currencies:

$$\downarrow T + \uparrow N + \downarrow S \Longrightarrow \downarrow e.$$

The given statment means that decreasing of trade balance (T) leads to growing of nonspeculative export of the capital (N) and decreasing of speculative import of the capital (S), the exchange rate of national currency (e) should decrease. As well as in a case of share market, the feedback between some variable, first of all between S and e is observed. In connection with that, the international capital searches for the strongest currency, i.e. the foreign investors will translate the means in currency of that country, which becomes stronger in relation to rests.

In [1], the assumption is introduced that trade balance and nonspeculative export of the capital do not depend on expectations of the participants concerning the future exchange rate of currency, and are determined by a today's condition of a system. Moreover, the important assumption is that for an open economy the influence of movement of the capital, by virtue of its volume, is much more than influence of the trade balance. Dynamics of movement of the speculative capital is set by the following relation:

$$\uparrow (e + i_{int}) \Longrightarrow \uparrow S.$$

It means that speculative capital is involved with becoming stronger national currency and high internal percentage rates. Thus the influence is much more for speculative movement of the capital. Therefore from the further reasonings i_{int} is excluded. The feedback in examined system expresses that the import of the speculative capital depends on force of national currency, which in turn depends on "a pure" flow of the capital, i.e. speculative import minus nonspeculative export (thus both of them can be negative, that will mean, that the country invests much abroad and receives back income, due to these investment).

From here

$$\uparrow e \Longrightarrow \uparrow S \Longrightarrow N + \uparrow e,$$

i.e. the growth of a rate results in import of the capital and, besides the further growth of a rate, in increasing burden of its service. The preference follows the trend, and if there is longer kept trend, the preference becomes stronger. If there is macroeconomic reason, on which the foreign capital began to come in the country, in due course this process only becomes stronger. There is an accumulation of "hot" money. In due course burden on service becomes long too large, it appears that the national currency cannot be kept from devaluation, and the process wavy goes in the opposite direction. According to the given reasonings, we believe:

a) the rate of currency e depends on movement of S and N. Thus

$$S(k) = S(t), t \in [\triangle t \cdot (k-1), \triangle \cdot k]$$

i.e. $S(k)$ and $N(k)$ are volumes of the comeing speculative and leaving nonspeculative capital accordingly for a certain interval of time.

b) dynamics of the speculative capital is defined by previous changes of rate of currency. Thus the delay for 1 period takes place.

c) the country bears burden on service of the capital imported into last 10 periods. Precisely the same as receives the due dividends from the capital which has been taken out from the country not longer of 10 periods back.

d) the "pure" flow of the capital is designated NCF (Net Capital Flow) and is defined as a difference between speculative import and nonspeculative export. The positive "pure" flow means inflow of the capital into the country, and negative — its flight. According to the accepted assumptions, we shall write down parities describing changes of the characteristics e, S and N in time (Soros equation for the currency market):

$$
\begin{aligned}
&e(t+1) = e(t) + a \cdot \frac{S(t+1)}{|S(t)| + |N(t)|} - c \cdot \frac{N(t+1)}{|S(t)| + |N(t)|},\\
&S(t+1) = S(t) + d \cdot (e(t) - e(t-1)),\\
&N(t+1) = \sum_{\tau=t-10}^{\tau=t} S(\tau) \cdot i_{ext},\\
&NCF(t) = S(t) - N(t).
\end{aligned}
$$

Here a connects e and S, c connects e and N, d is the external interest rate, which connects S and e. The given equations coordinate speculative and nonspeculative flows of the capital to a rate of national currency. On the basis of it, modelling of behavior of system were carried out supposing the values of a, c, d and i_{ext} are distributed on the following intervals: $a(t) \in [0.8, 1.2]$, $c(t) \in [0.8, 1.2]$, $d(t) \in [400, 700]$, $i_{ext}(t) \in [12.5\%, 15.5\%]$.

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Mathematical Modelling of Hydrogen Crystallization Process at Freezed Wall of Lazer Target

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key words: *mathematical modelling, hydrogen, crystallization, lazer target, nuclear fusion, Clapeyron-Clausius equation, heat conductivity equation, heat flow equation, singular*

A problem of production of the target for laser nuclear fusion is an actaul one. The laser target is a thin polystyrene shell with deuterium-tritium mixture cristallized on it's walls. In this work the model of desublimation of hydrogen isotopes inside a freezed spherical shell is preseneted. The essence of this model is that the temperature of the gas is not constant but given by Clapeyron-Clausius equation. The model implies a system of heat conductivity and heat flow equtions. It is perturbated in dimentionless variables. The problem is solved with help of temperature functional extention of series of powers of small parameter. Approximations of temperatures of substences are considered. Time when desublimation finishes is one of the most important chracteristics of the process. The formula for the time was obtained. It gives an opportunity to investigatage the process dependence on laser target parameters and external temperature.

A problem of future energetic lack is important. A possibility to solve it depends greatly on successful realization of the idea of practical use of laser nuclear fusion by world phisical society. In 1985 the USA National Laboratry in Livermore constructed a sophisticated laser setup (Nova Laser Facility) that is able to fire a laser target (LT) by synchronized lazer beams from all sides simultaneously. But it has almost no job till now because of absense of a verified technology of LT production and estimation.

The laser target is a polystyrene spherical shell with a layer of solid hydrogen isotopes freezed to extremely low temperatures on its walls. Process of transformation from gas substance to solid phase takes place under fast freezing almost without appearance of liquid, e.g. can be considered as desublimation. In this work the models of desublimation studied in [1,2] are made more precise. The difference of our model from the models descibed earlier is that the temperature of gas inside LT is not a constant but depends on the pressure. And the relation between temperature and pressure is given by Clapeyron-Clausius equation.

Write down Clapeyron-Clausius equation:

$$\frac{dp_g}{dT_g} = \frac{\lambda_{cr}}{T_g(v_g - v_{cr})},$$

where p_g is gas pressure, T_g is gas temperature, λ_{cr} is specific heat of sublimation, $v_i = \frac{V_i}{m_i} = \frac{1}{\rho_i}$, $i = g, cr$ is specific volume of the gas and solid substance (cryogenic layer).

We use in addition Mendeleev-Clapeyron equation and expantions in Taylor's series to get:

$$T_g = \ln\left(\gamma + \frac{1-\gamma}{(1-\frac{w(t)}{r_1})^3}\right)\frac{R}{\lambda_{cr}\mu}\left[\frac{1}{T_0} - \frac{R}{\lambda_{cr}\mu}\right]^{-1} T_0 + T_0, \qquad (1)$$

where $\gamma = \frac{v_g}{v_{cr}}$, T_0 is temperature when desublimation begins, $w(t)$ is cryogenic layer thickhess. It is related with time.

Temperature of gas depends only on time and doesn't change within coordinate. Temperatures of shell and solid hydrogen are descibed by heat conductivity equations

$$\rho_i c_i \frac{\partial T_i}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r} r^2 k_i \frac{\partial T_i}{\partial r}, i = cr, sh.$$

Boundaries shell-cryogenic layer and gas-cryogenic layer are descibed by heat flow equations

$$k_{sh}\frac{\partial T_{sh}}{\partial r}|_{r=r_1} = k_{cr}\frac{\partial T_{cr}}{\partial r}|_{r=r_1},$$

$$-\chi k_{cr}\frac{\partial T_{cr}}{\partial r}|_{r=r_1-w(t)} = \lambda_{cr}\rho_{cr}\frac{dw}{dt} - \chi k_g \frac{\partial T_g}{\partial r}|_{r=r_1-w(t)},$$

where r is distance from point inside the target to it's center, r_1 - inner radius of shell of LT, $T_i(r,t)$ is temperature of shell $(i = sh)$ and cryogenic layer $(i = cr)$, $k_i(T)$ is coefficient of heat conductivity, $c_i(T)$ is heat capacity, $\rho_i(T)$ is density.

Boundary condition for the inner wall of cryogenic layer is given by equation (1). For boudary cryogenic layer-shell claim the continuosity

$$T_{sh}(r_1,t) = T_{cr}(r_1,t).$$

At the external boundary of the shell we assume temperature to be constant

$$T_{sh}(r_1,t) = T_{ext},$$

where T_{ext} is external temperature.

Starting conditions have no need to be written because they fade away rapidly.

We make temperature, time and coordinates dimentionless in the given system of equations. In a new problem, heat conductivity equations are perturbated. We use temperature expansions of functional series of powers of small parameter. For the shell, we have

$$t_{sh}(x,\tau,\varepsilon_1,\varepsilon_2) = t_{sh}^{(r)}(x,\tau) + t_{sh}^{(s)}(x,\tau,\varepsilon_1),$$

where t_{sh} is dimentionless temperature, τ is dimentionless time, x is dimentionless coordinate, ε_i is small parameter,

$t_{sh}^{(r)}(x,\tau) = t_{sh}^{(0)}(x,\tau) + \sum\limits_{k=1}^{\infty} \varepsilon_1^k t_{sh}^{(k)}(x,\tau)$ is a regular composition,

$t_{sh}^{(s)}(x,\tau,\varepsilon_1) = \sum\limits_{k=1}^{\infty} \exp\left(-\frac{\mu_k(\varepsilon_1)\tau}{\varepsilon_1}\right) u_k(x,\varepsilon_1)$ is a singular composition.

The same expansion takes place for temperature of cryogenic layer.

Singular compositions in these series fading away rapidly. So that $t_i \approx t_i^{(0)}$, $i = cr, sh$. And $t_i^{(0)}$ is considered with methods used in [1]-[2].

The most important characteristic of desublimation process is a time it lasts. In our problem we can find a function $t(\bar{w})$ - time of cryogenic layer dimentionless thicness $t_{fin} =$

$$-\int\limits_0^{\bar{w}_{fin}} \frac{(1-\bar{w})^2 T_{ext}(\frac{2T_{ext}}{\sigma(1-\delta)} + 2(\theta_{sh} + T_0 + T_0 \frac{R}{\lambda_{cr}\mu}(\frac{1}{T_0} - \frac{R}{\lambda_{cr}\mu})^{-1} \ln[\gamma + \frac{1-\gamma}{(1-\bar{w})^3}])}{\lambda((T_{ext}+\theta_{sh})^2 - (\theta_{sh} + T_0 + T_0 \frac{R}{\lambda_{cr}\mu}(\frac{1}{T_0} - \frac{R}{\lambda_{cr}\mu})^{-1} \ln[\gamma + \frac{1-\gamma}{(1-\bar{w})^3}])^2)} d\bar{w},$$

where

$$\bar{w}_{fin} = 1 - \sqrt[3]{\frac{1-\gamma}{\exp\left(\dfrac{\lambda_{cr}\mu(T_{ext}-T_0)(\frac{1}{T_0} - \frac{R}{\lambda_{cr}\mu})}{T_0 R}\right) - \gamma}}$$

is cryogenic layer thicness when desublimation finishes. It is considered from phisical sense, e.g. understanding of a fact that t must increase.

The obtained formula gives an opprtunity to investigate the desublimation process dependence on laser target parameters and external temperature.

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Estimation of IBNR Using Full Information about the Development of Every Claim

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The problem of estimation of IBNR (incurred but not reported) claims reserves is quite important for an insurance company. Most of methods used in practice for estimation of IBNR claims consider aggregate claims information only. Hesselager (1994) suggested a new method which allows to use information about the development of each claim. According to this method, the claims generating process for a non-life insurance portfolio is modeled as a time-inhomogeneous Poisson process. The development of a claim from occurrence until final settlement is assumed to be a realization of a time-inhomogeneous, continuous-time Markov chain. This means that an unsettled claim is at any point in time assigned to a state in some state-space, and transitions between different states are assumed to be governed by a Markovian law. All claims payments are assumed to occur at the time of transition between states. We investigate the model with 3 states (IBNR, RBNS, Settled), develop expressions for the mean and the variance of IBNR, methods of estimation of unknown parameters of the model. Then we generate known loss experience from distributions with fixed parameters and compare the value of generated claims, reserves estimated by chain ladder method and reserves estimated by investigated method using all information about claim's development.

Consider the process of occurrence and development of claims. Every claim can be characterized by its amount, the moment of its occurrence, the moment when it is reported and the moment when it is settled. The problem is to estimate the value of incurred but not reported claims (IBNR). There is a great variety of different methods of estimation of IBNR claims reserve. All these methods differ not only in the method of approach but also in the amount of information used.

We consider the approach suggested by Hesselager [1] for the model with 3 states:

0 — IBNR,

1 — RBNS,

2 — Settled

and intensities of transition $\lambda_{01}(u)$ and $\lambda_{12}(u)$:

$$0 \xrightarrow{\lambda_{01}(u)} 1 \xrightarrow{\lambda_{12}(u)} 2$$

According to this method, the claims occurrence process for a non-life insurance portfolio is modeled by a time-inhomogeneous Poisson process. An unsettled claim is at any point in time assigned to a state in some state-space. Transitions between different states are assumed to be governed by a Markovian law. For our model we have homogeneous Markov chain. All claims payments are assumed to occur at the time of transition from state 1 to state 2.

The expected value and varience of IBNR claims, according to this approach, is

$$\mathrm{E}\, X_{IBNR}(\tau) = \frac{y_{12} \cdot \mu}{\lambda_{01}}(1 - e^{-\lambda_{01}\tau}),$$

$$\mathrm{Var}\, X_{IBNR}(\tau) = \left(\sigma_{12}^2 + y_{12}^2 \frac{(\lambda_{01} - \lambda_{12})^2}{\lambda_{01}^2}\right) \cdot \frac{\mu}{\lambda_{01}}(1 - e^{-\lambda_{01}\tau}),$$

where $X_{IBNR}(\tau)$ is the outstanding IBNR (at time τ) claims payments, μ is the intensity of a Poisson process, $\{K(t)\}_{t \geq 0}$ is the number of claims incurred during $[0, t]$, y_{12} is the average claim amount paid at time of transition from state 1 to state 2, σ_{12}^2 is the variance of the claim amount paid at time of transition from state 1 to state 2, $p_{mn}(x)$ is probability of transition from state m to state n during time x, $\lambda_{mn} \lim_{h \to 0+} p_{mn}(h)/h$ is intensity of transition from state m to state n.

We calculate transition probabilities $p_{mn}(x)$ by solving Kolmogorov's differential equations [4]:

$$p_{00}(s) = e^{-\lambda_{01} s},$$

$$p_{11}(s) = e^{-\lambda_{12} s}.$$

We need to estimate parameters y_{12}, σ_{12}^2, μ, λ_{01}, λ_{12}.

For estimation of μ, consider a Poisson process $\{K(t)\}_{t\geq 0}$ — the number of claims incurred during $[0,t]$. Let t_1 be the time of occurrence of the first claim, t_2 — the time of occurrence of the second claim and so on. Then the sequence of τ_i, where $\tau_1 = t_1\,, \tau_2 = t_2 - t_1\,, \tau_3 = t_3 - t_2\,, \ldots$, is the sequence of independent identically distributed random variables with exponential distribution. Having statistics of τ_i, we can use the method of maximum likelihood [5] to estimate the value of μ.

Let ξ_0 be the random variable for the time of being of the claim in the state 0. Then

$$p_{00}(x) = P(\xi_0 \geq x) = 1 - F_{\xi_0}(x).$$

We have $F_{\xi_0}(x) = 1 - e^{-\lambda_{01}x}$. Having statistics of ξ_0, we can use the method of maximum likelihood to estimate the value of λ_{01}.

Similarly the random variable ξ_1 can be defined. For it $F_{\xi_1}(x) = 1 - e^{-\lambda_{12}x}$. Having statistics of ξ_1, we can use the method of maximum likelihood to estimate the value of λ_{12}.

Consider chain ladder method [3], which is one of the oldest and widely spread for a long time methods.

Let C_{ik} denote the accumulated total claims amount of accident year i, $1 \leq i \leq I$, either paid or incurred up to development year k, $1 \leq k \leq I$.

So we know values, shown in the run-off triangle:

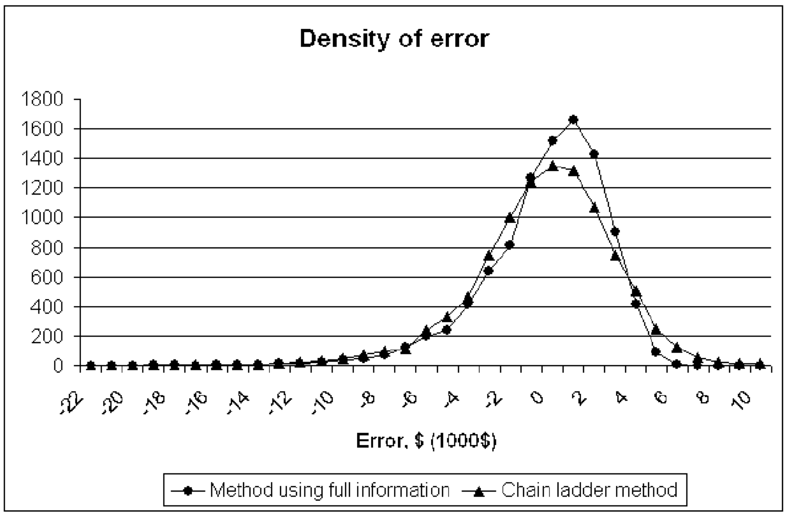
$$\begin{array}{ccccc}
C_{11} & C_{12} & \ldots & C_{1I-1} & C_{1I} \\
C_{21} & C_{22} & \ldots & C_{2I-1} & \\
C_{31} & C_{32} & \ldots & & \\
\vdots & \vdots & & & \\
C_{I1} & & & &
\end{array}$$

The present moment is the diagonal of the triangle. The values of C_{ik} for $i + k \leq I + 1$ are known to us and we want to estimate the values of C_{ik} for $i+k > I+1$, in particular the ultimate claims amount C_{iI} of each accident year $i = 2, \ldots, I$. Then $R_i = C_{iI} - C_{i,I+1-i}$ is the outstanding claims reserve of accident year i.

The chain ladder method consists of estimating the ultimate claims amounts C_{iI} by

$$C_{iI} = C_{i,I+1-i} \cdot f_{I+1-i} \cdot \ldots f_{I-1} \quad 2 \leq i \leq I\,,$$

where $f_k = \dfrac{\sum\limits_{j=1}^{I-k} C_{j,k+1}}{\sum\limits_{j=1}^{I-k} C_{j,k}}$, $k = 1, \ldots, I-1$, are the so-called age-to-age factors.



In order to compare these two methods the computer generates six accident years of known loss experience from distributions with fixed parameters with a method suggested by Stanard [2]. For each accident year a random number of losses is drawn from a normal distribution with fixed mean and standard deviation. For each loss the following random variables are drawn

- day of loss within accident year (uniform with minimum = 0, maximum = 364)

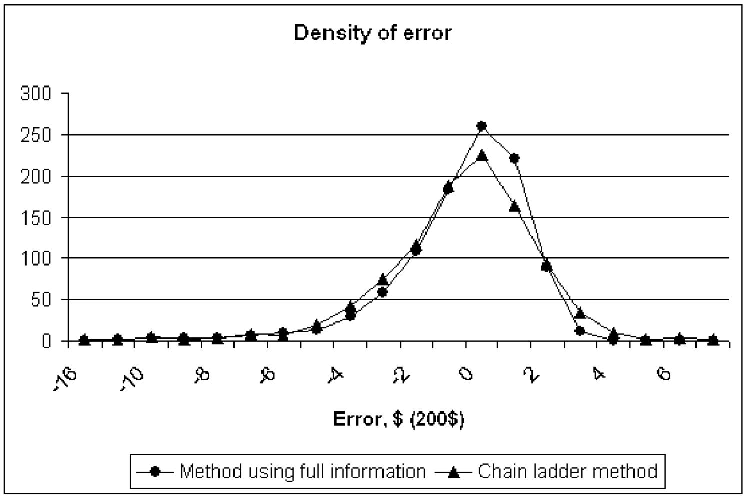
- report lag in days (waiting time between accident date and report date in days) (exponential with fixed mean)

- payment lag in days (waiting time between report date and payment date in days) (exponential with fixed mean)

- payment amount (lognormal with fixed mean and standard deviation).

Then estimation of IBNR was calculated by chain ladder method and by method using full information about development of claims.

For example, for the following set of parameters: mean of normal distribution = 1500, standard deviation of normal distribution = 5, mean of lognormal distribution = 10, standard deviation of lognormal distribution = 34, mean of report delay exponential distribution = 30,mean of payment delay exponential distribution = 60 and the number of iterations equl to 1000 we have:



Mean of error of the estimation, calculated by the method, using full information about development of claims equals 3,44;

Variance of error of the estimation, calculated by the method, using full information about development of claims equals 155000;

Mean of error of the estimation, calculated by the chain ladder method equals -22,2;

Variance of error of the estimation, calculated by the chain ladder method equals 201000.

For the following set of parameters: mean of normal distribution = 1000, standard deviation of normal distribution = 100, mean of lognormal distribution = 104, standard deviation of lognormal distribution = 348, mean of report delay exponential distribution = 30,mean of payment delay exponential distribution = 60 and the number of iterations equl to 10000 we have:

Mean of error of the estimation, calculated by the method, using full information about development of claims equals 146;

Variance of error of the estimation, calculated by the method, using full information about development of claims equals 10700000;

Mean of error of the estimation, calculated by the chain ladder method equals 59,3;

Variance of error of the estimation, calculated by the chain ladder method equals 13400000.

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The Game Insurer-Policyholder n the Stohastic Interest Rate Environment

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key words: *games theory, life insurance, optimal strategy*

An annual premium life insurance policy with a possibility of cancellation of the contract before the due date is considered. There is an option for the client to transfer the money to the bank and to obtain an additional profit from the higher market interest rate. This work describes a model of the optimal behaviour of the insurance company and the policyholder in conditions of variable market interest rate. The insurance company sells a policy to the client and he has an option to

cancel this contract at the beginning of each policy year. In this model, the client observes the current market situation and acts how it is more profitable for him at the moment. The insurance company must select its strategy at the moment of policy issuing. We have built the model that allows the insurance company to manage the behaviour of the policyholder and have formulated the rules that allow the company to obtain a positive gain from signing a contract. This paper uses a Vasicek model to describe the variability of the market interest rate. Both players have the possibility to estimate future values for the rate and calculate their estimated gains. If the profit of both players will be positive, the insurance contract will be signed.

Life insurance contract usually contains the cancellation option. This paper considers the situation when the policy can be cancelled because of additional gain that could be obtained by policyholder via money transfer to a bank. In case of early cancellation, the client will get a part of his policy reserve as a redemption. The challenge for the insurer is to manage the behaviour of the client to optimize own profit.

Let us introduce the following notation:

m — policy term in years (greater or equal than 2);

n — policy cancellation year (if $n = 0$, the policy is not signed, if $n = m$ the policy is not cancelled);

i — guaranteed interest rate used for pricing and reserving;

x_t — a part of the reserve that is returned to the policyholder in the case of cancellation at the year t;

$X = (x_t, t = \overline{1, m})$ — penalty scheme for early cancellation selected by insurance company;

${}_tV$ — policy reserve at the end of year t;

$v = \dfrac{1}{1+i}$ — discount coefficient;

b_p^q — a cost of one currency unit at time $p+q$, discounted to time p for the insurance company.

$\alpha^q * b_p^q$ — a cost of one currency unit at time $p+q$, discounted to time p for the policyholder.

Let us assume that $x_0 = 0$.

A game theory approach is considered. The players do their moves one after another. The insurer makes the first move when he tells the policyholder the penalty scheme X or the rules of penalties calculation during the policy due period. The policyholder makes the second move when he select the policy cancellation year.

The policyholder's and insurer's payoff functions are defined as a discounted to the end of the year m values of premium and payment stream:

$$\begin{aligned} G(n) = &-\frac{P}{\alpha^m * b_0^1 * \ldots * b_{m-1}^1} - \frac{P}{\alpha^{m-1} * b_1^1 * \ldots * b_{m-1}^1} - \ldots \\ &-\frac{P}{\alpha^{m-n+1} * b_{n-1}^1 * \ldots * b_{m-1}^1} + \frac{x_n *_n V}{\alpha^{m-n} * b_n^1 * \ldots * b_{m-1}^1} \end{aligned} \tag{1}$$

for the policyholder and

$$F(x,n) = \frac{P}{b_0^1 * \ldots * b_{m-1}^1} + \ldots + \frac{P}{b_{n-1}^1 * \ldots * b_{m-1}^1} - \frac{x_n *_n V}{b_n^1 * \ldots * b_{m-1}^1} \tag{2}$$

for the insurer.

The behaviour of the policyholder

Taking his decision, at the end of each year $t = \overline{1,m}$ the policyholder compares the expected value of what he could get if the money will be left at the insurance company, with what he can get from the cancellation and acts to maximize his expected profit G_t:

$$\begin{cases} G_m = x_m *_m V \\ G_{m-1} = \max(x_{m-1} *_{m-1} V; \ \ \alpha * b_{m-1}^1 * E_{m-1}[G_m] - P) \\ G_{m-2} = \max(x_{m-2} *_{m-2} V; \ \ \alpha * b_{m-2}^1 * E_{m-2}[G_{m-1}] - P) \\ \ldots \\ G_1 = \max(x_1 *_1 V; \ \ \alpha * b_1^1 * E_1[G_2] - P) \\ G_0 = \max(0; \ \ \alpha * b_0^1 * E_0[G_1] - P) \end{cases} \tag{3}$$

If for some t it is true that $G_t < G_{t-1}$ and $\forall i = \overline{1, t-1}$, $G_i = G_{i-1}$, then the client will cancel the policy at the year $t-1$.

At the zero time moment the exact values of G_i are unknown, it is only possible to estimate their means.

The behaviour of the insurer

The insurer tells the client the rules, according to that the penalty scheme X is going to be built during the policy due period. The scheme X is constructed as follows:

x_i, $i = \overline{1,n}$, is defined at the moment $i-1$ and depends on x_{i-1} like that:

$$x_i = \frac{x_{i-1} *_{i-1} V + P + \varepsilon_i}{\alpha * b^1_{i-1} *_i V} - \frac{\varepsilon_{i+1}}{_iV}. \tag{4}$$

The insurer tells the client the scheme $\varepsilon = (\varepsilon_i)$, $i = \overline{1, m}$, and the scheme X is built from ε. The choice of ε is a strategy of the insurer. The values ε_i are defined at the policy start, and they are known to the client at the time of policy signing.

Let us assume that either $\varepsilon_n < -\varepsilon_0 < 0$, or $\varepsilon_n > \varepsilon_0 > 0$, where $\varepsilon_0 = const$. Also, let us assume that by definition $\varepsilon_{m+1} = 0$.

Theorem 1. *If the penalty scheme X is constructed according to the system of ε using equations (4), then the policyholder, if acting according to the equations (3), will cancel the policy at the end of the year n, if $\varepsilon_{n+1} \le \varepsilon_{n+2} \le \varepsilon_{n+3} \le \ldots \le \varepsilon_m < 0$, and $\varepsilon_i > 0$, $\forall i = \overline{1, n}$. The policy will not be cancelled before the due time if $\varepsilon_i > 0$, $\forall i = \overline{1, n}$.*

Theorem 2. *If the penalty scheme X is constructed by the scheme ε according to the equations (4), then $G(k+1) > G(k)$.*

The gain of the insurance company

The strategy of the insurer is the selection of the scheme ε that defines the policy cancellation year. Each $|\varepsilon_i| \ge \varepsilon_0 > 0$.

According to the selected scheme ε the penalty scheme X is constructed during the policy due period according to the equations (4).

The insurer, by selecting the scheme ε at zero time moment, is trying to maximize the mean of his payoff function $F(k)$, where k is a policy cancellation year. The selection of the scheme ε defines the policy cancellation year $n(\varepsilon)$.

The mean of the insurer payoff function looks like follows

$$\begin{aligned} E_0F(x,n) = E_0\ [&\frac{P}{b^1_0 * \ldots * b^1_{m-1}} + \frac{P}{b^1_1 * \ldots * b^1_{m-1}} + \ldots + \\ &\frac{P}{b^1_{n-1} * \ldots * b^1_{m-1}} - \frac{x_n *_n V}{b^1_n * \ldots * b^1_{m-1}}. \end{aligned} \tag{5}$$

We assume that there is no arbitrage on the market, so it means that $E_0[b^1_0 * b^1_1 * \ldots * b^1_m - 1] = b^m_0$. Using this assumption we can derive that the payoff function can be represented as follows:

$$E_0F(x(\varepsilon),n) = \frac{\varepsilon_{n+1} * b_0^n}{b_0^m} + \frac{b_0^{n-1}}{b_0^m} * (P - \frac{P}{\alpha}) + \frac{b_0^{n-2}}{b_0^m} * (P - \frac{P}{\alpha^2}) +$$
$$+ \ldots + \frac{b_0^1}{b_0^m} * (P - \frac{P}{\alpha^{n-1}}) + \frac{1}{b_0^m} * (P - \frac{P}{\alpha^n} - \frac{\varepsilon_1}{\alpha^n}). \tag{6}$$

This value can be calculated at the zero time moment, because all values b_0^k are known at that time.

Construction of the optimal scheme ε

Since ε defines the policy cancellation year, then $E_0F(x,n) \to \max\limits_x$ is equal to:

$$E_0F(\varepsilon, n(\varepsilon)) \to \max_{\varepsilon}. \tag{7}$$

This problem is solved in two steps. First, we find the maximum of the mean of the payoff function in the each of the subsets $E_n = \{\varepsilon : \varepsilon_i > 0,\ \forall i = \overline{1,n},\ \varepsilon_{n+1} \le \varepsilon_{n+2} \le \ldots \le \varepsilon_m < 0\}$ — the sets of strategies ε, such that the policy is cancelled at the end of year n, and then we find the maximum from all possible strategies ε, by comparing the maximums in each subset E_n together.

Theorem 3. *If the penalty scheme X is constructed from the scheme ε using the equations (4) and $\varepsilon \in E_n$, than the gain of the insurer is maximized if he uses the strategy*
$\varepsilon_1 = \varepsilon_0,\ \varepsilon_i \ge \varepsilon_0,\ \forall i = \overline{2,n}, \varepsilon_{n+1} = \varepsilon_{n+2} = \ldots = \varepsilon_m = -\varepsilon_0.$

Theorem 4. *The insurer's payoff function is maximized if he uses the strategy*

$$\varepsilon_1 = \varepsilon_0, \varepsilon_i \ge \varepsilon_0,\ \ i = \overline{2,m}, \varepsilon_0 < \frac{P * (\frac{1}{\alpha^2} - \frac{1 - b_0^1}{\alpha} - b_0^1)}{b_0^2 - b_0^1 + \frac{1}{\alpha^2} - \frac{1}{\alpha}}. \tag{8}$$

The existence of the optimal solution of the problem

Let us call the solution a possible solution, if the gain of the policyholder in this solution is greater than zero. The possible solution with the highest gain of the insurer we shall call the optimal solution. The problem (7) with the selection of the optimal strategy according to the theorem 4 is equal to the following:
$F(m) \to \max\limits_{\varepsilon_0}$, where $\varepsilon_0 > 0$.

The optimal solution of this problem exists if the set of possible solutions is not empty, i.e. the set of possible values ε_0 is not empty.

Theorem 5. *The set of possible solutions for the problem is not empty.*

Thus we obtained insurer's optimal strategy defined by equations (4), (8) which allows to manage policyholders behaviour and maximize own profit.

Two-phase Method for Approximating the Edgeworth-Pareto Hull for Non-linear Models*

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key words:*multi-criteria optimization, non-linear models*

A design process consists of several phases. At the first phase, the conceptual design, multiple requirements on the quality of the design should be taken into account. Requirements in such aspects as technical perfection, economic efficiency, environmental acceptability, investment availability, etc., usually result in a conflict. It is needed to find feasible design parameters that result in a balanced combination of performance characteristics. Mathematical models can be used for describing the feasibility and outcomes of various combinations of design parameters (decision variables of the models). Multicriteria decision support tools can be used for assistance in the conceptual design process in the case of conflicting requirements.

Graphic techniques based on interactive visualization of the Pareto frontier [1] turned out to be an effective tool for decision support in the case of three to seven decision criteria. Information on the Pareto frontier supports direct identification of a preferred combination of criterion values. Then, the computer provides an associated decision. However, complications related to approximation of the Pareto

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frontier for non-linear models, usually given in a form of a black box (computational module), hinder a broad use of such a technique. Here we describe a new effective approach to approximation of the Pareto frontier for non-linear models, which is based on application of random search, local optimization and statistical testing the quality of the approximation.

Consider the problem of the following form:

$$\text{minimize} \quad f(\mathbf{u}) \text{ while } \mathbf{u} \in U$$

with *vector objective function* $f : \mathbf{R}^n \to \mathbf{R}^\mathbf{m}$, which is assumed to be continuous. The *feasible set*

$$U = \{\mathbf{u} \in \mathbf{R}^n \mid g_j(\mathbf{u}) \leq 0 \text{ for } j = 1, 2, \dots, J, \ \mathbf{u}^{lo} \leq \mathbf{x} \leq \mathbf{u}^{up}\}$$

is assumed to be nonempty and compact. Criterion vectors $\mathbf{z} = \mathbf{f}(\mathbf{u})$ for $\mathbf{u} \in U$ form the variety of feasible criterion vectors, that is, a *feasible criterion set* $Z = \mathbf{f}(U) \subset \mathbf{R}^m$.

Here, Pareto optimality is employed as the concept of optimality. A criterion vector $\mathbf{z}^* \in Z$ is said to be Pareto optimal (nondominated) if there does not exist a criterion vector $\mathbf{z}^{**} \in Z$ such that $z_i^{**} \leq z_i^*$ for all $i = 1, \dots, m$ and $\mathbf{z}^{**}$ is not equal $\mathbf{z}^*$. The variety $P(Z)$ of Pareto optimal criterion vectors is usually denoted as the *Pareto frontier* of the feasible criterion set Z. The *Edgeworth-Pareto Hull* (EPH) of Z is defined as $Z^* = Z + \mathbf{R}_+^m$, where $\mathbf{R}_+^m$ is the nonnegative orthant of the criterion space $\mathbf{R}^m$. It is important that the Pareto frontiers of the sets Z and Z^* coincide.

In this paper, we use an approach to nonlinear multicriteria problems, which is based on the approximation of the EPH [1]. We assume that some metric $d(\mathbf{p}, \mathbf{q})$ is introduced in $\mathbf{R}^m$. The EPH is approximated by the union of a finite number of cones $\mathbf{z} + \mathbf{R}_+^m$ with vertices located in criterion vectors $\mathbf{z} \in Z$ in the vicinity of the Pareto frontier. The variety T of such vectors is denoted as an *approximation base*. The union of cones $\mathbf{z} + \mathbf{R}_+^m$ with $\mathbf{z} \in T$ is actually the EPH of the approximation base, and so it is denoted by T^*. It is clear that the approximation set T^* belongs to Z^*. Therefore, the problem consists in selecting a relatively small number of criterion vectors $\mathbf{z} \in Z$, for which EPH is close to Z^*. The first idea how to solve this problem was introduced in [2].

The method introduced in this paper applies hybridization of random search with local optimization, which is a standard idea in global nonlinear single-criterion optimization [3]. The most important feature of our method is associated with the opportunity to test the quality of the current approximation of the EPH and to refine the approximation step-by-step. Traditionally, the quality of an approximation of the Pareto frontier is estimated on the basis of information about the Lipschitz constants of the criteria [4]. However, such estimates are very rough because of the roughness of the knowledge on Lipschitz constants. We apply several statistical tests of the approximation quality, which can be carried out without any knowledge on Lipschitz constants.

The most efficient test is based on the concept of *optimization-based completeness* of an approximation T^*, which is defined as

$$\eta_\varphi(\varepsilon) = Pr\{\mathbf{u} \in U \Rightarrow \mathbf{f}(\varphi(\mathbf{u})) \in (T^*)_\varepsilon\},$$

where $(T^*)_\varepsilon$ is the ε-vicinity of the set T^* and $\varphi(\mathbf{u})$ is the result of local scalar optimization of a specially developed objective function. The optimization process starts from a random point $u \in U$. The value of $\eta_\varphi(\varepsilon)$ equals one if the set $(T^*)_\varepsilon$ contains Z^*, and it is less than one in the opposite case. It turned out to be effective to use functions based on Chebycheff and on Germeier convolutions of criteria for scalar objective functions.

To estimate the optimization-based completeness of an approximation T^*, one has to generate a sample H_N, that is, N random uniformly distributed points from U and solve local optimization problems. Let $n(\varepsilon)$ be the number of such vectors $\mathbf{f}(\varphi(\mathbf{u}))$, that $\mathbf{f}(\varphi(\mathbf{u})) \in (T^*)_\varepsilon$. The statistics $\eta_\varphi^N = n(\varepsilon)/N$ is an unbiased estimate of the value of η_φ for a given ε. Moreover, it is valid [1] that $Pr\{\eta_\varphi > \eta_\varphi^N - \Delta\} \geq \chi$, where $\Delta = \sqrt{-\ln(1-\chi)/2N}$. The last relation helps to estimate the required size of the sample H_N depending on a confidence level χ and the value of Δ. Thus, a sufficient number N can be found as the minimal integer that satisfies $N \geq -\ln(1-\chi)/2(\Delta)^2$.

The approximation method consists of iterations, any of which is split into two steps. Let us consider the k-th iteration. Before it starts, an approximation base T_{k-1} must be given.

The aim of the first step is to test the quality of the approximation T^*_{k-1} and to decide whether it has a sense to stop the approximation process. Automatic stopping rules or human decision can be used. First, the approximation quality is evaluated using one or several of statistical tests. For example, the value of optimization-based completeness can be estimated. In the case of automatic stopping rules, the condition $\eta_\varphi(\varepsilon^*) < \eta^*$ is checked. Here $\varepsilon^* > 0$ and $0 < \eta^* < 1$ are some values specified by an expert in advance. In human decision process, an expert analyses the graph $\eta_\varphi(\varepsilon)$ for $\varepsilon > 0$. If the result of the testing satisfies the expert (or stopping rules are satisfied), the main part of the approximation process is completed. Then, one proceeds to the final iteration. In the opposite case, one has to proceed to the second step.

The aim of the second step is to construct a new improved approximation base T_k. The intehrated list is constructed. It contains the criterion points of T_{k-1} and the criterion points found in the process of the quality testing. The new approximation base T_k is then obtained by exclusion of dominated criterion points and filtration of the list to avoid close points. Then, the next iteration is started.

The final iteration consists in the final improvement of the approximation base by applying heuristic procedures. If needed, the quality of the final approximation can be estimated by using the quality tests, too.

The software that implements this method [5] can be interfaced with any nonlinear model given in the form of a black box. Due to such a feature, the software can be used for approximating the EPH and subsequent visualization of the Pareto frontier for any FEM, FDM or simulation module. For example, it was used in the framework of multicriteria search for a design of cooling equipment used in the process of continuous steel casting [6].

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Structural Properties and Methods of Finding Decisions in Optimization and Game-theoretical Resources Allocation Problem

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This paper is devoted to investigation of structural properties of the optimal strategies of players in the resources problem and to construction of the methods of solving large-scale linear programms. It is shown that considered non-cooperative n-person games can be reduced to the games on the product of convex sets and then to the square matrix games of very large dimension. So we get the linear programming problem with very large both numbers of variables and constraints. It is shown that these problems have such very important property – the rank of restriction matrices significantly smaller than their numbers of rows and columns.

We shall construct an algorithm for efficient solving linear programming problems with small rank. This constructed method is the generalization of known column generation technique in the simplex-method for problems with large both numbers of variables and constraints and with non-full rank matrices.

Then we shall discuss important particular cases of problems which can be solved by means of the proposed method.

Let us consider a non-cooperative n-person game , in which each player has some amount of resources (player i has K_i units of resources). The player allocates his/her resources to t places. The pure strategy of player i is a t-vector $x^i = (x^i_1, \dots, x^i_t)$, $x_\nu \in 0; K_i$,

$\sum_{\nu=1}^{t} x_v^i = K_i$. The component x_v^i is the amount of resources which player i allocates to place ν.

Let X_i be the set of pure strategies of player i. The game for players i and j on place ν is a matrix game with $(K_i + 1) \times (K_j + 1)$ payoff matrix P^{ij} and the total payoff of player i is the sum of his payoffs at all places in games with the other players. Game , is a separable zero sum game. Player i has $\left(\begin{array}{c} K_i + t - 1 \\ t - 1 \end{array} \right)$ pure strategies.

Let S^i be the set of mixed strategies for players i and $S = \prod_{i=1}^{n} S^i$. The aim is to find a Nash equilibrium point. Let us consider game ,$'$ – mixed extension of the game , . It is clear that the game ,$'$ is equivalent to , .

Let us consider two person zero-sum game Δ, in which the sets of strategies for both players are S.

$$\Phi(s, t) = \sum_{i=1}^{n} H_i(t||s^i)$$

(i) Game Δ is fair game.

(ii) Game Δ is equivalent to ,$'$.

The game Δ is a polyhedral game. The game Δ can be converted into a matrix game, we obtain the matrix game Δ' with a square matrix A the rank of which is significantly smaller than its size.

We introduce matrices B and H.

Proposition. Matrix A has the following representation: $A = BHB^T$. This representation implies that the rank r of matrix A is not greater than $\sum_{i=1}^{n} \left(\begin{array}{c} K_i + t - 1 \\ t - 1 \end{array} \right)$.

Matrix H can be represented in the form $H = RGR^T$.

Hence matrix A can be represented as a product of the matrices: $A = BRGR^T B^T$. The rank of A $r \leq t(\sum_{i=1}^{n} K_i + n)$. So we have to find optimal strategies for a matrix game with $(m \times m)$ matrix A, where

$$m = \prod_{i=1}^{n} \left(\begin{array}{c} K_i + t - 1 \\ t - 1 \end{array} \right).$$

Represent A in the product form: $A = PQ$, where $P = BRG$ and $Q = R^T B^T$.

The value of the game is 0, therefore the game is equivalent to the following feasibility problem:

$$\begin{aligned} Ay &\leq 0 \\ y &\in S_m, \end{aligned}$$

where S_m is simplex $S_m = \{y \in R^m : \sum_{j=1}^{n} y_j = 1, y_i \geq 0\}$.

The method operates with matrices and arrays of order not great than r. We define a set Ω as $\Omega = \{z \in R^r : z = Q_y, y \in S_m\}$. Since Ω is a polyhedron, there exists a matrix S and vector s such that $\Omega = \{z \in R^r : Sz \leq s\}$. We can write down the following problem:

$$\begin{aligned} Pz &\leq 0, \\ Sz &\leq s, \end{aligned}$$

which is equivalent to the previous system. If we have vector z which satisfies to this system, then we can construct vector y, the solution of the first system, and then we can receive required Nash equilibrium of the game , .

By solving this problem with the simplex method it is necessary to use repeatedly column generation technique. It is also necessary to solve some problem of finding extremum of linear function on discrete sets. These problem can be solved by means of recurrent relations of the dynamic programming. This method can be used for solution of separable non-cooperative zero-sum games, polymatrix games and games with degenerate payoff functions. This method was used for investigation of multistage resources allocation games.

Together with original game , we consider game , $_2$, in which one of the players is separated and the other players are united and form the second player.

The conception of equivalence of mixed strategies of the player is introduced. It is proved that the set of mixed strategies of a player is divided into classes of equivalence. Every class of equivalence contains mixed strategies possessing of special properties. In particular, there exist the equilibrium strategies possessing on the indicated properties in these games.

Economic-mathematical model in which it is necessary to find the competitive equilibrium of economic system with competition is considered. Solution of this problem gives us the competitive equilibrium in the system with k firms. This linear program can be solved by means of proposed generalization of column generation technique in the simplex-method.

Research of Relationship Between Economic Growth and Social Disparity: a Model with Heterogeneous Consumers

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key words: *economic growth, wealth distribution, heterogeneous consumers*

Considerable recent attention has been focussed on the relationship between inequality and economic growth, in particular, on the dependence of the dynamics of growth on distribution of wealth. In this paper, we study this dependence in a framework of an endogenous growth model with the concave consumption function. Individuals have identical behavioural equations but own different amounts of wealth.

We show that the set of equilibrium steady-state growth rates is an interval. Then we note that when they exist, unegalitarian equilibria are characterized by higher rates of growth than egalitarian ones and, moreover, higher equilibrium growth rates correspond to higher levels of inequality. Also we prove that each path converges either to an egalitarian or to one of unegalitarian equilibria. To what equilibrium a path converges depends on the initial distribution of wealth and the level of capital per capita. The model explains Pasinetti-type class savings behaviour endogeneously.

Introduction

In this work we study the relationship between social inequality and economic growth, in particular, we concentrate on the dependence of the dynamics of growth on distribution of wealth. We explore this dependence in a framework of an endogenous growth model with the concave consumption function (or, equivalently, the convex saving function). The assumption that the consumption function is concave dates back to Keynes who wrote that "...with the growth in wealth [comes] the diminishing marginal propensity to consume..." (Keynes, 1936, p.349). Empirical evidence (see, e.g., Lusardi (1996)) shows that the marginal propensity to consume is substantially higher for consumers with low wealth or low income than for consumers with high wealth or income. As was noticed by Stiglitz (1969), in the case of exogenous growth, if the saving function is convex, the distribution of income and wealth might tend toward a "two-class" equilibrium but his analysis was not detailed. A more detailed analysis of the convex saving function was proposed by Schlicht (1975) who showed that unegalitarian as well as egalitarian equilibria might be locally stable. In Bourguignon (1981) the welfare implications of the coexistence of egalitarian and unegalitarian stable equilibria were considered; it was proved that, when they exist, unegalitarian locally stable equilibria are Pareto superior to egalitarian ones. In this work, we combine the assumption that the consumption function is concave with an AK production function. Following Frankel (1962) and Romer (1986) we assume that technological knowledge grows automatically with capital.

We show that the set of equilibrium steady-state growth rates is an interval. Then we note that when they exist, unegalitarian equilibria are characterized by higher rates of growth than egalitarian ones and, moreover, higher equilibrium growth rates correspond to higher levels of inequality. Also we prove that each path converges either to an egalitarian or to one of unegalitarian equilibria. To what equilibrium a path converges depends on the initial distribution of wealth.

The Model

Two factors of production, capital K and effective labour L, are used to produce a single good according to a neoclassical production function $F(K, L)$. Capital does not depreciate, we assume Harrod-neutral technological progress and the population is constant over time. As for the state of technology, we assume following Frankel (1962) that it is proportional to the current economy-wide average of physical capital per worker. Therefore interest rate r, wage w and the amount of capital per unit of effective labour k are constant over time.

Suppose that at time $t = 0$ the population is divided into N different groups and groups differ only by their wealth holdings (calculated per unit of effective labour of the group). Let $0 < \alpha^j < 1$ denote the fraction of group j in the population.

Each group makes the consumption decision by means of an increasing a strictly concave consumption function $c(\cdot)\colon R_+ \to R_+$, which establishes the relationship between the wealth of a group per unit of effective labour and consumption per unit of effective labour. We denote by z_t the savings of the group per unit of effective labour of the group.

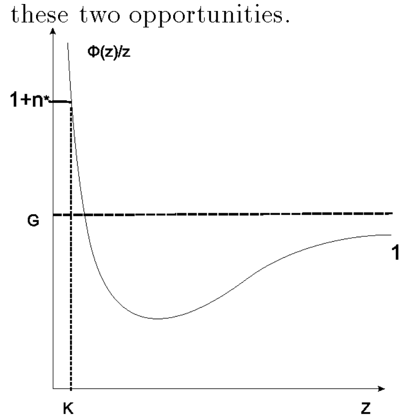
The function $\varphi(z)$ represents the relationship between savings of the groups in adjoining periods. For the group under consideration we get an equation $(1+n_t)z_{t+1} = \varphi(z_t)$, where by n_t we denote the growth rate of the economy, which is determined endogenously in the model.

Here is the system of equations, describing the dynamics of our model: for $t = 0, 1, \ldots,$

$$\begin{cases} k &= \displaystyle\sum_{j=1}^{N} \alpha_j z_t^j \\ z_{t+1}^j &= k \dfrac{\varphi(z_t^j)}{\sum_{i=1}^{N} \alpha_i \varphi(z_t^i)} \qquad j = 1, \ldots, N \end{cases}$$

Then we investigate steady-state equilibria and asymptotics of this system.

We prove that each group can find itself at one of two possible steady-state positions and the population can split at most into two families: spenders and savers, who are wealthier than spenders (per unit of effective labour). We also give the condition on k under which the equilibrium is dividing or non-dividing.Fig.1 and Fig.2. illustrate these two opportunities.



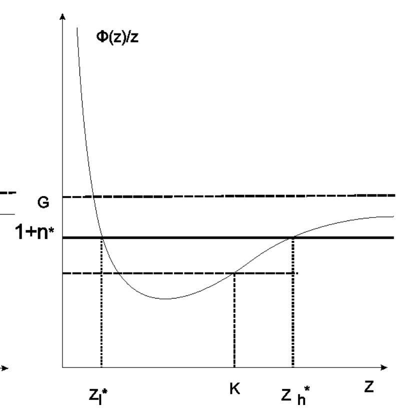


Fig.1 Non-dividing equilibrium Fig.2 Dividing equilibrium

We define an "index of inequality" in the economy and show that the growth rate of economy is related to inequality in the economy: the higher is the growth rate, the higher is inequality.

Then we study the asymptotics of the model and prove that each path $\{z_t^j\}_{j=1}^{N}{}_{t=0}^{\infty}$ converges to a dividing or non-dividing equilibrium. In the last proposition we give some conditions on k, that are sufficient for the stability of the dividing and non-dividing equilibria.

We also obtain that in any dividing equilibrium only the members of the wealthiest group (with the highest z_0^j) become savers and the others become spenders without dependence on their initial conditions.

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Social Conformity in the Behavior of Russian Voters

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Social conformity

It is common knowledge that one of the main benefits of democratic voting system is selection of alternatives favorable to the majority, provided all members of a society can make sensible decisions, choosing a variant that would benefit them most, or voting for a representative, whose political platform matches their views best.

But as one looks closer at how voting decisions are made, she may find that rationality is not their necessary attribute. Social conformity, defined as alignment of people's thinking or behavior with a societal or group norm is a widespread occurrence that provides an example for the rational choice theory failure. Substantial evidence for interpersonal conformity has been provided by social-psychological research. Such effects were also observed outside of small group situations. By Cialdini's principle of social proof, people tend to view behavior as correct to the degree that they see others doing it; when more people are doing something, additional people will do the same thing. People responsive to peer pressure may internalize social norm due to a threat of shame or embarrassment by society.

Social conformity in voting

Existence of social conformity in voting is somewhat alternative to the traditional rationalist theories (Aleskerov and Orteshook, 1995), where a person decides to vote when expected benefits of voting exceed its costs. A social conformity hypothesis suggests that people not just act in their narrow self-interest, but make decisions as members of a society, taking into account collective interests and public opinion.

Social conformity should be incorporated in voting analysis when there are reasons to believe that voting is attributed significantly to group or interpersonal pressure or when people tend to vote because of a widespread belief that voting is a civic duty. Evidence of voting as a response to a social norm has been reported several times during last decade, for instance, in Knack and Kropf, 1998.

The most popular evidence for conformity in voting for political parties is bandwagon effect, when people vote in line with what they perceive would be the choice of majority. The study of such effects, reported many times and even produced experimentally, suggests that group pressure and interpersonal conformity, may as well have an important impact on the election results, although sometimes in a less straightforward way. From the behavioral voting studies it is known that voter's choice is often correlated with his identification or affiliation with political parties, social groups or class; often voters tend to vote like their surroundings - friends, coworkers or in accordance with family traditions. Such choices would have little to do with rationality, provided these groups do vote alike due solely to peer pressure and not because of actually same interests. Recent research on political communication (Kenny 1998) suggests the former is quite likely.

A hypothesis about conformist behavior in voting

Russia is the country with long history of conformity-shaped voting behavior. In the Communist era unanimous votes were not uncommon and dissidence was often persecuted. So, a habit for conformity might still affect voting behavior to a substantial degree, and one might expect a positive outcome of a conformity test proposed by Coleman (2004). This test is based on several assumptions.

First, people who are conformists form a significant share of population, and extract the information about whether a norm is generally recognized by the society by observing relative frequency (or "predictability") of behavior of their neighbors. That is, the more the share of people who demonstrate a certain behavioral pattern among one conformist's surroundings, the more is his intention to behave likewise. To measure "predictability" of behavior as an indicator of conformity, Shannon's entropy measure is used. Given voter turnout, one can retrospectively calculate turnout entropy, and from the fractions of total vote for each of the parties or candidates — the entropy of voting behavior in this decision.

Second, conformist voters have similar information about election probabilities before an election (though it may less accurately represent society as a whole) and behave consistently in their two voting choices (vote or abstain and party choice). The latter implies positive correlation, if not proportionality in changes of the two entropy measures mentioned above.

Statistical analysis

The relation between turnout entropy and turnout is nonlinear and graphically turns out to be nearly parabolic, entropy being highest at 50% turnout. An assumption about proportionality of the two entropy measures yields that party entropy should also be parabolic, stretching upwards as the number of alternatives increase. Thus, the main assumptions that were empirically tested are that party entropy, regressed on turnout and squared turnout yield a parabola that opens downward, with a confidence interval for its maximum to at least include 50% turnout. A more complicated case, when a relationship is given by shifted parabola (with expected maximum at other than 50% turnout) can also be studied using non-linear regression models, although this is not the case with Russian election data.

In most cases clear and statistically significant relation is observed using linear model, suggesting strong conformist pattern in Russian voter's behavior.

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Moving Pareto Frontier for Dynamic Models *

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key words: *multicriteria dynamic optimization*

Animated Pareto frontier is a modern tool for on-line visualization of the Pareto frontier in multicriteria optimization problems [1, 2]. Usually, such an animation supports interactive human exploration of the criterion tradeoffs in the case of more than three criteria. The interactive visualization of the multicriteria Pareto frontier is based on the possible by the preliminary approximation of the Pareto frontier [1, 2]. In this paper, we describe a further development of the idea — animation of the Pareto frontier is used for decision support in the case of dynamic multicriteria problems.

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Let us consider a controlled system of differential equations

$$dx/dt = f(x(t), u(t), t), t \in [0, T], \tag{1}$$

where $u \in R^r$ are control variables, $x \in R^n$ are state variables, $f(x, u, t)$ is a given vector function.

Let us assume that constraints are imposed on values of the control and state variables

$$g(x(t), u(t), t) \leq 0, t \in [0, T], \tag{2}$$

where $g(x, u, t)$ is a given vector function.

Finally, the initial state $x(0)$ is assumed to belong to a given set $, (0)$ from R^n:

$$x(0) \in , (0). \tag{3}$$

By $, (\theta)$ we denote the reachable set for the time-moment $\theta \in [0, T]$, that is, the variety of the states, which can be reached by the system (1)-(3) precisely at a time-moment θ.

Let us consider a multicriteria decision problem with the criterion vector z related to the state x by a mapping

$$z = F(x). \tag{4}$$

It maps the states of the system into the linear criterion space R^m. We assume that the number of criteria is from three to seven. The set $Z(\theta) = f(, (\theta))$ that describes criterion vectors feasible at the time moment θ, is denoted by the feasible criterion set (FCS) for the time-moment θ.

It is assumed that, for any time-moment $\theta \in [0, T]$, decision maker is interested in minimization of all coordinates of the criterion vector z, that is, a criterion vector z^{**} dominates z^* if $z_i^{**} \leq z_i^*$ for all $i = 1, \ldots, m$ and z^{**} is not equal z^*. A criterion vector $z^* \in Z(\theta)$ is said to be Pareto optimal (nondominated) if there does not exist any criterion vector $z^{**} \in Z(\theta)$ that dominates z^*. The variety $P(Z(\theta))$ of Pareto optimal criterion vectors is denoted as the *Pareto frontier* of the feasible criterion set $Z(\theta)$. Since the feasible criterion set $Z(\theta)$ depends on time, the Pareto frontier $P(Z(\theta))$ depends on time, too.

In [1, 2], the Pareto frontier for a static model is visualized with the help of the Interactive Decision Maps (IDM) technique, in the framework of which the *Edgeworth-Pareto Hull* (EPH) of the FCS is approximated in advance, before the interactive visualization can start. In the case of multicriteria minimization problem, the EPH is defined as

$$Z^* = Z + \mathbf{R}^m_+,$$

where $\mathbf{R}^m_+$ is the non-negative orthant of the criterion space $\mathbf{R}^m$. It is important that the Pareto frontiers of the sets Z and Z^* coincide. The IDM technique provides an opportunity to display the Pareto frontiers on-line as frontiers of various collections of two-criterion slices of the EPH (decision maps). After the exploration of the Pareto frontier is completed, decision maker identifies a goal. The associated decision is found then by the computer. The IDM technique proved to be effective in both linear and non-linear cases [1-3].

In the case of the dynamic multi-criteria model (1)-(4), decision maker first of all has to identify a preferred time-moment $\theta \in [0, T]$ and only then a goal point z^* that belongs to the set $Z(\theta) = f(, (\theta))$. To support such an identification process, we use animation for displaying time dependence of the decision maps, which are generated by the IDM technique. By this decision maker is informed on the Pareto frontier $P(Z(\theta))$.

In this paper, we implement the idea for the case of a linear differential system (1) with convex compact constraints (2) imposed on the control only and convex compact initial set (3). The mapping (4) can still be nonlinear. Therefore, the differential system now has the form

$$dx/dt = A(t)x(t) + u(t), t \in [0, T], \tag{5}$$

where $A(t)$ is a given matrix. The constraints imposed on value of the control now have the form

$$u(t) \in U(t), t \in [0, T], \tag{6}$$

where $U(t)$ is a given convex compact set from R^r. Finally, the initial state $x(0)$ has still the same form

$$x(0) \in , (0), \tag{7}$$

but the set , (0) is now compact and convex.

To implement our idea in the form of a numerical procedure, we split the time period $[0, T]$ into a sufficiently large number N of steps by points $k = 0, 1, \ldots, N$ and consider a multi-step linear approximation of (5)-(7). In the framework of such an approximation, the differential equation is substituted for multi-step equation and compact convex sets are approximated by polytopes. Reachable sets $,_k$

for a multi-step linear system can be constructed using methods described in [1,2]. Then, the EPH Z_k^* can be approximated for any moment $k = 0, 1, \dots, N$ by the methods for approximation of the EPH for non-linear systems [3].

After approximation of the sets $Z_k^*, k = 0, 1, \dots, N$ is completed, human exploration of the problem can start. Decision maker specifies a certain decision map, that is, allocates criteria among different possible positions – on axes, at scroll-bars. Then, decision maps for such criterion allocation are constructed for all time-moments $k = 0, 1, \dots, N$ and displayed one after another. Such a display results in animation, which informs decision maker on dynamics of the decision maps and helps to choose an appropriate time-moment.

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On the Theory of Cooperative Games

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In the classical theory of cooperative games (TU games), an optimality principle is usually interpreted as a mapping, which assigns to any game (or any characteristic function) from a given set some nonempty subset of set of imputations in this game. However an optimality principle may also be interpreted as a mapping of a given set of games into itself. For all that, the new definition quite conforms to the previous one. They correlate with each other like concept of convex set and its reference function. At the same time, the new definition of optimality principle possesses a number of advantages.

John von Neumann and Oskar Morgenstern have long ago noted that the dynamic theory would be undoubtedly more complete and thus more preferable [1]. It may be explained, for instance, by the fact that a compromise is not achieved, in practice, in a moment but is a multistep process of concessions of the interested parties. The new definition of the optimality principle allows to construct the dynamic theory of cooperative games in which the process of restricting the region of compromises in original game is modeled by iterative sequence of games [2]. As this takes place, a transition from one game to another is performed on the basis of the selected optimality principle and it may be interpreted as a revision of coalitions' ambitions for their own part of common income or common profit.

A further advantage of the new definition of the optimality principle is that it allows to model such process of compromise achievement in which the players at different stage use various optimality principles. In this connection, it should be also noted that this new definition allows to construct new optimality principles on the basis of already existing ones by forming various superpositions.

Topical problems of the dynamic theory under consideration for the classical cooperative games which are connected with the concept of optimality principles are discussed in [3, 4].

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The Problem of Fair Bandwidth Sharing in Linear Network

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Let us consider a linear network (fig. 1).

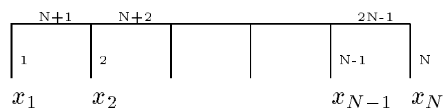

Figure 1: A linear network

The components of the network are:

- N nodes (e.g., workstations or gates to other networks) each of which is used by one of N network users;
- N transit nodes (routers);
- $L = 2N - 1$ links.

Each link $l = 1, \dots, L$ in the network has a capacity c_l.

Pairs of nodes in the network create connections to transfer data. Each connection between nodes $i < j$ uses links i, $i+N$, $i+N+1$, ..., $N+j-2$, $N+j-1$, j. Assume that each node $i = 1, \dots, N$ creates one connection to other node $j \neq i$ with x_i being a quota on amount of data to be transferred. Let $a_{ij} = 1$, if there is a connection between two nodes i and j, ans 0 – else. There is minimal data transfer requirement for each node $i = 1, \dots, N$: $x_i \geq \lambda_i$. Assume that total network capacity lets to satisfy all nodes' minimal reaquirements.

Denote by $T_l(x)$ an average time to pass a unit of data through the link l.

$$T_l(x) = \begin{cases} \left(c_l - x_l - \sum\limits_{p=1}^{N} a_{pl} x_p\right)^{-1} & \text{for } 1 \le l \le N \\ \left(c_l - \sum\limits_{\substack{1 \le p \le l-N, \\ l-N+1 \le q \le N}} (a_{pq} x_p + a_{qp} x_q)\right)^{-1} & \text{for } N+1 \le l \le 2N-1 \end{cases}$$

If connections use all the capacity of some link l, the time to pass a unit of data through this link equals ∞.

For each node $i = 1, \ldots, N$ define the network use utility function:

$$f_i(x) = u_i(x_i) - x_i \sum_{j=1}^{N} a_{ij} \left(T_i(x) + \sum_{l=min(i,j)+N}^{max(i,j)+N-1} T_l(x) + T_j(x) \right),$$

where $u_i(x_i)$ is a profit obtained by node from data transfer.

For each node $i = 1, \ldots, N$ we have to determine its quota on amount of data being transferred, which maximizes its utility function value. For that, according to general fairness criterion [1, 2, 3], the problem of conditional maximization must be solved:

$$\max_x \frac{1}{1-\alpha} \sum_{i=1}^{N} f_i(x)^{1-\alpha}, \alpha \ge 0, \alpha \ne 1,$$

$$\begin{array}{ll} \forall i = 1..N & x_i \ge \lambda_i, \\ \forall l = 1..L & T_l(x) \ge 0. \end{array}$$

Two special cases of the problem are considered: the parallel and the sequential data transfer models. The approximate solution pattern is proposed. Results of numerical experiments demonstrate changing of quotas being allocated to nodes and intended utility function values with variation of fairness parameter α.

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Optimal Control Problem by Discrete Fourier Filter*

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key words: *nonlinear PDE, optimization, Fourier filtering, spatial filtering*

In the present paper, we consider a class of the Fourier filtering models, which are described by initial value problems for nonlinear parabolic PDE. This is the model of the dynamics of additional phase modulation of a light wave passed through a thin layer of a Kerr-type nonlinear medium being a part of an optical feedback system with a Fourier filter in its feedback loop. The theorems of existence, uniqueness and Lipschitz-continuous dependence of initial data and parameters are proved.

We pose the optimization problem in the form of minimization of the terminal functional. The theorems of solvability of the minimization problems on compact and weak compact sets of the Hilbert space ℓ_2 are proved.

For the purpose of solving the minimization problem numerically, the conjugate problem is formulated, and the existence and uniqueness of the solution of the conjugate problem is proved.

Derived formula of gradient of the cost functional allows to perform numerical simulations and to investigate the capabilities and peculiarities of solving wide class of physical tasks in nonlinear optics.

Let $x = (x_1, x_2) \in \Omega$. Here Ω is a convex bounded domain in R^2 with a piecewise-smooth boundary $\partial\Omega \in C^2$, $|\Omega|$ is a Lebesgue measure of Ω; $L(X \to Y)$ is a space of linear bounded operators that map a Banach space X onto a Banach space Y; $H = L^2(\Omega)$ is a separable Hilbert space with a standard inner product $\langle\cdot,\cdot\rangle_H$ and norm $\|\cdot\|_H$, $H^s(\Omega)$ is Sobolev's space of order $s > 0$. We define

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an operator $\mathcal{A}$ as follows: $\mathcal{A}u = u - l_d^2(\partial^2_{x_1x_1}u + \partial^2_{x_2x_2}u)$, $\mathcal{D}(\mathcal{A}) = \{u \in H^2(\Omega) : \gamma(u) = 0\}$, where l_d is a positive constant and, from the physical sense, the boundary operator $\gamma(\cdot)$ is defined by one of the given formulas: $\gamma(u) = u|_{\partial\Omega}$, $\gamma(u) = \partial_\nu u|_{\partial\Omega}$, or the operator $\gamma(\cdot)$ corresponds to the periodical boundary conditions in a rectangle $\Omega = (0, l_1) \times (0, l_2)$. Let $\{e_n(x)\}_{n=1}^{+\infty}$ be an orthonormalized basis for the space H such that $\{e_n(x)\}_{n=1}^{+\infty}$ are the eigenfunctions of the operator $\mathcal{A}$. Let V be an energy space of the operator $\mathcal{A}$ and V^* be a conjugate space to V.

In the same way as in the papers [1], [2], let $\mathcal{F}_\rho(A)$ be a Fourier filtering operator that changes the Fourier series of a complex function $A(x)$ by the basis $\{e_n(x)\}_{n=1}^{+\infty}$ using a discrete filter

$$\rho = (\rho_1, \rho_2, \ldots, \rho_n, \ldots) \in \ell_\infty$$

by the rule:

$$\mathcal{F}_\rho(A)(x) = \sum_{n=1}^{+\infty} \rho_n \langle A, e_n\rangle_H e_n(x).$$

Here ℓ_∞ is the Banach space of bounded complex numeric sequences with the norm $||\rho||_{\ell_\infty} = \sup\limits_{n \in N} |\rho_n|$, and let ℓ_2 be the Hilbert space of bounded complex numeric sequences with the inner product $\langle \rho, \sigma\rangle_{\ell_2}$ $= \sum\limits_{n=1}^{+\infty} \rho_n \sigma_n^*$ and norm $||\rho||_{\ell_2} = \langle \rho, \rho\rangle_{\ell_2}^{1/2}$.

The considered class of the discrete Fourier filtering models is described by the initial value problems for nonlinear parabolic PDE, modelling the dynamics of additional phase modulation $u = u(x, t; \rho)$ of a light wave passed through a thin layer of a Kerr-type nonlinear medium being a part of an optical feedback system with a Fourier filter in its feedback loop [3], [4]:

$$\tau\partial_t u(x,t) + \mathcal{A}u(x,t) = F(u;\rho),\ t > 0, \tag{1}$$

$$u(x,t)|_{t=0} = u_0(x), \quad \gamma(u(x,t)) = 0. \tag{2}$$

Here $\tau > 0$ is the characteristic response time of the nonlinear medium. The function $F(u;\rho)$ is responsible for the specific arrangement of nonlinear optical system and is defined by:

$$F(u;\rho) = K_1|A_{in}|^2 + K_2 \mathrm{Re}\,(A^*_{in} A_{fb}(u;\rho)) + K_3|A_{fb}(u;\rho)|^2, \tag{3}$$

$$A_{fb}(u;\rho) = \mathcal{F}_\rho(A_{in} \exp\{i\,u\}), \tag{4}$$

where complex function $A_{in}(x)$ and real valued parameters K_1, K_2, K_3 are known.

Theorem 1. [2] *Let the initial condition $u_0 \in H$, $A_{in} \in V \cap C(\overline{\Omega})$, $\rho \in \ell_\infty$. Then, for any finite time interval $0 < t < T$, initial value problem (1)-(4) has a unique solution $u \in L^2(0,T;V)$, $\partial_t u \in L^2(0,T;V^*)$, which satisfies equation (1) for almost all $t \in (0,T)$ in the space V^*, and obeys the initial condition in the sense of the space $C([0,T];H)$. Also on any finite time interval $(0,T)$ the solution Lipschitz-continuously depends on initial data and Fourier filter.*

Suppose that an element $\rho_\infty \in \ell_\infty$ is fixed and an operator $\mathcal{B} \in L(H \to H)$ is chosen. Then the optimal control problem by the discrete Fourier filter is formulated in the form of the minimization problem for the terminal functional:

$$\mathcal{J}(\rho) = \|\mathcal{B}\left(u(T;\rho_\infty+\rho) - u_1\right)\|_H^2 \to \inf, \tag{5}$$

where $u(x,T;\rho_\infty+\rho)$ is the solution of initial boundary value problem (1)-(4) at the point of time $t = T$, target function $u_1 \in H$ is given. For example, from the physical sense the operator $\mathcal{B}$ can be chosen as follows: $\mathcal{B}v = v - \overline{v}$, where $\overline{v} = |\Omega|^{-1} \displaystyle\int_\Omega v(x)dx$.

In formula (5), the controlling filter $\rho \in \ell_2$ is choosing from the compact set

$$G_R = \{\rho \in \ell_2 : |\rho_k| \leq \frac{R}{k},\ k = 1,2,\ldots\}, \quad R > 0,$$

which is a well-known "Hilbert cube" from the space ℓ_2, or from the ball in the space ℓ_2

$$B_R = \{\rho \in \ell_2 : \|\rho\|_{\ell_2} \leq R\}, \quad R > 0,$$

which is the weakly compact set in the space ℓ_2.

Theorem 2. *Under the assumptions of theorem 1, for any fixed element $\rho_\infty \in \ell_\infty$ the infimum $\mathcal{J}_* = \inf\limits_{\rho \in G_R} \mathcal{J}(\rho)$ is finite, the set $G_{*R} = \{\rho \in G_R : \mathcal{J}(\rho) = \mathcal{J}_*\}$ is not empty, compact and any sequence $\{\rho_k\}$, minimizing functional $\mathcal{J}(\rho)$, converges to the set G_{*R}.*

Theorem 3. *Under the assumptions of theorem 1 and the assumption that all orthonormalized in H eigenfunctions $\{e_n(x)\}_{n=1}^{+\infty}$ of the operator $\mathcal{A}$ are uniformly by n bounded in the norm of the space $C(\overline{\Omega})$, for any fixed element $\rho_\infty \in \ell_\infty$ the infimum $\mathcal{J}_* = \inf\limits_{\rho \in B_R} \mathcal{J}(\rho)$*

is finite, the set $B_{*R} = \{\rho \in B_R : \mathcal{J}(\rho) = \mathcal{J}_*\}$ is not empty and is weakly compact in the space ℓ_2.

For the purpose of solving the minimization problem numerically, the conjugate problem is formulated:

$$-\tau\partial_t\psi + \mathcal{A}\psi = K_2 \mathrm{Re}\left(A_{in}\, i \exp\{i\,u\}\mathcal{F}^*_{\rho^*_\infty+\rho^*}\left(A_{in}\psi^*\right)\right) + \\ +2K_3 \mathrm{Re}\left(A_{in}\, i \exp\{i\,u\}\mathcal{F}^*_{\rho^*_\infty+\rho^*}\left(A_{fb}(u;\rho_\infty+\rho)\psi^*\right)\right), \qquad (6)$$

$$\psi(x,t)|_{t=T} = 2\mathcal{B}^*\left(\mathcal{B}\left(u(x,T;\rho_\infty+\rho) - u_1(x)\right)\right), \quad \gamma(\psi(x,t)) = 0, \qquad (7)$$

where $u = u(x,t;\rho_\infty+\rho)$ is the solution of direct problem (1)-(4), and in formula (6) symbol "*" denotes complex conjugate value, but in formula (7) symbol "*" is used for denoting the conjugate operator $\mathcal{B}^* \in L(H \to H)$.

Theorem 4. *Under the assumptions of theorem 1, for any discrete filter $\rho_\infty \in \ell_\infty$ and any $\rho \in \ell_2$ initial value problem (6)-(7) has a unique solution $\psi \in L^2(0,T;V)$, $\partial_t\psi \in L^2(0,T;V^*)$, which satisfies equation (6) for almost all $t \in (0,T)$ in the space V^*, and obeys the initial condition in the sense of the space $C([0,T];H)$.*

Theorem 5. *Under the assumptions of theorem 3, for any discrete filter $\rho_\infty \in \ell_\infty$ and any $\rho \in \ell_2$ the terminal functional $\mathcal{J}(\rho)$ is Frechet differentiable in the space ℓ_2, and for any $\delta\rho \in \ell_2$ the gradient of the functional $\mathcal{J}(\rho)$ is determined by the equality:*
$\langle \mathcal{J}'(\rho), \delta\rho\rangle_{\ell_2} =$

$$\tau^{-1}\int_0^T \langle \mathrm{Re}\left(\left(K_2 A^*_{in} + 2K_3 A^*_{fb}(u;\rho_\infty+\rho)\right) A_{fb}(u;\delta\rho)\right), \psi\rangle_H\, dt, \qquad (8)$$

where $u = u(x,t;\rho_\infty+\rho)$ is the solution of direct problem (1)-(4), $\psi(x,t)$ is the solution of conjugate problem (6)-(7).

The gradient of the functional $\mathcal{J}(\rho)$ allows to investigate a wide class of practically interesting problems in adaptive optics.

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A Class of Active-set Newton Methods for Mixed Complementarity Problems*

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key words: *mixed complementarity problem, weak regularity, error bound, Newton method*

We discuss the mixed complementarity problem (MCP), that is, the variational inequality on a generalized box. Basing on the identification of indices active at a solution of MCP, we propose a class of Newton-type methods for which local superlinear convergence holds under extremely mild assumptions. In particular, the error bound condition needed for the identification procedure and the nondegeneracy condition needed for the convergence of the resulting Newton method are individually and collectively strictly weaker than the property of semistability of a solution. Thus the superlinear local convergence conditions of the presented method are weaker than

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conditions required for the semismooth (generalized) Newton methods applied to MCP reformulations through complementarity functions. Moreover, they are also weaker than convergence conditions of the linearization (Josephy–Newton) method. The relations of different regularity conditions are studied. For the special case of optimal systems with primal-dual structure, we further consider the question of superlinear convergence of primal variables. We illustrate our theoretical results with numerical experiments on some specially constructed MCPs whose solutions do not satisfy the usual regularity assumptions. We also discuss a globalization scheme for a class of active-set Newton methods for solving MCP. We describe the basic hybrid globalization scheme and its convergence properties. Numerical experiments on some test problems are presented, including results on the MCPLIB collection.

The *mixed complementarity problem* (MCP) [5] is the variational inequality on a generalized box, that is

$$\text{find } x \in B \text{ such that } \langle F(x), y - x \rangle \geq 0 \text{ for all } y \in B, \tag{1}$$

where $F : \mathbb{R}^n \to \mathbb{R}^n$ and $B = \{x \in \mathbb{R}^n \mid l_i \leq x_i \leq u_i,\ i = 1, \ldots, n\}$, $l_i \in \mathbb{R} \cup \{-\infty\}$, $u_i \in \mathbb{R} \cup \{+\infty\}$, $l_i < u_i$ for all $i = 1, \ldots, n$. Equivalently, it can be stated as

$$\text{find } x \in B \text{ such that } F_i(x) \begin{cases} \geq 0, & \text{if } x_i = l_i, \\ = 0, & \text{if } x_i \in (l_i, u_i), \\ \leq 0, & \text{if } x_i = u_i, \end{cases} \quad i = 1, \ldots, n.$$

It is well known that many important problems can be cast in the format of MCP [6, 5]. As a special case of MCP, we mention the nonlinear complementarity problem (NCP), which corresponds to setting $l_i = 0$, $u_i = +\infty$, $i = 1, \ldots, n$. The systems of nonlinear equations are obtained by choosing $l_i = -\infty$, $u_i = +\infty$, $i = 1, \ldots, n$. Another important example is the primal-dual Karush-Kuhn-Tucker (KKT) optimality system: find $z \in \mathbb{R}^p$ and $\mu \in \mathbb{R}^m$ such that

$$\begin{array}{c} g(z) - (G'(z))^T \mu = 0, \\ \mu \geq 0, \quad G(z) \geq 0, \quad \langle \mu, G(z) \rangle = 0, \end{array} \tag{2}$$

where $g : \mathbb{R}^p \to \mathbb{R}^p$ and $G : \mathbb{R}^p \to \mathbb{R}^m$. The KKT system (2) can be written as an MCP if we set $n = p + m$ and

$$F(x) = \begin{pmatrix} g(z) - (G'(z))^T \mu \\ G(z) \end{pmatrix}, \quad x = (z, \mu) \in \mathbb{R}^p \times \mathbb{R}^m,$$

$l_i = -\infty$, $i = 1, \ldots, p$, $l_i = 0$, $i = p+1, \ldots, n$, $u_i = +\infty$, $i = 1, \ldots, n$. Under well-known assumptions, (2) represents the first-order primal-dual necessary conditions characterizing solutions in variational inequality or constrained optimization problems.

We start with deriving a new error bound (an upper estimate for the distance from a given point to a solution) for MCP based on a smooth reformulation of MCP and a 2-regularity condition [7].

Error bounds have many applications [10], among which is identifying active constraints in constrained optimization [4] (see also [5, Ch. 6.7]). In the context of MCP, those ideas correspond to identifying the sets of indices

$$\begin{aligned} A = A(\bar{x}) &= \{i = 1, \ldots, n \mid F_i(\bar{x}) = 0\}, \\ N = N(\bar{x}) &= \{i = 1, \ldots, n \mid F_i(\bar{x}) \neq 0\}, \\ N_l = N_l(\bar{x}) &= \{i \in N \mid \bar{x}_i = l_i\}, \\ N_u = N_u(\bar{x}) &= \{i \in N \mid \bar{x}_i = u_i\}, \end{aligned}$$

where $\bar{x}$ is some solution of MCP. If the specified sets can be correctly identified using information available at a point x close enough to the solution $\bar{x}$, then locally MCP can be reduced to a system of nonlinear equations (which is structurally much simpler problem to solve). In the sequel, we shall also use the following partitioning of the set of active indices:

$$\begin{aligned} A_0 = A_0(\bar{x}) &= \{i \in A \mid \bar{x}_i = l_i \text{ or } \bar{x}_i = u_i\}, \\ A_+ = A_+(\bar{x}) &= \{i \in A \mid \bar{x}_i \in (l_i, u_i)\}, \\ A_{0l} = A_{0l}(\bar{x}) &= \{i \in A_0 \mid \bar{x}_i = l_i\}, \\ A_{0u} = A_{0u}(\bar{x}) &= \{i \in A_0 \mid \bar{x}_i = u_i\}. \end{aligned}$$

Once all these index sets are identified, MCP (1) locally reduces to the following (overdetermined, in general) system of nonlinear equations:

$$F_A(x_{A_+}, l_{A_{0l} \cup N_l}, u_{A_{0u} \cup N_u}) = 0. \tag{3}$$

We propose a new class of active-set Newton methods for solving MCP, based on solving (3). Each iteration of the method consists of solving one system of linear equations. We note that when specified to the setting of KKT, this class is different from what has been discussed in [8]. Moreover, the nondegeneracy condition that we introduce here is weaker than the corresponding condition in [8]. Also, the new condition permits specific deterministic choice of parameters involved in reducing the MCP to a system of equations, while in [8] in general a generic choice of parameters had to be made (at least without strengthening somewhat the regularity assumptions). We show that the conditions needed for the identification of active sets and for convergence of the proposed local Newton method are weaker than semistability of the MCP solution [2, 5] (equivalently, the R_0-property of the natural residual). This implies, in particular, that the proposed method attains local superlinear/quadratic convergence under assumptions considerably weaker than what is needed for semismooth Newton methods (SNM) for MCP [1, 9] (BD-regularity of the reformulation is used). Even more remarkably, our assumptions are also strictly weaker than those needed for the linearization (Josephy-Newton) method [2] (which are semistability *and* hemistability of the solution). It should be also noted that in the latter methods, subproblems are linearized MCPs, which are computationally more complex than systems of linear equations in our methods. For the specific case of KKT, we consider the issue of superlinear convergence of primal variables, as well as quasi-Newton versions of the method.

We describe some possible scenarios of the *local* behavior of some known Newton-type methods and our method by applying them to some small test problems with various combinations of satisfied and violated regularity properties of the solution. Constructing artificial examples allows us to obtain a rather complete selection of irregular MCPs with "various degrees of irregularity", and to make reliable conclusions about the reasons for the observed performance of the algorithms.

To perform numerical experiments, we had to implement our method as a final stage of some globally convergent scheme. It seems difficult to suggest a globally convergent scheme directly related to the structure of our local algorithm. In some sense, this is a disadvantage. But on the other hand, our local approach can be combined with *any* globally convergent algorithm satisfying some requirements. In fact, the way we suggest to use our active-set method is precisely for improving the local convergence properties of standard algorithms (say, when they run into difficulties because of the lack of regularity of a solution). Our numerical experiments indicate that the resulting strategy fulfills the stated objective.

We report on our numerical experience based on the MCPLIB test problems collection (the newer version of [3]), and on some additional small examples, designed to highlight the case where various standard regularity conditions do not hold, and thus conventional methods may have trouble or converge slowly. This is precisely the case where the switch to our local algorithm can be particularly useful.

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Debt Policy as a Bubble: Scenario Approach

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There are several approaches towards a mathematical description of bubbles in literature. Here the Scenario approach is introduced. The main equations of the model and some tasks formulations are given by S.V. Dubovsky [1].

Model. Here the bubble is considered as the following financial scheme. The Organizer sells its commitments. According to them it engages to pay some concrete amount of money in the future. In this case their fulfillment takes place using only the revenue of new commitments cells [2]. Let us suppose without any loss of generality that the Organizer commitments are zero-coupon bonds.

Notations. Let us suppose that the bubble begins at the moment $t = 0$. $V(t)$ is the bubble Organizer revenue. $g(t)$ is the total face-value of the bubble securities sold exactly at the moment t. $\theta > 0$ is the fixed period of time, after which the moment of commitments fulfillment takes place, and counted off from the moment of their sale (i.e. a period of the bonds run). So, it is supposed that the bubble Organizer securities are bonds without coupon with equal time to run θ counted off from the moment of their sale. $g(t-\theta)$ is the total face-value of the bubble securities issued by the Organizer at the moment $t-\theta$ and retired at the moment t. $c_g(t) \in [0; 1]$ is the price (expressed in parts of the face-value) which is the real price of security sales at the moment t. The Organizer meets its commitments to investors using only the revenue of securities sells within the bubble.

Within the frame of this assumption it is clear that if during some periods of time "too small" amount of the Organizer securities is being sold, at some moment the Organizer won't have enough funds to fulfill its commitments, because the revenue $c_g(t)g(t)$, received by the Organizer at the moment t is less than the obligation $g(t)$ undertaken. Starting from economical meaning of quantities introduced and assumptions made one can write: $\frac{dV}{dt} = \begin{cases} c_g(t)g(t), t < \theta \\ c_g(t)g(t) - g(t-\theta), t \geq \theta \end{cases}$,

$V(0) = 0$.
Some generalizations of the model are introduced in [2].

Main idea. The following idea is introduced by the author (see [2] too). Some kinds of debt management policy may really be the policy of construction of a bubble during some period of time. It may be so deliberately or not. But in any case for an external observer (researcher, analyst) it is important 1) to determine if the policy is really bubble or not and 2) make a forecast of debts as if they are a bubble; because the behavior of the person (organization), who creates his (its) own debts, as if he (it) is a *bubble* Organizer is a *pessimistic* scenario of the future in question.

Main points of the technique. Main points of the technique for investigation of Russian public debts to determine, if they are a bubble or not, are given below. As specific example, Russian zero-coupon short-term state bonds are considered.

1. As an input data were used officially announced information: face values and auction interest rates of Russian GKOs (state short-term bonds; they don't have coupons) for the period of April 1994 — August 1998.

2. Interest rate was recalculated into price, noted in parts of the face-value. Because variance of price was high enough, monthly values of price were used later.

3. To determine the character of the function $g(t)$ several statistical tests were used. Firstly, correlations of $G(t)$ (the outstanding total face-value of the bubble securities in circulation by the moment t) and monomials $t, t^2, \ldots, t^6$ and, secondly, approximation of $G(t)$ as a 6-degree-polynomial were found.

4. After finding of the function $G(t)$ $g(t)$ is determined by the equation:

$$g(t) = \begin{cases} \dfrac{dG(t)}{dt}, t < \theta \\ \dfrac{dG(t)}{dt} + g(t-\theta), t \geq \theta, \end{cases}$$

which is inversed for the equation

$$\frac{dG}{dt} = \begin{cases} g(t), t < \theta \\ g(t) - g(t-\theta), t \geq \theta \end{cases}, G(0) = 0.$$

5. Calculations were made for two different values of θ: 3 and 6 months (this is limits for GKOs lifetime).

Results. It was obtained that Russian GKOs for the period 1994 — August 1998 can be considered as bubble. The time when this bubble became unprofitable is September-November 1997. This result can be considered as good post-estimation.

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The Specificity of Game Theory Problems with Joint Constraints by the Example of Electricity Auction*

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We consider the two-node model of electricity network with transmission constraint between the nodes. Sellers and buyers are located in these two nodes. All sellers compete in the sealed-bid auction to cover the demand. The marginal pricing is established in the auction, i.e. all sellers who win in the auction sell the power for the market-clearing price, which can be determined as the cost of the last loaded MW. The participant with the highest price in the bid for electricity production among those who won in the auction determines the price.

We suppose that the buyers are located in the first node and the sellers are divided and separated between the two nodes. Moreover, the flow on the line between these nodes is limited, that is the joint constraint on the producers' possibilities. We consider the corresponding game involving producers, where we analyze the strategy

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of bidding in the electricity auction. The goal of every participant is to be accepted in the market. When the producer submits bid he takes into account the value of production costs (this value as the price in the bid will be the "honest" participant's strategy). Supposing several variants of price determining (one price for both nodes or personal price for each node) we investigate the question of dominant strategies' existence. It is shown that in the presence of information about the whole demand volume, whole supply volume, the value of capacity limit and the values of production costs of other producers every participant can define his optimum strategy.

Problem of Coordination in Matrix Structure of Management

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key words: *The problem of coordination of decisions in subsystems and centers of multilevel organizational system with matrix management structure is considered*

The problem of coordination of decisions in subsystems and centers of multilevel organizational system with matrix management structure is considered. The results of this report are: - a numerical coordination method for linear multicriterion systems is built; - examples of coordination problem in two-level organizational system with matrix management structure are solved for it's element - triangle management system.

Improving Monetary Aggregates and Economic Growth

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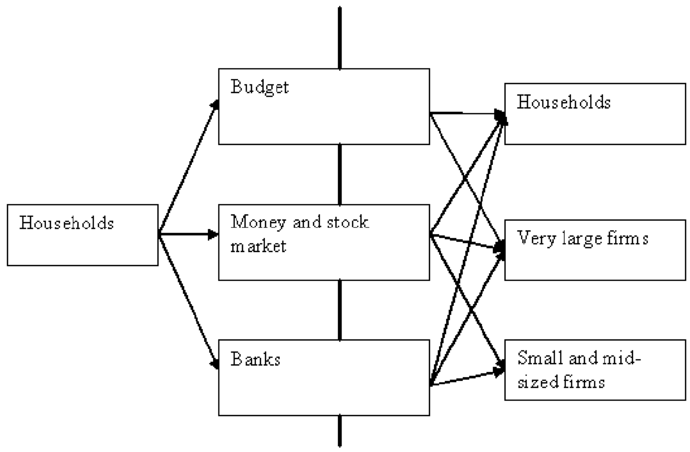
The goal of research is to define the role of structure of monetary aggregates in transition economy (based on Ukrainian, Russian and U.S. experience).

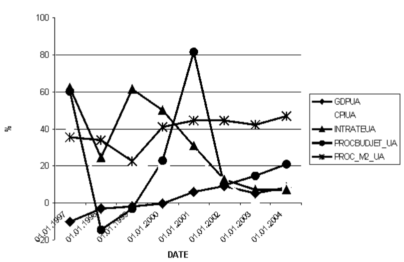
The problem on whether bond and/or equity funds should be added to M2 was discussed in many papers [1-7]. In CIS countries another issue it is more important to check whether the same part of the budget should be added to M2. Analysis of financial flows in economy shows that in transition economies a lion's share of GDP is generated by the budget funds.

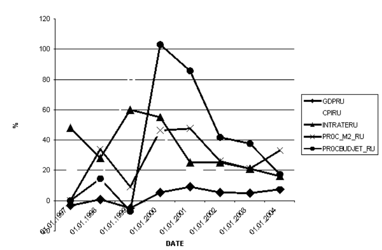


The relevant block on the diagram above is marked with gray color.

IS-LM model is considered to be an insufficient explanation of economic processes in transition economies. Among the reasons are unfinished privatization, high level of dollarization and bulk share of money in shadow economy. Author sees the solution in changing the method of monetary aggregate calculation.

Author introduce the hypothesis, that equation MV=PY should include not M2 aggregate, rather its adjusted number M2+ which will account for those budget expenses, that are disbursed for state subventions. The hypothesis stems from the following diagrams.





Here we use National Bank of Ukraine Bulletin data and Data on

Russia from the Banking Statistics Bulletin (Central Bank of Russia)

Data on Russia are from the Banking Statistics Bulletin (Central Bank of Russia)

The following abbreviations are used: GDP — dynamics of output (%), CPI — consumer price index, INTRATE — Central Bank's main refinancing rate, PROC_M2 —dynamics of monetary aggregate M2 (%), PROCBUDGET — dynamics of budget incomes (%).

It is evident that GDP high growth in Ukraine and Russia resulted from a rapid increase of both income and spending budget items (so this process was one of the main growth factors). Huge amount of financing was provided for the state unprofitable enterprises. To gauge properly the monetary policy in the country, we suggest the introduction of the monetary aggregate M2+, which is equal to M2+k * (budget incomes), where k is constant. Thus, the research is aimed at examining the empirical issues associated with whether such an augmented aggregate, called M2+, would be useful in estimation of monetary policy. Coefficient k whose value is not similar in different transition countries is a good indicator of the level of market relations in the country. It also discloses the part of financing that is allocated through the budget to support domestic production. The above-mentioned increase in both incomes and spending items of the budget allowed Ukrainian and Russian governments to boost the GDP growth without serious inflationary consequences as a result of distorted demand of retail trade sector where the goods and services are bought mainly for cash. We also assume that velocity of money V is constant and k coefficient changes slowly (k = k(t) и dk/dt = 0). Author suggests to consider the curve k(t), where t stands for time. Consider the Fisher equation as follows:

$$d_1(M - P) - d_2 y + d_3 R = \varepsilon_d, \qquad (1)$$

where d_i, $i = 1, 2, 3$, are coefficients, ε_d is stochastic factor, M is money growth, y is real GDP growth, R is central bank's main refinancing rate, and P is price level. To prove our assumptions the regression equations based on (1) are estimated. We consider R^2 to be a good validation of k. Thus the higher R^2 the more reliable is the relationship. The following values have been obtained: k=0.3 for Ukraine, k=0.8 for Russia. It implies that the level of market transformation in Ukraine is higher, than in Russia. One of our expected conclusions is that the real GDP growth in Ukraine and in Russia might not be so fast as currently estimated, rather the virtual GDP growth aimed at state budget financing. Stemming from this, a lot of mechanisms of GDP multiplication can be further introduced: for instance in 1999-2000 the mutual settlements for energy in Ukraine were made through the banks (other solution for GDP increase could have been the installment of cash desks on the bazars). The results allow forecasting the GDP dynamics more precisely. According to author's estimates, GDP in 2004 with account of the planned incomes and expenditures as well as money growth in Ukraine is forecasted to amount of 6.1% (if CPI = 6%, ΔM2 = 40%). Current political regimes in CIS countries make it more difficult to make a more detailed analysis as for example in USA when Enron and Arthur Andersen companies were inspected to disclose excessive non-existing profits.

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Constructing an Economical Description of a Polytope Using the Duality Theory of Convex Sets*

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key words: *multiple criteria optimization; interactive visualization of Pareto frontier*

1.Formulation of the problem. The Feasible Goals Method (FGM) is one of the methods for multiple criteria optimization. It is based on identification of the preferred goal in a graphic display of the Pareto frontier ([1], [2]). To visualize the Pareto frontier for three and more decision criteria, a special technique, called the Interactive Decision Maps (IDM) technique was developed. The main feature of the IDM technique consists in approximation of the variety of feasible criterion vectors (Feasible Criterion Set, FCS) and further interactive visualization of its Pareto frontier. Within the FGM/IDM, FCS of a convex model is approximated by the solution set of a finite linear inequality system. Complexity of the interactive visualization is determined by the number of inequalities in this system. In this connection, the following problem is of interest: constructing a simplified description of the solution set of a finite linear inequality system

$$Ax \leq b, \tag{1}$$

where A is a given matrix, b is a given vector, $x \in \mathrm{R}^n$. It means that we construct such a new description

$$A'x \leq b', \tag{2}$$

that contains smaller number of rows than original system (1), but which solution set is close in some sense to the solution set of (1). In

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particular, with the requirement of coincidence of the solution sets of the systems (1) and (2) the problem is known as the problem of exclusion of the redundant inequalities from the system (1).

The problem of constructing a simplified description was studied earlier using methods of linear programming (see, for example, [3, 4]). There are however the methods based on other ideas, for example, on the convolution of linear inequality systems [5, 6]. The method of Economical Description of a Polytope (EDP) proposed here is aimed at constructing a simplified description of (1) in the case of systems with the large number of inequalities (up to several millions). It is based on the use of the duality theory of convex compact bodies (CCB) and concepts of the duality theory of methods for approximation of CCB [2, 7].

Let us denote by C the class of CCB and by C_0 the subset of C, whose elements contain the point of origin in its interiority. For a CCB $C \in C_0$, we denote the polar set C^* with respect to the point of the origin of R^n as the set

$$\{x \in \mathrm{R}^n \colon < x,\ y > \leq 1,\ y \in C\},$$

where $< x, y > = \sum_{i=1}^{n} x_i y_i$ is the scalar product in R^n. Then $C^* \in C_0$ and $C^{**} = (C^*)^* = C$. Consider convex polytopes, whose vertices belong to the boundary ∂C of the approximated body C. Such the class of polytopes is denoted by $P(C)$. If a polytope $P \in P(C)$, then the conjugated polytope P^* is circumscribed over the conjugated body C^* (the facets of P^* touch C^*).

2. Description of the method. We assume that the solution set of the system (1) is a polytope that we designate by P. Let us assume also that $P \in C_0$.

The basic idea of the method consists in approximation of the polytope P^* conjugated to P. As it follows from the duality theory of convex compact sets, the polytope P^* is also a CCB, moreover $0 \in \operatorname{int} P^*$. Vertices of P^* correspond to facets of P, and vice versa. Thus, the inequalities of the system (1) determine the totality of points from R^n, whose convex hull is the polytope P^*.

The polytope P^* is approximated by a polytope $P^*(\varepsilon)$ with a relatively small number of vertices. The polytope $(P^*(\varepsilon))^*$ conjugated to the polytope $P^*(\varepsilon)$ is used as the approximation of the original polytope P. The facets of the polytope $(P^*(\varepsilon))^*$ correspond to the inequalities of system (2). The number of facets of the polytope

$(P^*(\varepsilon))^*$ equals to the number of vertices of the polytope $P^*(\varepsilon)$. It means that it is smaller than the number of inequalities in the system (1).

The polytope P^* is approximated with the aid of one of the adaptive iterative methods of the internal polyhedral approximation of CCBs – the Estimate Refinement method (ER) [8]. At the very beginning, a simplex $P_0{}^*$ is constructed, whose set of vertices is a subset of vertices of the polytope P^*. Then, a sequence of inscribed polytopes $P_k{}^*$ with the growing number of vertices is generated. The vertices are found by computing the support function of the polytope P^* for adaptively chosen directions. As the intermediate output of approximation of the polytope P^* we have representations of $P_k{}^*$ in the form of intersections of half-spaces

$$P_k^* = \{x \in R^n : D_k x \leq d_k\}, k = 1, 2, \dots, \tag{3}$$

where D_k and d_k are a matrix and a vector respectively. We will consider that the rows of the matrix D_k are calibrated.Then d_k^i, that is the ith component of the vector d_k, is the distance between the point of the origin and the hyperplane $\{x \in \mathrm{R}^n : D_k^i x = d_k^i\}$, where D_k^i is the ith row of the matrix D_k.

The process of approximation is usually stopped on reaching of certain predetermined accuracy. For some set $P = \{x \in \mathrm{R}^n : Dx \leq d\}$, such that $P \in C_0$, denote by $(P)_\delta$ the following set

$$\{x \in R^n : Dx \leq d + \delta\}, \tag{4}$$

where $\delta \geq 0$. By the accuracy of approximation of P^* by the internal polytope $P_k{}^*$ we will understand the minimum value $\delta \geq 0$, such that $P^* \subset (P_k{}^*)_\delta$.

Let $P^*(\varepsilon)$ be an approximating internal polytope of the polytope P^* that guarantees the required accuracy $\varepsilon > 0$. Then the conjugated polytope $(P^*(\varepsilon))^*$ is an external approximation of P, moreover, inequalities that make polytope $(P^*(\varepsilon))^*$, are determined by the vertices of $P^*(\varepsilon)$, namely

$$(P^*(\varepsilon))^* = \{x \in R^n : < y/||y||, x > \leq 1/||y|| \text{ , where } y \in M(P^*(\varepsilon))\},$$

where $M(P^*(\varepsilon))$ is the set of vertices of the polytope $P^*(\varepsilon)$.

Theoretical studies [8] shown that while approximating CCBs with a sufficiently smooth boundary the ER method constructs polytopes with the asymptotically optimal number of vertices. Although we can not refer directly to this result in case of approximation of polytopes, some experience of the use of the ER in applied problems [2] shows that while approximating polytopes described by the systems of type (1), with the large number of facets, good approximation is reached with the sufficiently small number of vertices of the approximating polytope. Because of this property of the ER method, the EDP method gives the possibility to construct polytopes with the relatively small number of faces, that approximate the solution set of the original system (1).

3. Accuracy of approximation. Let us examine the question of the accuracy of approximation of the polytope P with the polytope $(P^*(\varepsilon))^*$. By the accuracy of approximation of P by the external polytope $(P^*(\varepsilon))^*$ we will understand such minimum value $\delta \geq 0$, that $(P^*(\varepsilon))^* \subset (P)_\delta$.

Theorem 1. Let $\varepsilon \geq 0$ be the accuracy of approximation of the polytope P^* by the internal polytope $P^*(\varepsilon)$. Then the accuracy of approximation of P by the external polytope $(P^*(\varepsilon))^*$ is not less then $\kappa = (d_{max}/d_{min}) \cdot \varepsilon$, where d_{max} is the maximum value among all right sides in the description of the original polytope P in the form of the system of linear inequalities, and d_{min} is the minimum value among right sides in the description of the polytope $P^*(\varepsilon)$.

Remark. In the process of approximation of the polytope P^* by the sequence $\{P_k{}^*\}_{k\ =\ 0,1,\ldots}$, the value of $\kappa = \kappa(k)$ is known after each iteration and is monotonically nonincreasing with increment of k. Thus, the accuracy of approximation of the polytope P by the polytope $(P_k{}^*)^*$ can be checked from the process of approximation of the conjugated polytope P^* and does not decrease.

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Stochastic Models for the Uncertain Factors in Management of Bank Investment Portfolio

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In functioning of different financial institutions, there appears the necessity of solving the problem of a choice of the vector of current assets, i.e. investment portfolio, and the uncertain parameters, which are necessary for this task, and which are directly connected first of all to uncertainty of the prices of assets (securirues, real investments etc.), frequently occur. The result could be an example of the portfolio formation using state's short-term obligations.

The main question for the tasks of the given class is the construction of stochastic model of the price changing process, because the researcher, indeed, has only final numbers of the supervisions of the random prices realizations. Further there is one method to get the solution of the problem which was developed in CCAS during the research of the managing stochastic Markov's processes tasks.

Reflexive Market and Soros Equations

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The formalized approaches to the description of the reflective strategy in controlled systems are stated in the work. The models of G.Soros are used as substantial sending. The task about reflexive interaction of the characteristics of the share market (the quotations of the shares and earning per share) are considered. The offered computing procedure can with success be used in an arsenal of the financial engineers, when they examine the market, as reflective system. The part of this research was reported at a conference [7] "The 2002 IEEE World Congress on Computational Intelligence, International Joint Conference on Neural Networks. USA, Honolulu, Hawaii, May 12-17, 2002." Here the greater attention is given to initial substantial reasons at construction of the Soros equations.

In this paper we are describing the mathematical model, which reflects the main features of reflexive behavior of the curves, characterizing monetary evaluations of economic agents. Previously we write the following short thoughts. " ... all acts of conscious and unconscious life by origin are reflexes" I.M. Sechenov and "My mouth began to water as if I was one of Pavlov's dogs." George Soros. "The Crisis of Global Capitalism [Open Society Endangered]" [1].

The terms "a reflex, a reflexivity, a reflection, to reflect" are widely used in science and fiction, and the different authors put in these terms close, but nevertheless differing sense. In the theory of control, decision making and game theory reflexive behavior and reflexive control have been attracting researches' attention for a long time. See, for example, [2, 3] and speculations of Cournot quoted in [4]. In these papers, the main substantial motive is related to the attempts of active participants to anticipate, to foresee the actions or intentions of the others in order to improve or to form in general the control strategy. In many papers [3, 5], reflexivity is considered from the formal point of view as an element of the feedback in strategies, which adds to the decision making process modelling an enough adequate meaning on one hand, but on the other hand makes solving the set tasks more complicated.

Further we shall use the term "reflexivity" ("reflex") a) to designate the process of impact on an object without its conscious participation (this process can be named as the reflexivity of the first kind or Soros's cognitive function) and b) to designate the conscious process of object formulating *the idea* of surrounding world's action (the reflexivity of the second kind or Soros's participating function). On the basis of its ideas of the outside world (the reflexivity of the second kind) the object under consideration formulates its impact (control) on the other objects. In [3], the reflexive control was formed as $x(y)$, that is, as a function denoting the response of the first player x to the probable control of the second player y. The logical consistency of the task of decision making demanded the addition of a set of auxiliary conditions concerning the information interaction of the players. The solution to the possible contradictions was found in a way of introducing a notion "first player's right for the first move" and condition, that while choosing the concrete values of his controls the first player already knew the concrete choice of the second one, and the first player utilized his advantage in the way of dependence — strategy of behavior $x(y)$. If the situation differs from the mentioned above, and the first player has no advantage in obtaining the information, then the use of the strategy can be considered logically consistent only as some hypothesis of the first player. In further speculations, we are on the side of an abstract Observer of the events and describe their possible evolution. In [1, 8], G. Soros has developed the theory of reflexivity in application to economics on the whole and to stock market in particular. The epigraph to the present work is taken from [1, p.61]. George Soros assumes that "participants' views form the part of situation, to which they are related to", they may influence substantially the events and, in turn, are under the impact of the events. Such a mutual influence of participants G. Soros names "reflexive" influence.

The Soros model [8] of the reflexive behavior of stocks on the market assumes that:

1. *markets are always biased in one direction or another;*
2. *markets can influence the events that they anticipate.*

In the presence of the *global trends* (fundamental phenomena, historical events) and current evaluations (share quotations), the events on the stock market are interpreted in a reflexive style and somehow are opposed to and interconnected with the theory and the notion of equilibrium in economics.

After George Soros, let's consider two dynamic characteristics of an economic agent, acting on a stock market and subject to various impacts of the market forces. Let $x(t)$ be a share quotation and $y(t)$ be earnings per share. We assume that:

a) $y(t)$ is more inert; $x(t)$ is more mobile, with a greater variability;

b) mutual reflective interaction of x, y is manifested in positive and negative feedbacks;

c) the characteristics have a "growing" nature in accordance with the growth of the whole market, and their growth is characterized by the coefficient d;

d) coefficient a relates the rates of change of the x, y;

e) the characteristic y "attracts" the characteristic x and the "attractive power" is determined by the constant c;

f) the influence of x on y lags in time by one step, and coefficient b relates the rates of the corresponding changes of characteristics.

In accordance with accepted assumptions let's write the relations, describing the variations of the characteristics x, y in time by the *Soros equations*:

$$x(t+1) = x(t)(1+d) + a(y(t+1) - y(t)(1+d)) + c(y(t) - x(t)),$$

$$y(t+1) = y(t)(1+d) + b(x(t) - x(t-1)(1+d)),$$

where $a, b, c, d \geq 0$.

Let us consider now $d = 0$. Then we can see that under various proportions of the coefficients a, b, c and various constants A, B, C we can obtain the following simple types of solutions of the system of Soros equations : a) parabola; b) linear function; c) constant; d) exponent; e) sinusoid; f) tz^t.

In the case of $d \neq 0$ various curves may be obtained as the solutions of the system of the Soros equations in dependence of the proportions of the coefficients a, b, c and various constants A, B, C. The typical curves, which were determined in the computer experiments, are represented in [5-7]. But the investor subjectively conceptualize real markets have not so much deterministic as uncertain (*fuzzy*) character.

To examine behavior of the above characteristics on uncertain markets, let the coefficients be uniformly distributed variables; $a \in [2,3], b \in [0.2,1], c \in [0.1,1]$. Let these variables take on new values in every five units of time. So we'll describe the investor subjective anticipations. If $d = 0$, then, in the computational experiments with the Soros equations under the following conditions $x(0) = 1000, y(0) = 2000, y(1) = 2500$, we can obtain the various curves. Some of these curves are similar to the real curves in [1], which show the mutual reflective influence of *share quotations* and *earnings per share* on the real stock market.

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About Velocity of Money Circulation

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The notion of "velocity of money" arises in connection with classical exchange formula in quantity theory of money

$$< P, Y >= M \cdot V, \tag{1}$$

where $M \in \mathbb{R}$ is a monetary aggregate, $Y \in \mathbb{R}^A$ is gross domestic product at a period T, P is the price vector, and $A \subset \mathbb{N}$ is the set of products. The velocity is defined through (1) (see [1], for example) and, in practice, it is calculated from (1) as well. Then, in order to impart some sense to formula (1), it is usually written (see [2]) that V is defined by the structure of economy, hence it is slowly changing variable in compare with M and P. Therefore, as Y is slowly changing too, (1) represents the proportional dependence of the prices P on M.

This approach seems incorrect to me. An independent of (1) definition of velocity is suggested here.

According to the theory of money demand, there are two reasons for economic agents to keep money: (a) to meet current payment flow and (b) to buy financial assets at more profitable prices in the future (speculative reason). The agents are inclined to maintain some level of liquidity even in case of high alternative return because operational cost should be taken into account (see the Baumol-Tobin model in [1]). The theory of speculative demand for money was developed by J.M. Keynes [3]. Denote:

$\mathcal{N} \subset \mathbb{N}$ the set of economic agents (households, firms and government),

$\mathcal{M}$ is monetary aggregate $M1$, for simplicity, the other aggregates may be considered as well,

m_i — balance of the money accounts of agent i at the moment t,

s_i^0 — expenses of agent on real (not financial) assets at the period of time T foregoing the moment t. Let $\nu_i^0 = s_i^0/m_i$ be called agent i's *real velocity* of money, $i \in \mathcal{N}$, and $V_{\mathcal{K}}^0 = \sum\limits_{i\in\mathcal{K}} s_i^0 / \sum\limits_{i\in\mathcal{K}} m_i$ sub-group $\mathcal{K}$'s *real velocity* of money, $\mathcal{K} \subset \mathcal{N}$. Also denote $V^0 = V^0_{\mathcal{N}}$ real velocity for the whole economy.

$V^0_{\mathcal{K}}$ is the average *real velocity* for agents i from sub-group $\mathcal{K}$, it is free from personal chance. Of course, in practice it is enough to find average for a representative sample of sub-group $\mathcal{K}$. Parameter $V^0_{\mathcal{K}}$ characterizes inclination of $\mathcal{K}$ to liquidity, or in other words characterizes demand for money.

In order to determine our velocity to connect with macroeconomic parameters, we have to consider some other quantity, instead of real velocity. Let s_i be expenses of agent $i \in \mathcal{N}$ on consumption and accumulation of GDP. For a household, s_i does not include spending on secondary market goods and on services not to be taken into GDP account (for example, to hire a neighbor to cut lawn). For a firm, the difference is more essential: s_i does not include production (material and labor) spending, but only investments and unproductive consumption. For the government, s_i includes the orders of goods and services and does not include social transfers. It is more difficult to calculate s_i than s^0_i . However the ratio $\sum\limits_{i\in\mathcal{K}} s_i / \sum\limits_{i\in\mathcal{K}} s^0_i$ has to be stable for any sub-group $\mathcal{K}$; denote it by $\alpha_{\mathcal{K}}$.

Definition. Relation $\nu_i = s_i/m_i$ *is called agent i's* velocity *of money, $i \in \mathcal{N}$, and $V_{\mathcal{K}} = \sum\limits_{i\in\mathcal{K}} s_i / \sum\limits_{i\in\mathcal{K}} m_i$ is sub-group $\mathcal{K}$'s* velocity *of money, $\mathcal{K} \subset \mathcal{N}$; $V = V_{\mathcal{N}}$ is velocity for whole economy.*

Of course, $V_{\mathcal{K}} = \alpha_{\mathcal{K}} A V^0_{\mathcal{K}}$, $\mathcal{K} \subset \mathcal{N}$.

By the definition $V = \sum\limits_{i\in\mathcal{N}} s_i / \sum\limits_{i\in\mathcal{N}} m_i$. The sum $\sum\limits_{i\in\mathcal{N}} s_i$ of balances of all agents is monetary aggregate $\mathcal{M}$; the sum of all agents' expenses on consumption and accumulation of GDP is the value of GDP $< P, Y >$. Thus the formula (1) follows from our definition of velocity. So an independent of (1) definition of *velocity of money* is given as well as an alternative way to calculate it as average agent i's velocity. Agent i's velocity is completely under his or her control, and velocity V characterizes mass behavior of economic agents. This fact undermines monetary concept as monetary aggregate in many cases can considerably be changed because of velocity changing, without any influence on prices. The velocity certainly depends on various parameters in economy but this dependence is indirect only. It is an interesting but another theme. The other conclusions are as follows:

1. The velocity can be quickly changing quantity; it should be

measured every month or even every week.

2. It is possible to determine velocity separately for different groups of economic agents. Therefore it is possible to find (a) most insensible group to seasonal fluctuation of velocity; (b) most sensitive group which changes velocity earlier than others — to forecast demand for money in economy. Both groups are probably among the households.

3. Real velocity is better calculated and better characterizes the demand for money than usual velocity. This is why money serves for complete turnover in economy, not only for turnover of GDP.

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On Minimax Matrix Correction of Matrix Game

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Let two person zero sum game with the finite number of pure strategies of players (matrix game) be given by some $(m \times n)$ nonnegative payoff matrix A. It is well known that the problem of maximization of guaranteed gain of the first player can be reduced to the following problem of linear programming

$$\begin{gathered} v \to \max, \\ \sum_{j=1}^{n} a_{ij} x_j \geq v, i = 1, 2, ..., m; \sum_{j=1}^{n} x_j = 1; x_j \geq 0, j = 1, 2, ..., n. \end{gathered} \tag{1}$$

We assume that $\hat{v}$ determined as a solution of problem (1) is unacceptable in context of certain applied problem related to the considered matrix game. It is desirable to obtain some gain v^* such that $v^* > \hat{v}$. It is obvious that the system (1) of linear inequalities and equations with parameter v^* is inconsistent. Assume now that, within the framework of the applied problem, it is possible to correct elements of the matrix A. In other words, it is admitted to replace the matrix A by some non-negative matrix $\tilde{A} = A + H$. It is supposed that the $(m \times n)$- matrix of correction H is small, for instance, in sense of some matrix norm. In the present paper, the generalized matrix norm $\|H\|_{1,\infty} = \max_{x \neq 0} \frac{\|Hx\|_\infty}{\|x\|_1} = \max_{i,j} |h_{ij}|$ is considered, allowing to formulate the following problem of minimax matrix correction of the matrix game

$$\begin{gathered} \max_{i,j} |h_{ij}| \to \min, \\ \sum_{j=1}^{n} (a_{ij} + h_{ij}) x_j \geq v^*, i = 1, 2, ..., m; \\ \sum_{j=1}^{n} x_j = 1; x_j \geq 0, j = 1, 2, ..., n; \\ a_{ij} + h_{ij} \geq 0, i = 1, 2, ..., m, j = 1, 2, ..., n. \end{gathered} \tag{2}$$

We prove that column H^{*j} of matrix H^* that is a solution of problem (2), can be built as $H^{*j} = v^* \cdot 1_m + z^* - Ax^*$ if $x_j^* > 0$. Otherwise (if $x_j^* = 0$) the column H^{*j} is arbitrary under conditions $\left\|H^{*j}\right\|_{1,\infty} < \delta^*$ and $H^{*j} + A^j \geq 0$, where x^*, z^*, δ^* are the solutions of linear programming problem

$$\begin{gathered} \delta \to \min, \\ -\delta \leq v^* + z_i - A_i x \leq \delta, i = 1, 2, ..., m; \\ x_j \geq 0, j = 1, 2, ..., n; z_i \geq 0, i = 1, 2, ..., m; \delta \geq 0; \\ 1_n^T x = 1. \end{gathered}$$

Here, A_i and A^j are the row with number i and the column with number j of matrix A accordingly, 1_m and 1_n are vectors consisting of m and n ones accordingly. Herewith $\|H^*\|_{1,\infty} = \delta^*$.

Multicriteria Routing in Large Scale Data Networks

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key words: *network, routing, Nash equlibrium, effective solution*

Optimal routing in networks is a problem of choice of traffic optimal control by means of the data transmission routes. Since the resources of any network are restricted and the number of users is annually exponentially growing, optimization of transmission routes is requiered. In any case the routing problem is very large and multicriterial. It seems to have distributed solution in case of large scale data networks. For any pair of network users data transfer delay needs to be minimized. So routing algorithm should be based on the multicriteria optimization ideas. Such approach is presented in the paper.

Multicriteria routing task Multicriteria routing task is usually stated in mathematical programming form, see (Bertsecas et al., 1987) in spite of the fact that any multiproductive flows task is, by its nature, multicriteria. Vector optimization of network problems is more relevant, see (Fedorov, V.V. et al., 1996; Korilis et al., 1997; Vasilyev, 1998). In the paper results of multicriteria routing problem's study are given.

Let network structure be given by a connected graph. Users pairs (a, b) enumerated by the index $k = 1, 2, \dots m$ are located at some vertices $a, b, = 1, 2, \dots n, a \neq b$. Data transmission lines are represented by edges $l = 1, 2, \dots n$ of the graph. Messages are passed in the form of packets along the graphes routes.

Delays in communication channels are determined by nonnegative increasing continious functions $f_l = f_l(z_l)$ where z_l is the total data flow through channel $l = 1, 2, \dots n$. There are constraints $z_l \leq c_l, l = 1, 2, \dots n$ on the capacities of the lines.

Let total input traffic loads $\lambda_k, k = 1, 2, \dots m$ be given. For each kth users pair it is requiered to determine the number of routes $M_k = \{L_j^k\}$ and the transmission rates $\Lambda_k = \{\lambda_j^k\}$ over them. Therefore, the following conservation conditions hold:

$$\sum_j \lambda_j^k = \lambda_k, \lambda_j^k \geq 0, k = 1, 2, \dots m,$$

$$z_l = \sum_{k,j : l_j^k \in L_j^k} \lambda_j^k, l = 1, 2, \dots n.$$

Every message is to be divided into a set of packets to be passed by their own routes L with their own transfer delays:

$$\rho(l, z) = \sum_{l \in L} f_l(z_l).$$

This is the length of the route L which depends on network routing. Since any message should be constructed from its packets as parts, the whole transfer delay is determined by the worst of the transmission routes:

$$T_k(\{M_k\},\{\Lambda_k\}) = \max_{j:\lambda_j^k>0} \rho(L_j^k, z), k = 1, 2, \ldots m.$$

So, there is the vector quality index $T = (T_1, T_2, \ldots T_m)$. Any user strives to minimize its own delay function T_k. Therefore, it is the aim of routing optimization.

Routing $\{M_k, \Lambda_k, k = 1, 2, \ldots m\}$ is called optimal if it is Nash equilibrium point in the noncooperative game

$$\{T_k, M_k, \Lambda_k, k = 1, 2, \ldots m\}.$$

Transmission routes are called optimal if they are used in the optimal routing.

First of all routing algorithm should not cause traffic oscillations that spoil the transmission capacity of the networks. The Nash equilibrium is proved to exist in simple ring-like networks, see (Fedorov, V.V. et al., 1996). In networks of more complex topology, routing choice should be also based on the well known notion of effective (Pareto) solution. This principle doesn't contradict to the previous one in ring-like networks.

Both approaches were applied to create a fast optimization routing algorithm. Computing experiments were done with INTERNET models. Results of vector optimization of the EvroRings Network are presented. The algorithm substantially reduces vector data transfer delay.

Decomposition scheme for the routing task solution Let the initial network graph be replaced by a multilevel system of ring-like networks. The rings of the first level have to be selected so that every users pair can be united by a chain of the rings. Every pair of $p - th$ level rings ($p = 1, 2, \ldots P - 1$) having at least two common vertices is united to obtain a $(p+1) - th$ level ring the cross-pieces being removed. After that all communicating pairs should be distributed among the ring-like networks. Users pair is of the pth level if it can be linked up by a chain of p first level rings. First level pairs are called simple and pth level pairs, $p > 1$, - complex. Sought-for optimal transmission routes will be put together from the routes lying in rings of all levels as from pieces. Routes for the the complex pairs will be sought for among all tying up routes from its linking up chaines.

It is proved, see (Vasilyev, 1997, 1998) that all simple pairs have the shortest path optimal routes (in metric ρ). They are found by an iterative process which makes equal the lengths of all the transmission routes. The algorithm works in every ring of the multilevel system which is used for the better detour routes search. The process is proved to fastly converge to Nash equilibrium.

Choice of the complex pairs routing should be done in every tying up chain by means of effective solution search for all the pairs connected by the chain. In case of linear delay functions f_l, it is achieved as a solution of several LP tasks solution. The scheme permits distributed realization that is important for routing algorithms in large area networks.

The approach was successfully applied to EvroRings network routing study.The network contains 7 rings with $m \geq 1400$ communicating pairs. The optimal routing was fastly found substantially emproving initial vector delay T.

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Sociologist' Role in Enterprise Management

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Social research is used on Russian enterprises over 40 years. With the beginning of perestroika most of enterprises were forced to reject this service.

The main activity of a sociologist on an enterprise is based on direct interaction with both the management and the collective of the enterprise. The enterprise sociologist connects the staff of the enterprise with its management. Managers and workers can make a compromise in solving key problems only due to the sociologist. Therefore the aims of the sociologist are to study the problem of work collective, bring them to the notice of the management and solve for the good of the enterprise; to improve the socio-psychological climate in the collective.

Developing the labor motivation and satisfaction of labor, knowledge of the form and content of workers' free time, development of sociological level of managing are subject matter of the sociologist and very important factors in enterprise managing. Therefore making different managing decisions on the enterprise is effective when they are based on the results of social research on a certain problem.

In May of 2003 the management of the Tver enterprise "Post of Russia" initiated a social research on the subject of "Satisfaction of workers by the system of labor stimulation on the Tver post office". The research was made by questioning of 91 persons, with reliability of 0,68% and estimated error of 5%. There were the following results: 90,1% of workers guess that the enterprise management prefers financial stimulation; 69,3% of the enterprise workers are satisfied and rather satisfied with the existing system of labor stimulation; however, 31,9% consider it necessary to combine financial and moral stimulation; and only 15,4% of respondents understand stimulation only in financial sense. For the whole enterprise, 58,2% of workers prefer combined labor stimulation. The existing system of labor stimulation does not satisfy 17,6% (mostly from 18 to 29 years old). Social measures (like free medical care, medical accident insurance and health centers for a reduced price) are effective for 80,2% of respondent

Analysis of the findings shows the most actual for workers of the Tver enterprise "Post of Russia" methods of labor stimulation. The results of social research show that on this enterprise the most effective methods of moral and finance labor stimulation are the following: competitions among the divisions on the title of "The best worker of a region", "The best worker of a post office", "The best operator of the year" which allow to find leading workers who can be transferred to prestigious allotted work, can have additional vacations, free excursions, possibility to visit health centers for reduced price, get prizes; information in mass media about the results of the competitions; putting the photos of winners to the Honor board of the enterprise.

Managing on the enterprise was provided taking into account the results of the social research on labor stimulation. For attracting young workers, the management was suggested to include into the system of stimulation bonuses for first 5 year of work, payments for study in professional schools and colleges, housing for first 5 years of work, permission to use kindergarten, possibility to work on flextime for young mothers, partial payment for house.

The management of Tcer enterprise "Post of Russia" had changed the system of labor stimulation taking into account the sociologist's recommendation. According to quarterly financial reports, this allows to raise the productivity of labor, to attract many young workers and to increase income of the enterprise as a whole.

Possibility of a sociologist' work with the staff has multidimensional character and includes the following items: gathering information necessary for a management, studying and generalizing real social experience, exact estimation and showing negative moments of the life of labor collective, studying mistakes made by the management. Social research is not just gathering information necessary for effective management but also the form of participation of workers in the collective activity, it is a part of democracy. Mainly, the results of the conducted social research on enterprise allow making empirically reasoned and certainly efficient decisions. These decisions are based on the opinion of the most part of workers; therefore workers are waiting for these decisions from the management and are ready to follow them. The practice of sociologically investigated and reasoned decisions indicates their much more effective implementation to the enterprise. This helps the workers to adapt to innovation and increases the enterprise profit.

"VINTSERVING": Service Team of Situation Centre

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A situation centre (SC) is the very popular tool of operations research now. The great number of publications is dedicated to technical equipment and software of SC. Computer support of cooperative work is presented to much less degree. One can find the most interesting examples in the publications of E. P. Grigoriev and the same of A. N. Raykov. This publication is dedicated to cooperative work based on possibilities of SC.

The main idea of our technology was a virtual system information technology created by a group realising its own project. Virtuality means here that such system acts within some concrete group and its concrete project only. We have named a process of the creation of such virtual information technology as *Vintserving*. This name was created for the reason of producing an association identical to the notion introduced. Here we have an obvious analogy with windsurfing. *Vintserving* was created as some sort of a polyscreen environment, where an activity has been organized by a special team. This team includes an operator, a methodologist, and a psychologist.

The main components of the technology were offered in [1]. A very important part of it is reflexive analysis of V. A. Lefebvre. Certain applications were discussed in [2, 3, 4, 5]. We have put our technology into practice in the Omsk State Institute of Service, in Omsk State University, and in the British Council Resource Centre (Omsk, Russia) since 2001. Some of our projects concern monitoring of emergency situations, training systems, strategic planning and management of city education.

We can describe the role of each member of this team as follows. The operator creates all the prototypes and cognitive graphics. The methodologist controls the logic of the whole project and shaping the virtual system of the determinations. He creates cognitive maps of the project. The psychologist stimulates a process of creative thinking. Using TypeWatching he makes an analysis of any participant and the group as a whole.

The proposed technology uses an operations research approach, but it has certain essential differences. The first step is to determinate the main mission of the project and divide the group into two parts of stockholders and of spectators. Then the group creates the language of the project. The next step is a creation of some sort of a rapid prototype of the project. The prototype is the simplest version which contains the most complex element of the whole.

Our special tool is a screen-pictographics. There are two main differences from ordinary pictograph system. At first we represent our objects according to the present stage of the cycle of its own existence, that is, the number of elements of the image is equal to the number of stages of the cycle. Then we place all the images into the space of external system.

Reflexive analysis is used as a facility to navigate in space of exploratory position, subjects and terminology. The team creates the reflexive mirror, that is, a screen of cognitive images (such as Chernoff's faces) of each member of the group and the group as a whole.

Our experience shows that a service team is the key resource of the technologies under consideration. The Omsk State Institute of Service began training students in computer science and analytics. The first graduates will get their diplomas in 2006. They have some training as a service team of SC. The problem seems to be very important in order to improve all the components and the technology as a whole. So any critical remarks and offers will be very accepted.

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Research of Financial Operations at Innovative Designing

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key words: *innovative designing, uncertainty, innovation, innovative process, managing subject, customer, developer, financial operations, budgetary effect, profit*

In modern conditions of transition to market relations, the models connected with study of accepted decisions are of interest. These decisions depend on industrial activity, distribution of budgetary funds on realization scientific and production programs, etc.

In particular, such decisions appears in innovative designing, in which basis the concept *innovations* uses of results of scientific researches and the development directed to improving of industrial activity lays. Specificity of an innovation as goods consists in a high degree of uncertainty at reception of scientific and technical results, special character of financing, namely: *risk* time break between expenses and results, *uncertainty* demand [1].

In the present paper, *the innovation* is the use of results of scientific researches in development of new industrial system under the certain order. It is supposed that the customer and the developer are participants of innovative process, and the developer carries out functions of the innovative enterprise, and the customer — functions of the managing subject.

The innovative decision determining the sizes of investments of financial assets, the customer should accept. But at the conclusion of the contract the developer can not agree with the decision of the customer. Therefore two innovative decisions are possible:

(i) the decision offered by the customer – the innovative decision of the customer,

(ii) the decision comprehensible to the developer – the innovative decision of the developer.

On a course of innovative process the customer and the developer can pursue the different purposes.

In the present paper, financial operations which they should make for achievement of the purposes are investigated. Research is based on two models of acceptance of the innovative decisions corresponding to two types of the purposes, pursued by the customer and the developer:

the first type of the purposes consists in the customer's maximization of the utility function whereas the developer wants to maximize of the profit with the purpose of satisfaction of solvent demand of the customer [2],

the second type of the purposes consists in the customer's maximization of the budgetary effect whereas the developer want to maximize of the profit. It results in optimiation of criterion vector, each component of which describes the purpose pursued by the customer or the developer [3,4].

Approbation of these models is made by the example of development of single-channel system of mass service with refusals. On approbation two basic sources of statistical uncertainty are taken into account caused by casual character:

(i) an entrance stream of applications,

(ii) the process of service.

Therefore innovative decisions are based on predicted values as intensity of an entrance stream of applications and intensity of service of the most projected system.

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Multiobjective Optimization of Procedure of Statistical Classification of Supervision

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key words: *multiobjective, optimization, classification, multialternative procedure*

At absence of data on a priori distribution of probabilities on set of classes and payments for decision making for finding the decisions function, it is possible to use multiobjective optimization. We suggest the procedure of the statistical classification providing the maximal values of conditional probabilities for making correct decisions on all classes at once with bounded above conditional probabilities of making erroneous decisions.

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About Optimizing Stochastic Process Parameters on an Available Time Series for the Problem of Stochastic Optimal Control with Discrete Time

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The comparative efficiency analysis of solutions got under evaluation of stochastic parameters of the model by traditional statistical methods and under optimizing these parameters on an available time series is carried out for the problem of stochastic optimal control with discrete time. The properties of the solutions as the size of sample used tends to the infinity are investigated. The plan of computing experiments for the limited volumes of statistical data is developed.

Introduction. Practical problems of control in economic and engineering systems are often formulated as problems of decision-making under the uncertainty, which has regular nature but is complex for modeling. When such uncertainty is understood as a stochastic process it is usually presented as a multidimensional process, for which neither the structure nor parameters are known. Even if a model of the stochastic process has been formulated, obtained optimization problem is often too complicated to be solved analytically.

Many examples of problems can be indicated in decision making theory, in which uncertainty is related to uncontrolled nature factors and to interaction between large number of financial and economic institutions including financial portfolio management problems. In situations, when it is impossible to determine an optimal solution, the so-called method of optimization on time series is often used to determine a rational solution of the problem. Observation data on the uncontrollable factors are taken as a basis and such rules of the investigated object control are searched, which are effective on this data array. This approach implicitly assumes that since the uncertainty has a regular character, then if the control rules are optimal during some sufficiently long time period of time in the past it will be also optimal in the future. This idea appears to be rational especially in the cases when there is the necessity of a decision making and there are no other approaches to solve the problem. Nevertheless, this technique raises some questions and doubts, particularly, because control rules are usually being determined, as well as estimated on the same samples of stochastic parameters. When we construct an optimal control using time series, to what extend do we use systematic properties of the stochastic process, and to what extend do we just make adjustments using properties of available realization of the stochastic process that are non-significant for the future? The analysis of this problem seems to be important and represents an actual challenge.

Parametrical problem of stochastic optimum control with discrete time. Assume, that the controlled process proceeds in time with a step $t = 1, 2, \dots, \infty$. At any moment of time $t \geq 1$ the controlled object is in some state $A_t \in \hat{A} \subset \mathbb{R}^N$. At a choice of control u_t the object passes into the state $A_{t+1} = \varphi(A_t, \xi_t, u_t)$, where ξ_t — is a stochastic parameter. The factor $\xi \in \Xi \subset \mathbb{R}^M$ is understood as stationary Markov process with transitive function $\Phi(\xi_t|\xi_{t-1})$. We name the pairs $S_t = (A_t, \xi_t)$, included in argument of the function φ, as the states of process on a step t. The distribution $F^1(S_1)$ on the set of initial states of the process is given. We connect with each step of the process the estimated function of the control quality (payment) $h_t = h(S_t, u_t)$. It is required to maximize the average of the payments

$$Q = \lim_{n\to\infty} \frac{1}{n} E\left(\sum_{t=1}^{n} h_t(S_t, u_t)\right) \Longrightarrow \max_{u} \tag{1}$$

The control functions $u_t = u(S_t)$ are chosen as functions from a parametrical class $u(S;\alpha) \in \hat{U}_\alpha$. Thus, the task is reduced to searching the best value of the parameter α:

$$Q = \lim_{n\to\infty} \frac{1}{n} E\left(\sum_{t=1}^{n} h_t(S_t, u(S_t;\alpha))\right) \Longrightarrow \max_{\alpha}.$$

We call the formulated problem the *problem* 1. Let's associate with the problem 1 its discrete analogue — the *problem* $1D$, in which states of the object and stochastic and control parameters take on their values on the finite lattices: $A_t \in \{A^i\}_{i=1}^{I} = \hat{A}^D \subset \hat{A}$, $\xi \in \{\xi^j\}_{j=1}^{J} = \Xi^D \subset \Xi, u \in \{u^s\}_{s=1}^{S} \subset \hat{U}_\alpha^D$. The functions φ and Φ are modified accordingly. Consider the case when the solution of the problem 1 exists and the solution of the problem $1D$ is an approximate solution of the problem 1.

Parametrical problem of optimization on time series. Let a sequence of realizations of random value $\xi : \tilde{\xi}_1, \tilde{\xi}_2, \ldots, \tilde{\xi}_T \in \Xi^D$ and initial state of system $\tilde{A}_1 \in \hat{A}^D$ be known. Consider the following problem.

Problem 1R. *To maximize criterion*

$$\tilde{Q}^T = \frac{1}{T} \sum_{t=1}^{T} h_t\left(A_t, \tilde{\xi}_t, u(A_t, \tilde{\xi}; \alpha)\right) \tag{2}$$

on the set of the control functions $u_t = u(A_t, \xi_t)$ from a class $\hat{U}_\alpha^D$, under condition of $A_1 = \tilde{A}_1$

Theorem 1. *If there exists an everywhere optimal control function in the problem $1D$ then the optimal value of the criterion of the problem $1R$ tends to the optimal value of the criterion of the problem $1D$ as the size of the used sample increases unrestrictedly.*

Problem of optimization of the stochastic process parameters on the time series. If the available data do not allow to construct trustworthy model of the stochastic process or if the problem 1 is too complex to be solved strictly, then the stochastic process under investigation is frequently replaced with the other one that is simpler than the initial process but reflects its essential features in opinion of the researcher. Parameters of this auxiliary process $\beta \in \mathbb{R}^N$ are adjusted on the available time series and the problem 1 with the modified stochastic process is solved. Let's call it the problem $1M$.

Let us designate by $Q(u)$ the value of criterion (1) of problem 1 under control function u. Consider the parametrical set of the problem such as $1M$, for which the adjusted factors of the modified stochastic process are used as parameters. The solution of a problem $1M$ at fixed values of the factors β defines a control function u^β and a value of the criterion $Q(u^\beta)$. Let's state the problem of maximization the value $Q(u^\beta)$ on the set of adjusted parameters β. We call it the problem $1A$. Let's choose some traditional statistical method for evaluation of stochastic parameters and consider the problem $1M$ under parameters determined by this method. Let's call this problem $1S$. As above we associate problems $1M$, $1A$ and $1S$ with their discrete analogues, they are the problems $1MD$,$1AD$ and $1SD$.

The efficiency of the control function received by the solution of the problem $1MD$ can be estimated on the sample $\tilde{\xi}_1, \tilde{\xi}_2, \dots, \tilde{\xi}_T$ by calculating the value of criterion (2). Let's state the problem to find such values of the stochastic parameters, which maximize this estimate. We call it the problem $1MR$.

Let us designate the optimal control functions of the problems $1SD$ and $1MR$, using array of the historical data $\tilde{\xi}_1, \tilde{\xi}_2, \dots, \tilde{\xi}_T$, by $u^{1SD}(\tilde{\xi}_1, \tilde{\xi}_2, \dots, \tilde{\xi}_T)$ and $u^{1MR}(\tilde{\xi}_1, \tilde{\xi}_2, \dots, \tilde{\xi}_T)$, respectively. Using the theorem 1 it is easily to prove the following statement.

Theorem 2. *If in the problem $1AD$, there exists an everywhere optimal control function u^{1AD} then the values $Q(u^{1MR}(\xi_1, \xi_2, \dots, \xi_T))$ a.s. converge to optimal value of criterion $Q(u^{1AD})$ for the problem $1AD$ as T grows unrestrictedly. At that $Q(u^{1SD}(\tilde{\xi}_1, \tilde{\xi}_2, \dots, \tilde{\xi}_T)) \leq Q(u^{1AD})\ \ \forall \tilde{\xi}_1, \tilde{\xi}_2, \dots, \tilde{\xi}_T$.*

As the stochastic model of the problem $1S$ is no more than one of elements of heuristic procedure, there are no reasons to think that the values $Q(u^{1SD}(\tilde{\xi}_1, \tilde{\xi}_2, \dots, \tilde{\xi}_T)$ converge to value $Q(u^{1AD})$ as the size of the sample increases. Therefore, it is possible to assert the following. The solution of the problem $1MR$ as the method for solving the problem 1 gives, under increasing volume of the sample $\tilde{\xi}_1, \tilde{\xi}_2, \ldots, \tilde{\xi}_T$, generally speaking, more effective control functions than the solution of the problem $1M$ with parameters of the stochastic model estimated on the same sample by any traditional statistical methods.

General plan of computing experiments. However asymptotic preference of one of the methods does not give formal reasons to consider it as an effective method for solutions of applied problems, in which samples are always limited and frequently unsufficiently great. In applied tasks with unstated structure of stochastic processes, the theoretical estimation of efficiency of the method based on the solution of the problem 1MR seems very complex. Therefore, to estimate the method under consideration, it is offered to carry out practical experiments. Such experiments can be realized on the basis of the computer modeling under the following outline.

The mathematical models of controlled systems including the models of random factors in the form of Markov process are built up. This Markov process simulates the reality and nature. By means of this process, the data, which simulate time series, are generated. Then, from positions of the researcher, who does not know the real structure of stochastic process and deals only with the time series, the various variants of optimization problems on time series are solved. The experiments are to be carried out at different sizes of the arrays simulating time series. The results, i.e. received control functions and their estimations on the available time series, are compared with their "real" efficiency, i.e. efficiency on the basic Markov process.

Certainly, such experiments can not prove the efficiency of the method under consideration in strict mathematical sense, but it seems that their results to be essential from the point of view of their further application.

Mathematical Model for Determination of Enterprise Pollution Quotas

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Pollution of atmosphere is the most serious environmental problem for health of people in the short-time and mid-time prrospect. It is more difficult to be saved from polluted air, than from the polluted water. The paper presents two mathematical models for determination of enterprise pollution contribution i.e. quotas on polluting substances emission.

Optimization model. This model provides a complex approach to all enterprises polluting the atmosphere in a region. Therefore it takes into account requirements of expenses minimization and the relative enterprise importance factors. Using of optimization methods for determining the enterprise pollution quotas significantly improves results for enterprise resources saving.

Capacitor model. This model uses computational time idle algorithm and improves essentially results of widely used methods for determining the enterprise pollution quotas. In capacitor model, region quotas determination problem is shown to widely known class of optimum capacity loading (a rucksack, a train, a ship) at the maximum utility.

The considered mathematical models have been realized in module "Kvotirovanie" of a program complex "Prisma-region" on a Visual Studio C++ base. Comparison of the basic methods for determining the enterprise pollution quotas is viewed, and results of calculations for the given mathematical models are submitted. Examples of program realization show sufficient efficiency and practical importance of the created models.

Multibounds Heuristic Algorithm for Design of Scheduling M Tasks on N Equal Processors

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Task definition: for each of M tasks and N processors (with equal characteristics), time $t_{i,j}$ needed for working of task i on processor j is known. We want to find (construct) scheduling that minimize the total time of processors' working.

This task is formulated as follow:

$$T \to \min$$

$$\begin{cases} \sum_{i=1}^{M} t_{i,j} x_{i,j} \leqslant T, \qquad j = \overline{1,N} \\ \sum_{j=1}^{N} x_{i,j} = 1, \qquad i = \overline{1,M}, \\ x_{i,j} = \{0,1\},\ i = \overline{1,M},\ j = \overline{1,N}, \end{cases}$$

where T is time of all tasks to be finished; $x_{i,j}$ is Boolean variable, ($x_{i,j} = 1$, if task i is distributed on processor j, otherwise $x_{i,j} = 0$).

We also will use vector $z[M]$, ($z_i = 1$, if task i is distributed on one of the processors, and $z_i = 0$ otherwise).

This task is known to be NP-full. Some heuristics algorithms for solving it are given in literature ("greedy" algorithm, for example). But if we apply multibound procedure (as we can see some lines below), we can improve our scheduling in comparison with "greedy" algorithm.

The heuristic algorithm uses a modified stage (c) of the following subalgorithm: we use not an ideal bound ($ideal\text{-}time = (\sum_{i=1}^{M})/N$) but calibrated *ideal-time*, i.e. we increase this bound in some share. The subalgoritm performs with different values of calibrarion from 2% to 10% by step 1% and selects the best result (for diffent values of input data the best share of calibration is also different).

Subalgoritm of scheduling.

a) Sorting $t_{i,j}$ in decreasing order ($t_i \geqslant t_{i+1}$ $\forall i = 1, \dots, M-1$).
b) Calculate the *ideal-time* for all tasks to be finished. $ideal\text{-}time = (\sum_{i=1}^{M})/N$.
c) Distribute task on processor (from the first to the last) while total sum of $t_{i,j}$ for current processor is not more than *ideal-time*. $\forall j = 1, \dots, N$ determine $x_{i,j} = 1$ while $\sum_{i=1}^{M} x_{i,j} t_i < ideal\text{-}time$ for current j.
d) Searching the number of processor (k^*) such that

$$k^* = \underset{k=1,N}{\operatorname{argmin}} \sum_{i=1}^{M} x_{i,j} t_i$$

e) $x_{i,k^*} = 1 : i = \operatorname{argmin} t_{i,j}) \forall i = 1, \dots, M, j = 1, \dots, N | z_i = 0$
f) Repeating (d) and (e) while all tasks are not distributed ($\sum_{i=1}^{M} z_i < M$).

Optimal Scheduling of Incomplete Communication Graph Multiprocessor Systems

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The problem of finding feasible preemptive schedules in real-time multiprocessor systems is considered. This problem is very important for designing complicated computing systems, economic and ecological monitoring systems. The problem of finding feasible schedules given durations of executions of tasks and directive intervals has the most extensive field of application. Known algorithms for this problem assume the following conditions:

1. All processors communicate each other and all tasks can switch over to other processor directly.

2. Time delays related to interruption of execution of tasks and switching them from one processor to another are not taken into account.

We investigated the cases that do not meet one or both conditions 1, 2. We proved that the problem is NP-complete for some special cases.

The main result of this work is a polynomial algorithm for the case, when the communication graph is an arbitrary connected graph, and time delays related to interrupton of execution of tasks and switching them from one processor to another are not taken into account. In the strict sense the problem is solved only for the case, where

$$GCM(b_1, b_2, \dots, b_n, f_1, f_2, \dots, f_n, t_1, t_2, \dots, t_n) \geqslant 2m.$$

Here GCM is the greatest common measure, $N = \{1, 2, \dots, n\}$ is the set of tasks, m is the number of processors, $(b_i, f_i]$ is the directive interval, t_i is duration of execution of task i. We can mention however, that the problem in general case can be reduced to this case by an appropriate choice of the time unit. We have proved that the problem is NP-complete if it does not meet this condition. Based on this result we constructed some heuristic algorithms that solve a generic problem without restrictions 1 and 2. The special case of this problem which takes into account the delays is reduced to a problem of Boolean and integer-valued linear programming.

In the first part of this work we investigate the next problem. Without loss of generality we assume that $\min b_i = 0$, $\max f_i = T$, $i \in N$. It is supposed that all parameters are integer. Interruption of execution of tasks and switching them from one processor to another are permitted. Interruption and switching are not related with time delays. We define the communication graph as a graph with nodes symbolizing processors and edges symbolizing ties between them. It is supposed that communication graph is connected but is not necessarily complete. This problem is called the main problem.

As stated above we have elaborated a polynomial algorithm for the main problem. The computational complexity of this algorithm is restricted by $O(n^4m^2+n^3lnT)$. We have also proved that the problem is NP-complete if communication graph is not connected and NP-complete in strict sense if the time delays related to interruption of execution of tasks and switching them from one processor to another are taken into account (even if we have only one processor).

In the second part of this work we consider a system consisting of m identical processors with the complete communication graph. Each processor has its own memory but the common database. There are n tasks and to solve any of them we must load necessary information from the database. If we solve this task on several processors we must perform all this loading on each of them.

We prove that this problem is NP-complete and reduce it to the problems of Boolean and integer-valued linear programming. The number of variables and restrictions in the problem of Boolean linear programming is bounded by a polynomial of m, n, lnT. In the problem of integer-valued linear programming there are only $nm+m$ variables and $nm+n+2m+1$ restrictions.

Effective Scheduling Algorithms for Multiprocessor Real-time Systems

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The problem of finding feasible schedule in the multiprocessor hard real-time system is considered. It is formulated in the following way:

In a real-time system, consisting of m processors, each processor $j(j=1,...,m)$ is specified by its speed s_j and the amount of memory V_j. There are n tasks, each of them specified by its release time r_i, due date d_i, length (processing complexity) p_i and the amount of data v_i needed to be uploaded into processor memory for the i-th task execution, $i=1,\ldots,n$. The execution of task i on processor j requires p_i/s_j time, $i=1,...,n$, $j=1,...,m$. Processors can work on only one task at a time and each task can be executed by at most one processor at a time. One may suspend the execution of a task before its completion and resume its execution at a later time, possibly on a different processor. Such preemptions require λ_j time for interrupting a task on j-th processor, $j = 1, ..., m$. System processor's communication graph (describing the possibilities of switching the tasks from one processor to another) is arbitrary. The problem is to find out if there exists a feasible schedule and if it does, indicate such schedule.

The formulated task is NP-complete, that is why various simplifications and special cases, still having significant practical importance, are considered. Another approach proposed is the construction and application of heuristic algorithms.

A special case of the original scheduling problem, where processor's communication graph is full, there are no memory limitations, and tasks interruptions require no considerable time, is examined. An exact algorithm reducing the problem to finding a maximum flow in a special network is used. Such flow in fact distributes the parts of tasks for the sequence of time intervals, defining the conditions for the series of local scheduling problems such that all tasks have the same release times and due dates. The problems are solved by a known effective polynomial algorithm. In spite of the fact that the proposed algorithm finds an exact solution in a polynomial time $O(m^3n^3)$, this time turns out to be too big for practical application of an algorithm for high dimensionality cases. Therefore two significantly faster heuristic algorithms were constructed. They both use the following idea: moving in time from minimal r_i to maximal d_i, at each release time or the time of execution completion of some task we assign the task with the earliest deadline to the fastest of the processors available at the moment. The difference between the first and the second heuristic algorithm is the following: when scheduling tasks in the first algorithm, we consider only currently free processors, interrupting the least priority tasks only if there are no processors available, and in the second algorithm we each time reassign the tasks to all system's processors according to their priorities (deadlines), faster processors being assigned the most priority tasks. The complexity of these two algorithms is, correspondingly, $O(mn)$ and $O(n^2 \log_2 n)$. In spite of the theoretically and experimentally proven fact that the second algorithm solves the problem correctly on a considerably wider set of conditions than the first one, it generates the feasible schedule with significantly greater number of tasks interruptions ($2(n-1)m$ against $(n-1)$). This begins to affect its correctness when it is applied to the feasible schedule problem with non-zero time required for tasks interruption. Three options for the calculation model of the processor's task interruption time are considered. For each model the boundary conditions of the mean interruption time, at which the first algorithm becomes more correct, than the second, were experimentally found.

Then the feasible schedule problem in its original statement is considered. However, for the simplification reason, it is assumed that all the processors are working in discrete synchronized time units. An algorithm was constructed for this special case, which reduces the problem to finding a multicommodity flow in a specially built network, its correctness proven and its complexity calculated. This algorithm is not polynomial, therefore the possibility of applying various heuristics is discussed.

For a single processor case of the original problem, however, far more efficient exact polynomial algorithm was constructed. This time the processor's work is not digitized into time units. A number of statements describing this case were proven, and the algorithm was constructed based on them. The algorithm starts with finding an optimal tasks order, then it constructs a feasible schedule moving back in time from the latest due date to the earliest release time, then having one more correcting pass forward in time. The complexity of this algorithm is $O(n^2 \log n)$.

Then we discuss another type of scheduling problem called job-shop scheduling problem. In this problem, there are n jobs and m machines. Each job could be done on a machine for specified processing time, and there are no predefined release times and due dates for each job. At a moment, a machine can process only one job. A processing of a job cannot be interrupted for relocating it on a different machine. Measure of effectiveness for the constructed schedule is a total processing time. The job-shop scheduling problem lies in the class of *NP*- complete problems and will hardly be solved with polynomial algorithms. So finding of effective approximate algorithms is an important task.

The aggregation approach consists of three fundamental procedures: decomposition, calculating, and linking. Decomposition is the process of partitioning a large problem into two or more subproblems, calculating is a process of finding admissible schedules for the subproblems, and linking is a process of constructing (aggregating) the schedule for the initial problem using the schedules calculated for the subproblems. Various procedures differ with respect to the decomposition strategy, calculating and linking algorithms.

Authors carried out a research aimed to find an effective hybrid aggregation approach, in which the calculation procedure is taken with heuristic algorithms, and applying it to different job-shop scheduling problems. The following scheduling problems were considered.

Problem 1. The machines are different in functionality and performance. That is the job's processing times are given as a matrix $||\gamma_{ij}||$.

Problem 2. The machines are identical in functionality and performance. That is the job's processing times are given as a vector $||\gamma_i||$.

For *Problem 1* the following approach is suggested. The problem is first solved using a greedy (fast) heuristic algorithm. Then the problem is partitioned in such a way that a subproblem involves rescheduling the jobs that were allocated on a part of machines, on that particular machines. For the rescheduling, the pseudo-polynomial algorithm is used. Using directive interval bisection method, the pseudo-polynomial algorithm provides required precision (for integer problems it can be used for constructing optimal schedule). The linking involves the concatenating of the subproblems' schedules.

For the *Problem 2* the described above approach can be modified for better effectiveness. The so-called *multi-level aggregation approach* includes the following. After the decomposition and calculating steps, the jobs allocated on one of the machines (for each of the subproblems) are considered as a 'bigger' job with the processing time equal to the sum of the ones of the jobs. Let's call it *aggregation*. So for the set of the aggregated ('bigger') jobs the steps of decomposition and calculating are repeated. The aggregation process may repeat several times until further decomposition is not effective (the number is called the *level* of the algorithm). The linking involves reconstruction of the initial jobs out of the aggregated.

Consider two special cases of *Problem 2.* The job's processing times could be sorted in such a way that they follow arithmetical progression dependence or close to it. For these cases both the length of schedule constructed by greedy algorithm and the measure of closeness are defined by the authors.

The described approach shows high effectiveness and underlies the computational programs which have been run on multiprocessor and cluster systems.

About a Ruin Probability of an Insurance Company

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key words: *ruin probability, insurance, non-Cramer model*

A simple computational algorithm for an unruin probibility of an insurance company for the Lundberg-Cramer model, when the both distribution functions are Erlang's ones, is presented. Besides, the Lundberg-Cramer model, when claims are connected in a Markovian chain, is considered. A system of integral equations (analogous Cramer's equation) is constructed. A non-Cramer model, in which every client initiates an arrival of a claim is considered also. A state of a capital is considering at moments $n\tau$, what permits to reduce the non-Cramer model to a Markovian variant of the Cramer model.

After pioneer articles by Lundberg and Cramer [1], [2] there appeared many papers, (see the bibliography in [3]), in which their model (L-C) is developed and generalized. An evolution of a capital of an insurance company is a random walk

$$u_k = u + \sum_{i=1}^{k}(c\tau_i - X_i) = u + \sum_{i=1}^{k}\eta_i$$

in the model (L-C), where t_i are moments of a claim arrival, $\tau_i = t_i - t_{i-1}$, c is an intensity of premiums, X_i are claims with a known distribution function $F(x)$, $g(t)$ is a distribution function of τ_i. All the random values are independent. If $P_n(u)$ is an unruin probability when $t < t_n$, then there is the following reccurent relation

$$P_n(u) = \int_{-u}^{\infty} P_{n-1}(u+y)d\Phi_{c\tau-X}(y), \tag{1}$$

where $\Phi_{c\tau-X}(y)$ is a distribution function of $c\tau - X$. If $n \to \infty$ in (1), then we shall obtain an integral equation for an unruin probability in infinite interval:

$$P(u) = \int_{-u}^{\infty} P(u+y)d\Phi_{c\tau-X}(y) \tag{2}$$

A solution of (2) as the Laplace's transform is obtained in [1], if $g(t) = \lambda e^{-\lambda t}$.

1. Let the both distribution functions be Erlang's ones (or Erlang's mixes). Then by the Laplace's transform of (2) we obtain a solution as a fraction with the numerator containing some unknown constants. The number of the constants is equal to the number of the roots of the denominator in the right halfplane and these roots are the roots of the numerator also [4]. Thus, computation of an unruin probibility is reduced to computation of the roots of the denominator in the left halfplane.

It should be pointed out, that Erlang's mixes are good approximations for a wide class of distribution functions.

2. Now let's consider the main model (L-C), but the intervals between claims are of several types (let it be m), and they are connected in a Markovian chain with a known transition matrix $\{\pi_{i,j}\}$. The type of an interval defines a type of a claim that is its distribution function. Let $P_j(u)$ be an unruin probability in the infinite time interval, provided that u is an initial capital and a claim of the j-th type was the first. Then the followig system of integral equations may be put down

$$P_j(u) = \int_{-u}^{\infty} \sum_{k=1}^{m} \pi_{j,k} P_k(u+y) d\Phi_{j,k}(y), \quad j = 1, ...m. \tag{3}$$

A description of the capital dynamics for $0 < t < t_1$ should be added to the system as analogy with an initial state for Markovian chains.

3. Now we shall consider a dynamic non-Cramer's ruin model, in which every client initiates an arrival of a claim. A state of the capital is being analized at the moments $t_n = n\tau$. A stream of clients is the simplest one with the parameter λ, every client brings in a random premium and initiates an arrival of a random claim. Using the queueing theory, a client is put into service with an exponential service time interval. This system has an infinite number of service devices. In order to calculate a capital of a company at the moment t_n, it is necessary to know the number of arrived clients and serviced clients. If u_n and u_{n+1} is a capital at the beginning and the end of the interval correspondingly, then $u_{n+1} = u_n + X_n - Y_n$, where X_n and Y_n are positive and negative increments obtained during the interval τ. Unruin probabilities for two adjacent moments are interrelated in

the following manner

$$P_{n+1,j}(u) = \sum_k \int_0^\infty \varphi_{j,k}(x-u) P_{n,k}(x) dx. \qquad (4)$$

If $n \to \infty$ in (4), then we obtain a system of integral equations for $P_j(u)$, $j = 1, 2, \ldots$, which are unruin probabilities in the infinite interval under assumption that u is the initial capital and j clients were without service at the end of the first interval

$$P_j(u) = \sum_k \int_0^\infty \varphi_{j,k}(x-u) P_k(x) dx, \quad j = 1, 2, \ldots.$$

Some initial conditions should be added to the system. The system consists of an infinite number of equations. It may be done finite one in different ways, for example in the following way: if the queue is equal to some N, the next clients are rejected.

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Optimization Methods with Finite-Step Inner Algorithms*

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key words: *optimization, optimal control, regularization, numerical methods*

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In the theory of infinite-dimensional optimization, in particular, in optimal control problems, available methods (for example, gradient methods) usually exhibit weak convergence with strong convergence with respect to control. Thus, an important problem is to develop stable numerical methods with respect to control. To this end, various regularization methods are applied. The results suggested deals with an analogous task. Specifically, we propose strongly convergent (in argument) methods for a class of convex problems with inequality constraints.

For many available optimization methods, issues related to their implementability and efficience remain important. Such issues include the development of numerical methods and algorithms not involving infinite inner procedures and a search for and formulation of stopping rules. The methods suggested in this work deals with these tasks. Two numerical methods with finite-step inner procedures are proposed for solving convex infinite-dimensional minimization problems with inequality constraints. The methods are based on regularization, gradient projection, constrained gradient and dual techniques. For the methods, stopping rules are obtained, estimates for the rate of convergence in the functional are proved and strong convergence to a normal optimal element is shown. The results presented here are effective for problems with convex functionals and quadratic inequality constraints.

Exact Penalty in One Feed-back Optimal Control Problem*

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The problem of reducing a constrained mathematical programming problem to an unconstrained one has been given a great deal of attention. In most cases such a reduction is performed with the help of so–called penalty functions. At present the theory of Penalization is well developed and widely used (see, e.g., [1–2]).

The exact penalization approach is most interesting and elegant but it generally requires solving a nonsmooth problem even if the

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original one was smooth. However, recent developments in Nondifferentiable Optimization give some hope that these difficulties will be overcome. To be able to reduce a constrained optimization problem to an unconstrained one via exact penalization it is suitable to represent the constraining set in the form of equality, where the function describing the set must satisfy some conditions on its directional derivatives (or, in general, on its generalized directional derivatives).

In the present report we show how to describe the constraints — given in the form of differential equations — by a (nonsmooth) functional whose directional derivatives satisfy the required properties. This problem is reduced to a nonsmooth unconstrained optimization problem.

Let $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $t \in [0,T]$, $T > 0$ fixed, $f : \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R} \to \mathbb{R}^n$ be differentiable with respect to x and u. The functions f, $\dfrac{\partial f}{\partial x}$, $\dfrac{\partial f}{\partial u}$ are assumed to be continuous on $\mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}$.

Consider the following system of differential equations, depending on the control $u \in \mathbf{v}$ where $\mathbf{v}$ is the class of piecewise continuous (in general, measurable) on [0,T] vector functions:

$$\dot{x} = f(x,u,t), x(0) = x_0. \tag{1}$$

Let $C[0,T]$ be the class of n–dimensional vector functions $z(t)$ continuous on $[0,T]$. Consider the set

$$\Omega := \{[z,u] \mid z \in C[0,T],\ u \in V : \varphi(z,u) = 0\},$$

where

$$\varphi(z,u) := \left[\int_0^T \left(z(t) - f(x_0 + \int_0^t z(\tau)d\tau, u, t)\right)^2 dt\right]^{1/2}.$$

Note that $\varphi(z,u) \geq 0 \ \ \forall z \in C[0,T]$, $\forall u \in V$.

It is shown that if $\varphi(z,u) > 0$ then φ is differentiable (in some sense) at $[z,u]$. If $\varphi(z,u) = 0$ then φ is directionally differentiable (in some sense) at $[z,u]$ (even subdifferentiable). Let us consider the problem of minimizing the functional

$$\mathcal{I}(u) = \int_0^T F(x(t,u))dt,$$

where $x(t,u)$ is the solution of (1) with $u \in V$, and $F(x)$ is a smooth function.

This problem is equivalent to that of minimizing the functional

$$\varphi(z,u) = \int_0^T F(x_0 + \int_0^t z(\tau)d\tau)dt$$

subject to the constraint $\varphi(z,u) = 0$.

Theorem. *If φ is Lipschitz on $C[0,T] \times V$ then there exists a $\lambda_0 \geq 0$ such that for any $\lambda \geq \lambda_0$ the set of minimizers of φ on the set $\Omega = \{[z,u] | \varphi(z,u) = 0\}$ coincides with the set of minimizers of the function*

$$\psi_\lambda(z,u) = \varphi(z,u) + \lambda\varphi(z,u)$$

on the entire space $C[0,T] \times V$.

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Parameter Estimation by Maxmin Method of Likehood Function

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This article is motivated by the need for more effective methods of optimum decision- making problem in a task of estimation of parameters when a priori uncertainty exists regarding of additive obstacle components maintained in equation of observation. The solution of the task is found upon guaranteed result principle. According to this principle in presented paper a method has developed for treatment of sample-date in order to estimate vector of parameters with using likehood function. The novelty of the solution is in transformation of maximinimum task to full-problem of eigenvalues for positively definite matrix.

We consider the following linear equation

$$L = FA + \xi + \vartheta, \tag{1}$$

where L is sample-date, F is $N \times n$ matrix, A is vector of parameter estimations of size n, ξ is random vector of errors of sampling observations of size N, this vector is normally distributed with zero expectation and covariance matrix K_ξ of size $N \times N$, and ϑ is vector of size N. It is known that elements ϑ_r, $r = \overline{1, N}$, of this vector satisfy following condition:

$$|\vartheta_r| \leq \Delta_r, \quad r = \overline{1, N},$$

where $\Delta = (\Delta_1, \Delta_2, \ldots, \Delta_N)^T$ is a given vector.

We would like to notice that the equation written above is analog to the equation of observation in vector-matrix form. In connection with such a priori uncertainly of values of vector ϑ we shall use maximinimum principle for parameters estimation of vector A in order to guarantee accuracy of their calculating. Then according maximum likehood method an estimation of vector A is calculated from following criterion

$$I = \max_{|\vartheta| \leq \Delta} \min_{A} (L - \vartheta - FA)^T Q (L - \vartheta - FA), \tag{2}$$

where is inverse covariance matrix for K_ξ that is $Q = K_\xi^{-1}$.

For fixed vector ϑ minimum of (2) is achieved when

$$A = G^{-1} F^T Q (L - \vartheta), \tag{3}$$

where $G = F^T Q F$.

Substituting expression (3) in functional (2) we obtain

$$I = \max_{|\vartheta| \leq \Delta} (L - \vartheta)^T C (L - \vartheta), \tag{4}$$

where $C = Q - QD - D^T Q + D^T Q D$ and $D = F G^{-1} F^T Q$.

Since matrix Q is positively definite then matrix C is positively definite too. If we reorder terms of the expression (4) and ignore term $L^T C L$ which does not depend on ϑ then functional (4) is reduced to following form

$$\vartheta^T \tilde{N} \vartheta - 2 L^T C \vartheta. \tag{5}$$

Let matrix P be the matrix of eigenvectors of matrix C. Since matrix C is positively definite then all its eigenvalues λ_i, $i = \overline{1, N}$ are positive, that is $\lambda_i > 0$.

Now we use Py instead of ϑ. Then quadratic form (5) may be written like this

$$\lambda_1 y_1^2 + \cdots + \lambda_N y_N^2 + \beta_1 y_1 + \cdots + \beta_N y_N, \tag{6}$$

where β_i, $i = \overline{1, N}$, is element of vector $\beta = -2L^T CP$.

We have equality

$$\vartheta_1^2 + \cdots + \vartheta_N^2 = y_1^2 + \cdots + y_N^2 \qquad (7)$$

for vectors ϑ and y. It follows from (7) that this equality will be fulfilled if $\vartheta_i^2 = y_i^2$, $i = \overline{1, N}$. But since $|\vartheta_i| \leq \Delta_i$, $i = \overline{1, N}$, then for y_i, $i = \overline{1, N}$, we have

$$|y_i| \leq \Delta_i, \quad i = \overline{1, N}. \qquad (8)$$

In this case maximum of expression (6) must be equal to sum of its components

$$\lambda_i y_i^2 + \beta_i y_i, \quad i = \overline{1, N}, \qquad (9)$$

with constraints (8).

Maximum of expression (9) is achieved at the following values of variables

$$\vartheta_i = y_i = \begin{cases} \Delta_i, & \text{when } \beta_i \geq 0, \\ -\Delta_i, & \text{when } \beta_i < 0, \end{cases}, \quad i = \overline{1, N}. \qquad (10)$$

Now if we substitute elements of vector from (10) into equation (3) then we obtain maximinimum estimation of vector A and guaranteed value for covariance matrix of estimations of vector A is equal $K_\xi = G^{-1} F^T Q [K_\xi + \Delta\Delta^T](G^{-1} F^T Q)^T$.

Thus we obtain the following result.

Theorem. If sample-date (1) has be with a priori uncertainty of additive obstacle vector ϑ then optimum estimation of vector A is guarantee its values. Such estimation is calculated from expression (3) under condition that vector ϑ is determined from (10) and accuracy characteristic of its components is determined from (11)

Multicriteria Optimization Methods in Information Process Engineerings of Control

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key words: *multicriteria problem, guarantee result method, criteria normalization*

In the given paper the formalized statement of primary goals of innovational activity — optimal choice of the investment project and creation of an optimal portfolio of potential investors — as tasks of multicriteria optimization is submitted. The algorithm of their solution developed by the author is offered, on the basis of a method of guaranteed result and normalization of criteria.

Economic Motivation of Coalition Formation in the International Military Campaigns

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key words: *cooperative games, coalition formation, coalition stability, military campaign*

A simple model of coalition formation by the economically motivated countries is considered. It is found that it can be profitable for the countries with weak economies to participate in such a coalition, while the better off countries can incur losses in this case. The structure of a stable coalition is determined.

It is shown for the members of anti-Iraqi coalition of 2003 that the characteristics of military forces for the countries with weak economies satisfy this model's predictions, while the ones for almost all well-developed NATO countries do not.

The experience of anti-Iraqi coalition formation in the military campaign of 2003 shown that the participation of at least some countries in it may be caused by economic motives. We could see that despite the resistance of the major economies in Europe — France and Germany, several Eastern Europe and CIS countries with relatively weak economies, some of which are not NATO members, supported this operation.

Here we construct a simple model of coalition formation by the economically motivated countries. We show that it can be profitable for the countries with the weak economies to participate in such a coalition, while the better off countries can incur losses in this case. We determine the structure of the stable coalition in this model.

The model includes N countries, each of them can become a member of a certain coalition which is formed in order to carry out a military operation. Each country is characterized by its economic potential λ_i which represents an aggregate measure of the country's welfare.

Each member of the coalition provides for the campaign a military force which is characterized by military potential θ_i. This value measures the ability of the military force to control occupied territory and to maintain order on it. As a proxy for this measure we take the number of the troops of a given country.

We assume that the expected gain from the campaign which is carried out by the coalition K is an increasing concave function $V(\theta_K)$ of the coalition military potential

$$\theta_K = \sum_{i \in K} \theta_i.$$

This function represents the overall economic gain net of the cost of the military campaign maintenance. We also introduce another type of costs which are related to the reconstruction of the occupied country's economy. We assume that the costs of this type are fixed and equal to c.

If the coalition K has more than one member the problem of the gains and costs sharing arises. We assume that the costs of military campaign and its gains are distributed according to the military potentials of its members, while the costs of reconstruction are borne primarily by the economically developed countries.

Then the total expected gain of the ith country in case of participation in the coalition K is

$$u_i(K) = \frac{\theta_i}{\theta_K} V(\theta_K) - \frac{\lambda_i}{\lambda_K} c. \tag{1}$$

In the case of non-participation the country's gain is zero.

The gain of ith country can be rewritten in a form

$$u_i(K) = \frac{\theta_i}{\theta_{-i} + \theta_i} V(\theta_{-i} + \theta_i) - \frac{\lambda_i}{\lambda_{-i} + \lambda_i} c, \tag{2}$$

where θ_{-i} and λ_{-i} are total military and economic potential of the coalition $K \backslash \{i\}$.

By differentiation of (2) with respect to θ_i, λ_i, θ_{-i} and λ_{-i}, one can obtain the following result.

Proposition 1. *1) The gain of ith country (2) is an increasing function of its military potential θ_i and a decreasing function of its economic potential λ_i.*

2) The gain of ith country (2) is an increasing function of the total economic potential of the other coalition members λ_{-i} and a decreasing function of the other coalition members military potential θ_{-i}.

This simple result shows the reason why the weakly-developed countries are willing to participate in a military campaign, while the better off countries do not.

The participation condition of the ith country in the coalition K is

$$\frac{\theta_i}{\theta_K} V(\theta_K) - \frac{\lambda_i}{\lambda_K} c \geqslant 0. \tag{3}$$

If the values of θ_K and λ_K are fixed then the condition (3) defines the set of parameters (λ_i, θ_i) of possible members of K. The boundary of this set is the line defined by

$$r_i = \frac{\theta_i}{\lambda_i} = \frac{\theta_K}{\lambda_K} \frac{c}{V(\theta_K)} = r_K \frac{c}{V(\theta_K)}.$$

We refer to value r_i as ith country's militarization level. It follows from the proposition 1 that it is more profitable to participate in the same coalition for a country with higher militarization level.

Let's study the possible structure of the coalition which can form according to this model.

The first condition this coalition should satisfy is the participation condition (3) for each member of K. We call such coalition *weakly stable*.

The weak stability condition implies that the campaign carried out by coalition K has non-negative economic effect

$$V(\theta_K) \geqslant c, \tag{4}$$

which gives us necessary condition for existence of weakly stable coalitions.

The second stability condition is the absence of incentives for non-member countries to join the coalition K. Because the gain of a non-member is zero the corresponding condition is of the form

$$\frac{\theta_j}{\theta_K + \theta_j} V(\theta_K + \theta_j) - \frac{\lambda_j}{\lambda_K + \lambda_j} c < 0, \text{ for all } j \notin K. \tag{5}$$

The coalition K which satisfies both condition (3) for each $i \in K$ and the condition (5) is called here *stable*.

The notion of stable coalition in this problem is equivalent to Nash equilibrium in the following game. There are n players, each of them has two strategies: "to cooperate" and "to be alone". The ith player's gain is equal to zero if he chooses to be alone and is equal to $u_i(K)$ if all members of K (and only them) choose to cooperate and $i \in K$. Then the pure strategies of the subset K of players who choose to cooperate form Nash equilibrium if and only if K is a stable coalition.

It is possible to find some examples when there is no stable coalition in this problem. Fortunately, the necessary and sufficient condition of the existence of a stable coalition can be formulated.

Proposition 2. *Let the players be ordered according to their militarization level. The stable coalition exists if and only if for some $k \leqslant n$ there exists a weakly stable coalition of k countries with greatest militarization levels.*

The proof of this proposition allows to show explicitly the structure of such a coalition, that is pointed out in the corollary.

Corollary. *Each stable coalition is the subset of $k \leqslant n$ countries with highest militarization levels.*

So, the model considered predicts that the economically motivated countries form the coalitions of the specific structure: it consists of one or several "donor" countries with well-developed economy which bear the major part of costs, and several countries with weak economy but high militarization level which try to get some economic benefits from the participation in the campaign.

It is interesting to look at the anti-Iraqi coalition structure of 2003 military campaign from the point of view of this model. For this purpose we use the number of troops of each member of the coalition to measure its military potential θ_i and the aggregated economic indicators from the World Bank database for the economic potential λ_i.

We find that most of the countries with weak economies in the coalition which took part in the military action (67 %) had higher militarization levels than the leaders of this campaign, the United States and Great Britain. At the same time almost all NATO countries which participated in this campaign had significantly lower militarization levels.

Such a situation indicates that the participation of CIS and Eastern Europe countries with the weak economies in this coalition could be motivated by possible economic benefits, while the most of NATO countries had other incentives to join the coalition.

Optimization Based Flow Control in Communication Networks with Moving Nodes

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The class of network flow control problems is rather large and thoroughly investigated (see, e.g., [1]). Recently, the development of the so-called *elastic* computer and telecommunication networks which admit the variable data transmission rate (such as networks using the TCP/IP protocols and ATM networks) caused a lot of new problems. Most of them imply the optimization of some performance criterion which describes the *total utility of the network*. A lot of problems is related to the performance of *wireless networks* which have several specific features such as the absence of predefined physical links between the nodes, the mobility of nodes, and the boundedness of the batteries capacity. Because of the stochastic character of the nodes movement and absence of any central decision maker, the usual solution methods for such problems are *simulation based*. However, we believe that *optimization based* approach may be useful for the assessment of the network effectiveness and for working out of some control solutions.

In this paper, we consider several optimization problems for wireless networks and propose the corresponding solution techniques. We treat the nodes of a wireless network as moving objects located in some bounded plane area. We suggest to consider the movement of each node as an independent *Markovian chain*, i.e., given the location of a certain point at a given time moment we can obtain the probabilities of its location at the next moment. For simplicity, in this paper, we implement the particular case of this model which is based on the so-called transition probabilities. Obtaining the probability distribution for each node location enables us to calculate any other variable such as the average distance for a pair of nodes, the probability that a node is "heard" by another one, etc. Regardless of the way we calculate the probability distribution, we thus define the *network topology.*

Each node can act as an *origin*, a *destination* or a *transmitter* of data packages. We first focus on evaluating the network parameters which are related to flows distribution and energy consumption. The corresponding optimization problems can be viewed as certain generalizations of the well known problem of maximal flow.

For any i, j, we denote by f_{ij} the flow from node i to node j. Evidently, $\sum\limits_{j\in I(i,q)} f_{ij}$ is the total flow *sent* by node i; $\sum\limits_{j:i\in I(j,q)} f_{ji}$ is the total flow *received* by node i, where $I(i,q)$ denotes the set of nodes which hear node i within the prescribed probability q. For short, we denote these flows by $f_i^{(1)}, f_i^{(2)}$, respectively. The battery capacity consumption for node i is $\alpha_i \cdot f_i^1 + \beta_i \cdot f_i^2$. Without loss of generality, we assume that the sets of origins, destinations and "pure" transmitters do not intersect. (Otherwise, we enumerate the nodes which combine two or three roles as separate nodes.) We denote the mentioned sets by O, D, and T, respectively. We write the flow balance constraints:

$$\sum_{o\in O} f_o^{(1)} = \sum_{d\in D} f_d^{(2)} \tag{1}$$

$$f_i^{(1)} = f_i^{(2)}, \forall i \in T, \tag{2}$$

and the flows nonnegativity constraints

$$f_{ij} \geq 0, \forall i \neq j;\ \ f_{i,i} = 0, \forall i. \tag{3}$$

We consider the following basic problems:

a) *Maximization of the total flow via the network*

$$\sum_{o\in O} f_i^{(1)} \to \max$$

subject to (1) – (3), and the following constraint on energy consumption:

$$\alpha_i \cdot f_i^{(1)} + \beta_i \cdot f_i^{(2)} \leq \gamma_i,\ \forall i,$$

where each γ_i is the battery capacity which provides the transmission level u_i for node i.

b) *Minimization of energy consumption*

$$\sum_i \alpha_i \cdot f_i^{(1)} + \beta_i \cdot f_i^{(2)} \to \min$$

subject to (1), (2), (3), and the constraint on the total originating flow:

$$\sum_{o \in O} f_o^{(1)} \geq \xi.$$

Since the problems stated above are *linear programming problems*, we can obtain their precise solutions within a finite number of steps of a linear programming technique (see, e.g., [2]). We can also formulate the corresponding dual problems and solve them approximately in order to obtain satisfactory feasible solution within an acceptable time interval. In [3], we investigate the computational aspects for network flows optimization problems. The above problems enable us to estimate the characteristics of a network a priori, without conducting any experiments or computer simulation.

In practice, there often occur various problems of operating control of networks. Below we show that the latter can be formulated as generalization of the above problems. Namely, introducing flow variables for each particular origin-destination pair, we obtain extensions of the above optimization problems, whose solutions enable us to construct routing tables throughout the network. The new problems are also either *linear programming* problems or can be reduced to those, we can solve them by the linear programming techniques. Taking into account the specificity of the matrix of constraints coefficients, we can apply the *decomposition procedures.* In [3] we propose the so-called *adaptive approach* to solve the linear programming problems of network control, which implies the step-wise constructing of the set of constraints (or variables) of the problem. This approach is applicable to the problems described in this paper. This enables us to reduce the dimension of the problem, which is being solved on each step, and to accelerate the convergence of the algorithm by using the previous optimal solution as an initial one at a current step. In [3] we also describe the *dual approach*, which can be used if we need to obtain some feasible solution within a short time interval. Ibid, the results of numerical tests are described.

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Import Duties and International Trade Competition*

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Consider n identical countries, denoted by $i = 1, \ldots, n$. Each country has a government that sets an import duty, a firm, that produces output for home consumption and export, and consumers who buy at the home market from either the home firm or foreign firm. If the total quantity at the market in country i is Q_i, then the market-clearing price is $P_i(Q_i) = a - Q_i, a > Q_i$. The firm in country i produces h_{ij} items for market in j's country. Thus, $Q_i = \sum_{j=1}^{n} h_{ij}$. The firms have a constant marginal cost, c, and no fixed costs. Thus, the total cost of production for firm i is $C_i(h_{i1}, \ldots, h_{in}) = c \sum_{i=1}^{n} h_{ij}$. The firms also incur tariff costs on exports: if firm i exports h_{ij} to country j, when government j has set import duty t_j, then firm i must pay $t_j h_{ij}$ to government j.

This situation may be described by game:

$$, = \langle X_i, Y_i;\ t_i \in [0; \infty), h_{ij} \in [0; \infty);\ F_i, G_i;\ i, j = 1, \ldots, n\rangle,$$

where X_i is government, Y_i is firm; t_i is government's strategy (import duty), h_{ij} is firm's strategy (quantity of production from i's country to j); F_i is government's payoff function, and G_i is firm's payoff function.

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The timing of the game is as follows. First, the governments simultaneously choose import duties, $t_1, \dots, t_n$. Second, the firms observe the import duties and simultaneously choose quantities for home consumption and for export, $(h_{11}, \dots, h_{1n}), \dots, (h_{n1}, \dots, h_{nn})$. Third, payoffs are profit to firm i and total welfare to government i, where total welfare to country i is the sum of the consumers' surplus[1] enjoyed by the consumers in country i, the tax for the profit earned by firm i, and the tariff revenue collected by government i from firms $j:\ j \neq i$:

$$G_i(h_{11}, \dots, h_{nn};\ t_1, \dots, t_n) =$$

$$\left(a - \sum_{k=1}^{n} h_{ki}\right) h_{ii} + \sum_{j \neq i} \left(a - \sum_{k=1}^{n} h_{kj}\right) h_{ij} - -c \sum_{j=1}^{n} h_{ij} - \sum_{j \neq i} t_j h_{ij}$$

$$F_i(h_{11}, \dots, h_{nn};\ t_1, \dots, t_n) = \frac{1}{2} k_i Q_i^2 + s_i G_i + t_i \sum_{k \neq i} h_{ki},$$

where s_i is profit tax in i's country, $s_i \in [0;1]$, and k_i is a constant, $k_i \in [0;1]$.

Theorem. In the game , , there is a single equilibrium in pure strategies $(t_i^*, h_{ij}^*,\ i, j = 1, \dots, n)$. These strategies are:

$$h_{ii}^* = [n(1-k_i) + 3 + k_i]T;\ h_{ij}^* = [2 - 2s_j + k_j]T, j \neq i;$$

$$t_i^* = [n(1-k_i) + 2s_i + 1]T,$$

where $T = (a-c)/[n(4 - 2s_i - k_i) + 4 + 2s_i + k_i]$.

The basic model with $n = 2$ was taken from the book [1].

If we consider the price function $P_i = \dfrac{a}{Q_i}$ and $n = 2$, it is possible to show that

$$h_{ii}^* = a[(6c - k_i p_i^{lim})(k_i p_i^{lim} - 2c(2s_i - 1))]/[16c^3(2 - s_i)^2];$$

$$h_{ij}^* = a[(k_j p_j^{lim} - 2c(2s_j - 1))^2]/[16c^3(2 - s_j)^2], j \neq i;$$

and

$$t_i^* = [2c(2c(s_i + 1) - k_i p_i^{lim})]/[k_i p_i^{lim} - 2c(2s_i - 1)],$$

where p_i^{lim} is a maximal price which a consumer in $i's$ country can pay for an item of the good.

[1] If a consumer buys a good for price p when she would have been willing to pay the value v, then she enjoys a surplus of $v - p$. Given the inverse demand curve $P_i(Q_i) = a - Q_i$, if the quantity sold on market i is Q_i, the aggregate consumers surplus can be shown to be $(\frac{1}{2})Q_i^2$.

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Multicriteria Game Model for Two-sided Auctions*

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Two-sided auction of buyers and sellers is treated as a two-person multicriteria game. The first player (the Seller) consolidates sellers at an auction of a single good. Each ith seller has his own cost a_i for manufacturing one unit of the good. If the auction price be c^* and the seller sell V_i^* units then his payoff f_i is equal to $V_i^*(c^* - a_i)$.

Values V_i^* and c^* depend on the auction result, which in its turn is determinate by the strategies of all the sellers (x) and buyers (y) at the auction. The first player aims in the game to maximize the vector-function

$$f(x, y) = \{V_i^*(c^* - a_i) |\ i \in I\}$$

over the set X of sellers' strategies $x = (x_i |\ i \in I)$, where I is the set of sellers and $x_i = (V_i, c^i)$ contains the volume and its price for sale that the ith seller supplies at the auction.

Similarly the second player (the Buyer) consolidates all the buyers $(j \in J)$ at the auction. The strategy $y_j = (V_j, c^j)$ of the jth buyer contains the volume and its price in his bid. The buyers' strategy is described by the vector $y = (y_j |\ j \in J) \in Y$.

Let a_j be the price of one unit of the good that represents its utility for the jth buyer. Then his payoff g_j is equal to $V_j^*(a_j - c^*)$ where V_j^* is the quantity which the jth buyer has bought at the auction that depends on (x, y). The second player aims to maximize the vector-function

$$g(x, y) = \{V_j^*(a_j - c^*) |\ j \in J\}$$

with respect to $y \in Y$.

Evidently the first player tries to maximize c^*, but the second player tries to minimize it. However the multicriteria case under

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consideration is not reduced to a simple antagonistic game. Also this game is not zero-sum multicriteria game. An approach to the game solution is proposed in the paper.

Digital Libraries with Human interFACE: Information Storage, Retrieval, and Analysis Toolkit

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1. The statement of the problem. Society and its institutions (political, social, scientific, educational, professional, and so on) produce the hutch amount of information. And modern people are forced to navigate in this information. But the real difficulties are to do it efficiently. Digital libraries are the principal way. The problems, structures, and technologies of scalable digital libraries' effective usage and creation both for small (thousands) and large (thousands of millions) documents' collections are under discussions in the paper.

To get information a person (user) wants, it is necessary to solve the set of subsequent problems:

1) a user forms inquiry to express one's needs in information;

2) a user determines the field of search (for example, one's personal computer, particular data base or digital library, Internet, English language Internet);

3) the search & retrieval tools analyze the inquiry and the description of the search field and produce formal inquiry and scenario of the search;

4) the search & retrieval tools fulfil the search;

5) analyzing the results of the search to determine the consistency of retrieved documents and inquiry. (Possible but different things are to give user thousands documents containing some words from inquiry or twenty documents of direct interest for the user);

6) the search & retrieval tools form and present to the user the results of the search on the base of their analysis;

7) now a user should remake inquiry and/or description of the search field in a hope to get necessary information in the right place and in a right time and try the hole circle once more.

Developing good digital library means creation of the tools and data structures to solve problems 1)-6) in a way avoiding multiple solving of the problem 7) by the user who wants to get information. Here, critically important are tools and data structures which can get the integral information about the sets of documents in the digital library. The user and search & retrieval tools can efficiently solve the above problems on the basis of this information. And only then user can analyze and interpret collected information for decision-making, research, work, experience, and entertainment.

The situation is that problems 1)-6) are difficult and bad solutions of at least one of the problems really lead to the impossibility to satisfy the information needs of the user. So to get right information to the user, Digital Libraries need efficient data analysis tools and data structures well suited to the analysis tools. For example, usually it is a difficult problem for the user to express realistically his/her information needs as a list of key words or as a leaf in a classification tree used for representing the semantics of a subject domain of the user's interest. And if user fails to do this, no search & retrieval tools can help to get right information. But if the user does, the tools should be in consistence with the data one can used to express information needs and data are used to describe the documents in the search field.

2. The Ideal Digital Libraries functions and data structures. The analysis of the textual information search & retrieval shows that development of the Ideal Digital Library means the creation of the toolkit for the solutions of the set of interrelated problems:

- to control of the search field;
- to help user in forming the inquiry;
- to analyze the users' inquiry;
- to search and retrieve information with high precision and recall;
- to analyze the retrieved documents;
- to form and to present ordered results of the search to the user;
- to create secondary information (secondary resources) to support efficient solving of the above problems and applied problems of the user (from both simple and comprehensive indexing of the documents to the generation of the user's pattern for individualization of the search and analysis of information).

Thus the main functions of the Ideal Digital Library for the sufficient support of the its human interface are:

- to search and retrieve information;
- to help in forming the inquiry;

- to analyze information resources and to make their descriptions;

-to create secondary precise, expressive and repeatable information to describe individually single document and to describe integrally the collections of documents;

- to get information to help user in solving applied problems on the base of the information itself or on the base of the secondary resources.

For the information search, retrieval, and analysis in the Ideal Digital Library one needs to have: - different search mechanisms (semantic search based on the computational characterization of the documents or on metadata using some thesauri or ontology — the sorts of secondary information, contextual search by words presenting in the document, not semantic metadata search — author, title, source, data of publication of the document, for example),

- the effective support for the user's forming of the inquiry,

- the possibility to incorporate the user patterns — information about user and user's behavior (evaluation of retrieved documents) during search and analysis of retrieved information and standard scenarios of search during information retrieval,

- the ordering of retrieved information by its consistency with inquiry and a sort of explanation of this ordering,

- the specific media to facilitate analysis of collected information by the user.

To form precise selectable and expressive inquiry one needs special services based on secondary information described both documents and the sets (specific collections) of documents. The usage of interesting for the user document as an inquiry and easy and comfortable finding of one interesting document in search field by using the secondary information — this is an effective way to retrieve information.

In the case of large-scale digital library or not large but personal library the development of precise, expressive and repeatable secondary information can be very expensive and time consuming for society, corporation, or person. The common way to develop secondary information is to use some standard descriptions of the documents prepared by the experts in the subject domain (non expert can do no useful description). But requirements to create precise, expressive and repeatable secondary information are contradictory. That's why society, corporation, and person need computational services to convert the collection of the texts in computer or in computer network to the Ideal Digital Library with functions discussed above.

It is wonderful that one can compute the complex secondary information to support all the functions of the Ideal Digital Library on the basis of single data structure. The efficient way to do it and to develop and use digital library with human interface is technology Key to Texts (K^2T). In this technology, we use only the dictionaries and indexes of the words and the key words of the documents as the secondary information.

3. Technology Key to Texts — the Digital Libraries toolkit. The base of our technology is the algorithm of construction semiotic pattern of a text — weighted set of words, semiotic mostly strongly connected among themselves in the text. It is wonderful that when a man makes the analysis and interpretation of this computed set of words, their intelligence and connection with subjects, contents and sense of the analyzed text is obvious. This list usually includes 20-30 words for the different volume intelligent texts (from the half of the page till the mane dozens of pages). The computations of the semiotic pattern of the text do not require any semantic information and knowledge of language grammar. These computations use original metrics to extract semiotic connected words in the texts. This metrics (proposed by M. Kreines) uses only combinatorial statistics of the words in the analyzed text and in some set of the texts, representative for language in which the analyzed text is written. The choice of reference set of the texts is equivalent to the formulation of positions of the man, who wants to perceive the concrete text. It is possible to limit such choice to the reference texts of a certain group of carriers of language, for example, professional or political. The problem of forming the reference set of the texts can be treated as implicit forming of a subset of language adequate to the subject perceiving the text.

In the technology K^2T, the text is a self-similar structure. If monothematic text is divided into two nearly equal parts, there semiotic patterns will be very similar to each other and to the semiotic pattern of the text. This self-similarity is the base for computing information to describe individually single document and to describe integrally the collections of documents.

The semantic interpretation of semiotic pattern of the text by the user of the technology is based on two basic hypotheses: 1. semiotic characteristics (semiotic connections of words in the text) determine semantics of the text; 2. to understand or to get a sense of the concrete text it is necessary to determine a reference set of the text, in which context it is necessary to perceive the concrete text.

In essence, it is practically folklore axioms in the linguists, philologists and psychologists societies. It is enough to recollect two classical formulations: the man is a style and the man is a text.

Validity of the formulated hypotheses proves to be true by high efficiency of the computing analysis of the texts in technology K^2T.

The technology K^2T assumes that it is necessary to identify uniform various forms of each word (for example, one noun in various numbers or a verb in various times, singular or plural number). Such identification enables us to take into account the concrete grammatical forms of the words for construction of semiotic patterns of the text. For this purpose, the knowledge of the morphology of the language is used. In our technology this procedure (so-called lemmatization) is based on the specific morphological analysis, which allows with high reliability to recognize various forms of concrete words of the given language. Now lemmatization (morphological analysis of the words) is working for Russian and English texts.

The semiotic pattern of the text — the weighted set of words really is unique precise, expressive and repeatable secondary information characteristic of the text. We have looked this in our experiments with Reuters Research Corpus. It includes about 800000 texts of Reuters news. In many thousands of experiments we conducted with the Corpus we got semiotic similarity of two texts more then 96% only for practically identical texts.

The dictionaries and indexes of the words of the documents and the words of the semiotic patterns of the documents (key words) are the main secondary information structures for the technology K^2T. This secondary information allows implement scaleable uniform toolkit for solving all main information storage and retrieval problems and digital libraries' functions:

- computational semantic indexing and annotation of the texts,

- development efficient tools for making inquiry and navigation into the sets of documents, including possibility to use interesting for the user document as an precise, selectable, expressive, reliable, and repeatable inquiry,

- computational integral description of the documents and collections of documents,

- high precision and recall information search and retrieval,

-precision and repeatable analysis of consistency of retrieved documents and inquiry, similarity of documents and ordering documents by their consistency to each other or to the inquiry,

- semantic classification and grouping of documents, control (if necessary) the process of classification and grouping the documents by human experts,

- dynamic analysis of enriched collections of documents to identify new items and new connections between subjects.

The base for K^2T digital library toolkit is the computation of the semiotic pattern of the text. This secondary information (generated by the technology itself) is used by K^2T as the main data to solve all the problems of information search, retrieval, and analysis. The structures of data for K^2T include texts — primary information (not obligatory), the dictionaries of the words, lemmas, the indexes of lemmas in the texts, the semiotic patterns of the texts and the indexes of lemmas in the semiotic patterns.

To make the large-scale corpus of texts interactively searchable for user, K^2T implements new type of navigation tool — adaptive interactive thesaurus (AIT). AIT is interactive construction based on the semiotic pattern of the texts of the corpus. The reader begins here work with AIT by any word from the semiotic pattern of any text of the corpus (interactive step). For the reader's usability we join the lists of words in alphabet order and use some other ways to navigate through long list of the words. The second step is done by the computer system. It presents to the reader the list of all words included in the semantic patterns of any documents together with the word the user have selected (adaptive step). Now reader selects next word of interest. And computer system presents new list of the words included in the patterns of documents with two words of reader's selection. We believe this is similar to the thesaurus because the words from the lists are key words for the texts under consideration.

AIT is a tool to solve standard hard information retrieval problems: how to construct the query with not empty set of the search results, how to construct the query with reasonable set of the search results, how to arrange effective user's feedback for search system and user.

The same time AIT is a tool to develop new services:

- aggregating large scale collections of the texts into secondary resources for efficiently information and essential knowledge retrieval (including terminological and logical structure of the subject domain),

- extracting novelty in the subject domain,

- finding the cross links in different subject domains.

We present data structures for scalable technology K^2T, scaleable hardware organization for K^2T effective realization, experience of its usage for construction and application of digital libraries, possibility to use K^2T in solving all the problems described above, and applications of technology K^2T in social and political information analysis.

Generalized Congestion Games*

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key words: *strategic game, Nash equilibrium, congestion game*

Strategic games are considered where the players participate in the functioning of certain "objects." The state of each object depends on the number of participants; the utility of each player is a continuous, symmetric, and strictly increasing function of the states of relevant objects. A list of such functions ensures the existence of a Nash equilibrium regardless of other characteristics of the game if and only if each of them is additive up to a monotonic transformation.

The notion of a congestion game was introduced by Rosenthal (1973). The class of congestion games played a central role in Monderer and Shapley's (1996) theory of potential games. Here we show that additive aggregation of intermediate utilities in congestion games is, in a sense, necessary for their nice properties.

A *generalized congestion game* , may have an arbitrary (finite) set of players N whereas the sets of strategies and utility functions satisfy certain structural requirements. There is a finite set $\mathcal{A}$ of *objects*; Rosenthal called them "factors." Each strategy is a subset of $\mathcal{A}$, $X_i \subseteq 2^{\mathcal{A}} \setminus \emptyset$. With every $\alpha \in \mathcal{A}$, an *intermediate utility function* is associated, $\varphi_\alpha : \mathbb{N} \to \mathbb{R}$.

Given a strategy profile $x \in X = \prod_{i \in N} X_i$, we denote, for each $\alpha \in \mathcal{A}$, $N(\alpha, x) = \{i \in N |\ \alpha \in x_i\}$: the set of players having chosen α at x. The "ultimate" utility functions of the players are built of

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the intermediate utilities:

$$u_i(x) = U_i^{x_i}\big(\langle\varphi_\alpha(\#N(\alpha,x))\rangle_{\alpha\in x_i}\big), \tag{1}$$

where $i \in N$, $x \in X$, and $U_i^{x_i}$ is a numeric function defined on the appropriate subset of $\mathbb{R}^{x_i}$. Congestion games are a particular case of this scheme, where each $U_i^{x_i}$ is just the sum of its arguments.

The concept of a *universal aggregator* will be used; it is perceived as an infinite sequence of functions $U^{(m)} : \mathbb{R}^m \to \mathbb{R}$, $m = 1, 2, \ldots$, each of which is assumed continuous, symmetric (w.r.t. any permutation of the arguments), and *strictly increasing* in the sense of

$$\big[\forall s[v'_s \geq v_s] \cap \exists s[v'_s > v_s]\big] \Rightarrow U^{(m)}(v') > U^{(m)}(v). \tag{2}$$

We say that a player $i \in N$ in a game , *uses a universal aggregator* U if the appropriate $U^{(m)}$ is substituted into (1):

$$u_i(x) = U^{(\#x_i)}\big(\langle\varphi_\alpha(\#N(\alpha,x))\rangle_{\alpha\in x_i}\big) \tag{3}$$

for every $x \in X$. The assumed symmetry of $U^{(m)}$ ensures an unambiguous meaning of (3), which is difficult to achieve without the symmetry.

Theorem 1. Let N be a finite set with $\#N \geq 2$; let $\langle U_i\rangle_{i\in N}$ be a list of universal aggregators such that every function $U_i^{(m)}$ is symmetric, continuous, and strictly increasing in the sense of (2). Then every generalized congestion game, where N is the set of players and each player i uses the aggregator U_i possesses a Nash equilibrium if and only if both following conditions hold:

(i) there is a continuous and strictly increasing mapping $\nu : \mathbb{R} \to \mathbb{R}$ and a continuous and strictly increasing mapping $\lambda_i^m : m \cdot \nu(\mathbb{R}) \to \mathbb{R}$ for every $i \in N$ and $m \geq 1$ such that

$$U_i^{(m)}(v_1, \ldots, v_m) = \lambda_i^m\Big(\sum_{s=1}^{m} \nu(v_s)\Big) \tag{4}$$

for all $v_1, \ldots, v_m \in \mathbb{R}$;

(ii) for every $i \in N$ and $m, m' \geq 1$, either $\lambda_i^m(m \cdot \nu(\mathbb{R})) \cap \lambda_i^{m'}(m' \cdot \nu(\mathbb{R})) = \emptyset$ or there is a constant $\bar{u}_i^{mm'} \in \mathbb{R}$ such that

$$\lambda_i^{m'}(u) = \lambda_i^m(u + \bar{u}_i^{mm'}) \text{ for all } u \in \mathbb{R} \tag{5}$$

(hence $\lambda_i^m(m \cdot \nu(\mathbb{R})) = \lambda_i^{m'}(m' \cdot \nu(\mathbb{R}))$).

The sufficiency part is all but identical with Rosenthal's (1973) theorem. An important role in the necessity proof is played by the famous Debreu–Gorman Theorem on additive representation of separable orderings. The proof remains valid, virtually without any modification, if each $U^{(m)}$ is assumed defined on R^m, where R is an open interval (bounded or not) in $\mathbb{R}$, e.g., $R = \mathbb{R}_{++}$. If R is not open (e.g., if only integer-valued φ_α are considered), the proof collapses. I have no idea whether the theorem itself remains valid in this case.

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Lipschitz Stable Stationary Points Under Canonical Perturbations of NLP's or Variational Inequalities

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key words: *Lipschitzian solutions, stability, reduction to quadratic objectives, non-invariance w.r. to quadratic approximations of constraints*

For standard optimization programs in finite dimension, linear variations of the objective and RHS-perturbations of the constraints will be considered. An analytical criterion will be presented, in terms of original data and under a constraint qualification weaker than MFCQ, which ensures that related stationary points are locally unique and Lipschitz near a reference point. We show that (in contrast to the same behaviour for KKT-points or under LICQ) this stability property does not only depend on a couple of derivatives of the involved functions at the reference point (even for convex problems), and describe the stability by means of stationary points for a family of quadratic programs, too. We also discuss some particular cases (linear-quadratic constraints) and relations to other stability notions. Basic statements of the talk have been developed by joint work with D. Klatte.

Decomposition in Variables for Nonlinear Optimization Problems*

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1. A block mathematical programming problem with coordinating variables is considered: to find

$$\min_{y,x}\left\{\sum_{q=1}^{Q} f_q^0(y,x^q) : f_q^i(y,x^q) \le 0, i=1,...,I_q, q=1,...,Q\right\}, \quad (1)$$

where $f_q^i(y,x^q)$ - are convex proper functions of the vector (y,x^q), $y \in E^L$, $x^q \in E^{N_q}$, $i=0,...,I_q$, $q=1,...,Q$.

Let coordinating variables y be fixed.

Denote $D_q(y) = \left\{x^q \in E^{N_q} : f_q^i(y,x^q) \le 0, i=1,...,I_q\right\}$ and define the function $\Phi^q(y)$:

$$\Phi^q(y) = \begin{cases} \min\left\{f_q^0(y,x^q) : x^q \in D_q(y)\right\}, y \in W_q, \\ +\infty, y \notin W_q, \end{cases} \quad (2)$$

where W_q is a set of such values of the vector y, for which the optimization problem in (2) has a solution.

The following problem, equivalent to the initial one (1), is solved in decomposition schemes: to find

$$\min\left\{\sum_{q=1}^{Q} \Phi^q(y) : y \in E^L\right\}. \quad (3)$$

Properties of functions $\Phi^q(y)$ are studied in [1].

Here we describe the following results: procedures for calculating ε -subgradients of function $\Phi^q(y)$, a regularisation of the problem (1) permitting to build the functions, analogues to the functions $\Phi^q(y)$ and with finite values in the whole space E^L.

2. If it does not cause ambiguities, we omit the index q in notations for functions $\Phi^q(y)$, $f_q^i(y,x^q)$, and sets $W_q, D_q(y)$. Let

$$D = \left\{(y,x) : f^i(y,x) \le 0, i=1,...,I\right\}$$

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be a non-empty closed set. It is assumed that $D \in \text{int dom} f^i, i = 0, ..., I$.

Theorem 1. *Let* $(\overline{y}, \overline{x}) \in \text{int dom} f^i$, $i = 0, ..., I$, $\overline{y} \in \text{int } W$, $\overline{x} \in D(\overline{y})$, *the set* $D(\overline{y})$ *satisfy the Slater condition; for some numbers* $\overline{u}_i \geq 0, i = 1, ..., I$, ε_i*-subgradients* $g^i = (g^i_y, g^i_x)$ *of functions* f^i, $i = 0, 1, ..., I$, *calculated at the point* $(\overline{y}, \overline{x})$, *satisfy the relations* $g^0_x + \sum_{i=1}^{I} \overline{u}_i g^i_x = 0$. *Then* $\Phi(\overline{y}) \geq f^0(\overline{y}, \overline{x}) - \overline{\varepsilon}$, *and the* $\overline{\varepsilon}$*-subgradient of the function* $\Phi(y)$ *at the point* $y = \overline{y}$ *is equal to* $g^{\overline{\varepsilon}}_{\Phi}(\overline{y}) = g^0_y + \sum_{i=1}^{I} \overline{u}_i g^i_y$, *where* $\overline{\varepsilon} = \varepsilon_0 + \sum_{i=1}^{I} \overline{u}_i(\varepsilon_i - f^i(\overline{y}, \overline{x}))$.

The sequence of points $\{x_1, x_2, ..., x_m\}$ from E^N is given, $x_m \in D(\overline{y})$, the values of $f^{ik} = f^i(\overline{y}, x_k)$ and subgradients $g^{ik} = (g^{ik}_y, g^{ik}_x)$ of functions $f^i, i = 0, ..., I, k = 1, ..., m$ are calculated in every point $(\overline{y}, x_k)$.

Consider a linear approximation of the problem (2) for the fixed $y = \overline{y}$: to find

$$\min_{x,\xi} \xi, \tag{4}$$

subject to

$$f^{ik} + (g^{ik}_x, x - x_k) \leq \xi, k = 1, ..., m, i = 0, \tag{5}$$

$$f^{ik} + (g^{ik}_x, x - x_k) \leq 0, k = 1, ..., m, i = 1, ..., I. \tag{6}$$

Theorem 2. *Let* ξ^*_m *be an optimal value,* $\overline{u}_{ik}$ *be optimal values of dual variables,* $k = 1, ..., m, i = 0, ..., I$, *in the problem (4)-(6). Then* $g^{\overline{\varepsilon}}_{\Phi}(\overline{y}) = \sum_{k=1}^{m} \sum_{i=0}^{I} \overline{u}_{ik} g^{ik}_y$, *where* $\overline{\varepsilon} = f^{0m} - \xi^*_m$.

This theorem permits to formulate stopping rules for approximate solution of the problem (2).

The use of the proposed results will be especially effective in the cases when the initial problem (1) splits into sub-problems such that:

- the main part of these sub-problems has a special structure, and there exist effective algorithms to solve them,

- the number of general type sub-problems is rather small, and they have moderate dimensions.

The program realisation was used to solve test problems. Two methods were used to find an approximate solution to the problem (2): the modified linearisation method by B.N. Pshenichny [3] and r-algorithm [4]. The r-algorithm was also used to solve the master problem (3). The results of numerical experiments are discussed.

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On Compensation of Disturbances in Nonlinear Controlled Systems*

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A dynamical system described by a nonlinear (with respect to a phase variable) differential equation is considered. This equation may be an ordinary one with time delay or a parabolic one. A trajectory of the system, $x(t)$, $t \in [0, T]$, depends on the input action $v = v(t)$ varying in time as well as on the control $u = u(t)$ (the system is linear with respect to both u and v). A priori u, v and x are unknown. We only know that $v(t)$ is a square integrable function. This function is

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treated as a disturbance that should be compensated as far as possible by action on the output x. Thus, the problem in question consists in constructing an algorithm of choice of a control $u(t)$ compensating a disturbance $v(t)$.

This problem was investigated by many authors. One of approaches to solving it is to choose a control u as a feedback loop that annuals or weakens (with respect to an appropriate cost functional) acting of a disturbance. As an example of this approach, the theory of H_∞-control [1] can be mentioned. In this theory u is a feedback control. Another approach to solving the problem of compensation of disturbances was suggested by Wonham M [2]. This approach is developed by the theory of almost invariant subspaces. In the case when "instant" constraints on u and v are given (for example, we assume that

$$u(t) \in P, \quad v(t) \in Q \quad \text{for all} \quad t \in [0, T], \tag{1}$$

P and Q are closed bounded sets), the well-known in the theory of positional differential games method of stable paths [3] can be used for solving the problem under investigation.

In the present paper, we consider the case when a priori information (in the form of a set of constraints) is absent. We use ideas of the method of dynamical inversion developed in [4] in order to identify (with a small delay) an input v and then, by choosing u, to compensate the action of v. Thus, we construct a feedback regulator which is capable to adapt to any real input v. Although we cannot completely eliminate the influence of disturbance, nevertheless, we present solving algorithms which are robust with respect to measurements of an output x at discrete time moments as well as to observation errors. Note that a similar method was used in [5] in the case when a priori information on integral constraints imposed on a pair (u, v), i.e., on inequalities

$$\int_0^T |u(\tau)|^2 \, d\tau \le \mu, \quad \int_0^T |v(\tau)|^2 \, d\tau \le \nu, \quad \mu \le \mu$$

is available. Under constraints of the form (1), the problem in question was solved in [6]. This problem was considered for linear with respect to a phase variable systems in [7].

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Projection Two-step Variable Metric Methods

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1. This report discusses projection two-step variable metric methods (VMM) for unconstrained minimization (see, for example, [1]). and estimates the rate of convergence. Consider the minimization problem on convex closed set $Q \subset E^n$

$$f(\mathbf{x}) \longrightarrow \inf, \quad \mathbf{x} \in Q \subset E^n, \tag{1}$$

where E^n is n-dimensional Euclidean space normalized by scalar product, $\|\mathbf{x}\| = (\mathbf{x}, \mathbf{x})^{1/2} \forall \mathbf{x} \in E^n$; $f(\mathbf{x}) \in C^{1,1}(E^n)$ is a convex function and

$$\inf f(\mathbf{x}) = f_* > -\infty, \mathbf{x} \in Q; Q_* = \{\mathbf{x} \in Q : \quad f(\mathbf{x}) = f_*\} \neq \emptyset; \tag{2}$$

function $f(\mathbf{x})$ has prolate level surfaces (when multi-step methods can be successfully used). For solving problem (1) we use generalized two-parameter projection two-step variable metric methods. We will construct iterative VMM based on the idea from [2] developed in [3] for designing continuous variable metric method. For that purpose we introduce a new metric in the space E^n by scalar product $(\mathbf{B}(\mathbf{x})\mathbf{u}, \mathbf{u})$, where $\mathbf{B}(\mathbf{x}) : E^n \longrightarrow E^n$ for every fixed $\mathbf{x} \in E^n$ is a self-conjugate positive definite linear operator which modifies the metric of the space E^n. We denote by $P_Q^{B(x)}[\mathbf{v}]$ the projection $\mathbf{w} \in Q$ of the point $\mathbf{v} \in E^n$ to the set Q in the new metric. In the constructed space, we consider two-step two-stage projection VMM

$$\begin{aligned} &\text{1st stage) } \mathbf{y}^k = \mathbf{x}^k - \mathbf{x}^{k-1}; \mathbf{z}^k = \mathbf{x}^k + \alpha_k \mathbf{y}^k; \\ &\text{2nd stage) } \mathbf{x}^{k+1} = P_Q^{B(x)}\left[\mathbf{z}^k - \gamma_k \mathbf{B}(\mathbf{z}^k)^{-1}\mathbf{f}'(\mathbf{z}^k)\right], k = 0, 1, 2, ..., \end{aligned} \tag{3}$$

where $\mathbf{x}^0 \in E^n$ is an initial point, $\mathbf{x}^{-1} = \mathbf{x}^0$, $\alpha_k > 0$, $\gamma_k > 0$ are parameters of the method, $\mathbf{B}(\mathbf{z}^k)^{-1}$ is the inverse operator to the $\mathbf{B}(\mathbf{x})$ at the point $\mathbf{z}^k$, its matrix is positive by definite and symmetric. The iterative formulae (3) determines the family of the variable metric methods for different types of choice of the parameters for the method and for operator $\mathbf{B}(\mathbf{x})$.

2. The special case of the method (3) for $f(\mathbf{x}) \in C^{2,1}(E^n)$ and $\mathbf{B}(\mathbf{x}) = \mathbf{f}''(\mathbf{x})$ is two-step two-stage VMM of the second order

$$\begin{aligned} &\mathbf{y}^k = \mathbf{x}^k - \mathbf{x}^{k-1}; \mathbf{z}^k = \mathbf{x}^k + \alpha_k \mathbf{y}^k; \\ &\mathbf{x}^{k+1} = P_Q^{B(x)}\left[\mathbf{z}^k - \beta_k \mathbf{f}'(\mathbf{z}^k)^{-1}\mathbf{f}'(\mathbf{z}^k)\right], k = 0, 1, 2, ..., \end{aligned} \tag{4}$$

where $\mathbf{x}^0 \in E^n$ is a starting point, $P_Q[\mathbf{v}] = P_Q^{\mathbf{f}''(\mathbf{x})}[\mathbf{v}]$ is a projection of the point $\mathbf{v}$ to the set Q ; $\alpha_k > 0$, $\beta_k > 0$ are parameters of the method (4), $\mathbf{f}'(\mathbf{z}^k)^{-1}$ is the inverse matrix for the matrix of the second derivatives of function $f(\mathbf{x})$ at the point $\mathbf{z}^k$. This matrix satisfies to inequalities

$$m||\mathbf{u}||^2 \leq (\mathbf{f}''(\mathbf{x})\mathbf{u}, \mathbf{u}) \leq M||\mathbf{u}||^2, 0 < m \leq M, \forall \mathbf{x}, \mathbf{u} \in E^n. \tag{5}$$

In the following theorem, conditions for convergence of the method (4) are given.

Theorem 1. *Assume that: 1) the set $Q \in E^n$ is convex and closed; 2) $f(\mathbf{x}) \in C^{2,1}(E^n)$ is a convex function; 3) the operator $\mathbf{f}'(\mathbf{x}) \forall \mathbf{x} \in E^n$ meets (5); 4) there is convex function $\varphi(\mathbf{x}) \in C^{1,1}(E^n)$ such that* *its gradient* $\varphi'(\mathbf{x}) = \mathbf{f}''(\mathbf{x})^{-1}\mathbf{f}'(\mathbf{x})\forall\mathbf{x} \in E^n$*; 5) the parameters of the method (4) meets conditions*

$$0 < \alpha_k < 1/3, 0 < \beta_k < (1 - 3\alpha_k)/[L(1 + \alpha_k^2)]. \tag{6}$$

Then for any initial point $\mathbf{x}^0 \in E^n$*, the sequence* $\{\mathbf{x}^k\}$ *found by the method (4),(6), converges to a point* $\mathbf{x}^* \in Q_*$ *in the norm* E^n *and*

$$||\mathbf{x}^k - \mathbf{x}^*|| \longrightarrow 0, \qquad f(\mathbf{x}^k) \longrightarrow f(\mathbf{x}^*), \qquad k \to \infty.$$

Now, we estimate of the rate of convergence of the method (4).

Theorem 2. *Assume that all conditions of theorem 1 are satisfied and, in addition: 1) for the sequence* $\{\mathbf{x}^k\}$ *found by the method (4),(6), there exists number* $N > 1$ *such that* $\beta_k = 1$ *for* $k \geq N$*; 2) the matrix of the second derivatives of function* $f(\mathbf{x})$ *meets Lipschitz condition* $||\mathbf{f}'(\mathbf{v}) - \mathbf{f}''(\mathbf{u})|| \leq R||\mathbf{v} - \mathbf{u}|| \quad \forall \mathbf{u}, \mathbf{v} \in E^n, \quad R = const > 0.$ *Then, for any initial point* $\mathbf{x}^0 \in E^n$*, the sequence* $\{\mathbf{x}^k\}$*, converges to the solution of problem (1)* $\mathbf{x}^* \in Q_* \subset Q$ *at a quadratic rate. The estimate for the rate of convergence is* $||\mathbf{x}^{k+1} - \mathbf{x}^*|| \leq c||\mathbf{x}^k - \mathbf{x}^*||^2$*, where* $c = R(\kappa + 2L\alpha)^2/(m\kappa^2)$*,* $\kappa > 0$ *is a strong convexity constant of the function* $f(\mathbf{x})$*.*

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About the Computation of Equilibrium Points in General N-person Games

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The general problem of the existence of equilibrium points in the mixed extention of n-person games was poved by Nash. However the relation of bimatrix games with the nonlinear programas was derived by Lemke-Hobson. Beside such a connection they provide a constuction procedure to construct for these games an algorithm, which is rather efficient to obtain at least one equilibrium point. Some other appoaches were developed by Wilson, Kuhn, Vorobev and Rosenmüller. However many of them are not effective from the actual computation point of view. In this paper, we present a new apprach for having the relationship of nonlinear programming with a great variety of n-person games related with equilibrium points. Such a characterization is one to one with the solution of the correponding programming. Besides we provide an effective algorithm for computing them. Moreover we try to study the friendly equilibria points due to the senior author of this paper. Finally some relation with the recent work of McLennon is expected.

Introduction Even though Nash's theorem asserts the existence of equilibrium points for the mixed extension of a normal game, it does not tell us how to find them. Even in the case of two matrices or two person games many algorithms have been proposed by Vorobev [10], Kühn [2] and Mangasarian [4] to determine all equilibrium pairs, they are more of theoretical interest than for actual computation. The algorithm proposed by Lemke and Howson [3] seems until now to be one of the most effective for finding an equilibrium pair.

In the case of n-person games, there have been some attemps in order to get the computation of an equilibrium point. Let us mention the works of Rosenmüller [8], Sobel [9], Wilson [11] and Garcia, Lemke and Luethi [1] about computation of equilibrium in n-person games. This last reference is relationed with the simplicial approximation of equilibrium points. This suggested the constructive, combinatorial approach of Scarf for finding, among other things, approximations to fixed-points of continuous mappings. However, the effective computation even in simple cases and low dimension is still open and very intricate questions, which have to be attacked and solved in the future.

This paper deals with a new approach based on the rather effective algorithm of Lemke and Howson to general n-person games. The definition of general n-person games is going to be accordingly in the next pages.

A new approach We assume that the readers shall know the basic definitions about n-person games given in Burger and the material in chapter VII of Parthasarathy-Raghavan [7] regarding the bi-matrix case. Here we follow the notation presented in the last book.

At the first step, we consider the problem of a 3-person game to extend the method of Lemke and Howson.

Let A, B, C be payoff matrices of dimensions $m \times n$, $n \times s$ and $s \times m$ respectively; x and x^0 vectors in R^m, y and y^0 vectors in R^n, and z and z^0 vectors in R^s. By $A > 0$ we mean that all entries in A are positive and by $x \geq 0$ that all entries in x are nonnegative.

Let $e = (1, 1, 1, ..., 1)$ be appropriate vector and E the appropriate matrix with all entries unity.

Definition. A point (x^0, y^0, z^0) is an equilibrium point of the 3-person game if:

$$\begin{array}{lclclclcl}
x^0\, A\, y^0 & \geq & x\, A\, y^0 & & \forall x & & (x,e)=1 & & x \geq 0 \\
y^0\, B\, z^0 & \geq & y\, B\, z^0 & & \forall y & & (y,e)=1 & & y \geq 0 \\
z^0\, C\, x^0 & \geq & z\, C\, x^0 & & \forall z & & (z,e)=1 & & z \geq 0
\end{array}$$

We present the following results.

Theorem 1. *A point (x^0, y^0, z^0) is an equilibrium point for the 3-person game iff for some scalars p, q, r:*

$$\begin{array}{lclcl}
(x^0, (A\, y^0 + C^T\, z^0)) & = & p + r & & A\, y^0 \leq p\, e_0 \\
(y^0, (B\, z^0 + A^T\, x^0)) & = & q + p & & B\, z^0 \leq q\, e_0 \\
(z^0, (C\, x^0 + B^T\, y^0)) & = & r + q & & C\, x^0 \leq r\, e_0.
\end{array}$$

Theorem 2. *A point (x^0, y^0, z^0) is an equilibrium point iff $(x^0,\ y^0,\ z^0,\ p,\ q,\ r)$ is a solution to the problem:*

$$\max[(x, Ay + C^T z) + (y, A^T x + Bz) + (z, Cx + B^T y) - 2(p + q + r)]$$

subject to

$$\begin{array}{l}
A\, y \leq p\, e;\ B\, z \leq q\, e;\ C\, x \leq r\, e;\ x,\ y,\ z \geq 0 \\
(x, e) = (y, e) = (z, e) = 1.
\end{array}$$

Let us consider the convex sets

$$\begin{aligned} S &= \{(x,r) : C\,x - r\,e \leq 0,\ x \geq 0,\ (x,e) = 1\}, \\ T &= \{(y,p) : A\,y - p\,e \leq 0,\ y \geq 0,\ (y,e) = 1\}, \\ R &= \{(z,q) : B\,z - q\,e \leq 0,\ z \geq 0,\ (z,e) = 0\}. \end{aligned}$$

Definition. Point (x^0, y^0, z^0, p, q, r) is called an extreme equilibrium point if (x^0, r) is an extreme point of S, (y^0, p) is an extreme point of T and (z^0, q) is an extreme point of R, and further (x^0, y^0, z^0) is an equilibrium point for A, B, C.

Theorem 3. *Any equilibrium point of a 3-person game is a convex combination of some extreme equilibrium points.*

From here we could describe all the extreme points of S, T and R. The extreme equilibrium points are those vertices which satisfy

$$(x, Ay + C^T y) + (y, A^T x + Bz) + (z, Cx + B^T y) - 2(p + q + r) = 0.$$

We have just extended this method for general n-person games having cycles in the interaction among the players. Even if this is a relevant approach result, we are far for solving all the n-person games.

Further considerations about the real algorithm for the computation are obtained and the general procedure is going to be published elsewhere.

We would like to emphasize that we may get similar results for games with rational payoff functions studied by Marchi [5] in a possible approach for studying and computing friendly equilibrium points (Marchi [6]) it seems natural but presents much further difficulties.

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Communication Leading to Nash Equilibrium Without Acyclic Condition

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key words: *awareness, belief, no speculation theorem*

Introduction. The purpose of this paper is to present the pre-play communication-process leading to a mixed strategy Nash equilibrium of a strategic form game.

The stage sets up as follows: The players start with the same prior distribution on a finite state-space. In addition, they have the private information given by the non-partition structure corresponding to the modal logic **S4**. Each player communicates privately his/her

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conjecture about the other players' actions through messages according to a protocol, and the receiver of the message updates his/her conjecture. When a player communicates with another, the other players are not informed about the contents of the message. Suppose that all players are expected utility maximizers. Then

Main theorem. *The players' predictions about the other players' actions regarding as the future conjectures converge in the long run, and those convergent conjectures constitute a mixed strategy Nash equilibrium of the game.*

The concept of Nash equilibrium has become central in game theory, economics and its related fields. Yet a little is known about the process by which players learn if they do. Recent papers by J. S. Jordan [2] and E. Kalai and E. Lehrer [3] indicate increasing interest in the mutual learning processes in Bayesian games which leads to Bayesian equilibrium.

As far as Nash's fundamental notion of strategic equilibrium (see J.F. Nash [5]) is concerned, R.J. Aumann and A. Brandenburger [1] gives epistemic conditions for Nash equilibrium. However it is not clear just what learning process lead to Nash equilibrium. The present paper aims to fill this gap. The pre-play communication process according to a protocol is proposed which is a mutual learning process leading to a Nash equilibrium of a strategic form game as a cheap talk. The emphasis is on that any topological assumption on the communication graph is not required. T. Matsuhisa [4] proved the theorem under the assumption that the graph contains no cycle.

The Model. Let Ω be a non-empty set called a *state-space*, N a set of finitely many *players* $1, 2, \dots n$ $(n \geq 2)$, and let 2^Ω be the family of all subsets of Ω. Each member of 2^Ω is called an *event* and each element of Ω called a *state*. Let μ be a probability measure on Ω which is common for all players.

Information and Knowledge. An *information structure* $(P_i)_{i\in N}$ is a class of mappings P_i of Ω into 2^Ω. It is called an *RT-information structure* if for every player i the two properties are true: for each ω of 2^Ω,

Ref $\quad \omega \in P_i(\omega)$; $\qquad$ **Trn** $\quad \xi \in P_i(\omega)$ implies $P_i(\xi) \subseteq P_i(\omega)$.

Given our interpretation, player i for whom $P_i(\omega) \subseteq E$ knows, in the state ω, that some state in the event E has occurred. In this case we say that in the state ω the player i knows E. By i's *knowledge operator* we mean the mapping $K_i : 2^\Omega \to 2^\Omega$ defined by $K_i E = \{\omega \in \Omega | P_i(\omega) \subseteq E\}$. This is the set of states of Ω in which i knows that E has occurred. We note that K_i satisfies the following properties[1]: For every E, F of 2^Ω,

N $\quad K_i\Omega = \Omega$ and $K_i\emptyset = \emptyset$; $\qquad$ **K** $\quad K_i(E \cap F) = K_iE \cap K_iF$;

T $\quad K_iF \subseteq F$; $\qquad$ **4** $\quad K_iF \subseteq K_iK_iF$.

The set $P_i(\omega)$ will be interpreted as the set of all the states of nature that i believes to be possible at ω, and K_iE will be interpreted as the set of states of nature for which i believes E to be possible. We will therefore call P_i i's *possibility operator* on Ω and also will call $P_i(\omega)$ i's *possibility set* at ω. An event E is said to be i's *truism* if $E \subseteq K_iE$ We should note that the RT-information structure P_i is uniquely determined by the knowledge operator K_i such that $P_i(\omega)$ is the minimal truism containing ω; that is, $P_i(\omega) = \bigcap_{\omega \in K_iE} E = \bigcap_{\omega \in T = K_iT} T$.

Game and Knowledge. By a *game* G we mean a *finite* strategic form game $\langle N, (A_i)_{i\in N}, (g_i)_{i\in N}\rangle$ with the following structure and interpretations: N is a finite set of players $\{1, 2, \dots, i, \dots n\}$ with $n \geq 2$, A_i is a finite set of i's *actions* (or i's pure strategies) and g_i is an i's *payoff-function* of A into $\mathbb{R}$, where A denotes the product $A_1 \times A_2 \times \cdots \times A_n$, A_{-i} the product $A_1 \times A_2 \times \cdots \times A_{i-1} \times A_{i+1} \times \cdots \times A_n$. We denote by g the n-tuple $(g_1, g_2, \dots g_n)$ and denote by a_{-i} the $(n-1)$-tuple $(a_1, \dots, a_{i-1}, a_{i+1}, \dots, a_n)$ for a of A .

A probability distribution φ_i on A_{-i} is said to be i's *overall conjecture* (or simply i's *conjecture*). For each player j other than i, this induces the marginal on j's actions; we call it i's *individual conjecture* about j (or simply i's *conjecture about* j.) Functions on Ω are viewed like random variables in a probability space (Ω, μ). If $\mathbf{x}$ is a such function and x is a value of it, we denote by $[\mathbf{x} = x]$ (or simply by $[x]$) the set $\{\omega \in \Omega | \, \mathbf{x}(\omega) = x\}$.

An RT-information structure $(P_i)_{i\in N}$ with a common-prior μ yields the distribution defined by $\mathbf{q}_i(a, \omega) = \mu([\mathbf{a} = a] | P_i(\omega))$; and i's overall conjecture defined by the marginal $\mathbf{q}_i(a_{-i}, \omega) = \mu([\mathbf{a}_{-i} = a_{-i}] | P_i(\omega))$ which is viewed as a random variable of φ_i. Here we assume that $[a_i] := [\mathbf{a}_i = a_i]$ is i's truism for every a_i of A_i. The payoff functions $g = (g_1, g_2, \dots, g_n)$ is said to be *actually played* at a state

[1]According to these we can say the structure $\langle \Omega, (K_i)_{i\in N}\rangle$ is a model for the multi-modal logic $\mathbf{S4}_n$.

ω if $\omega \in [\mathbf{g} = g] := \bigcap_{i \in N}[\mathbf{g}_i = g_i]$. The i's action a_i is said to be *actual* at a state ω if $\omega \in [\mathbf{a}_i = a_i]$.

Player i is said to be *rational* at ω if each i's actual action a_i maximizes the expectation of his actually played payoff function g_i at ω when the other players actions are distributed according to his conjecture $\mathbf{q}_i(*, \omega)$. Formally, letting $g_i = \mathbf{g}_i(\omega)$ and $a_i = \mathbf{a}_i(\omega)$, $\mathbf{Exp}(g_i(a_i, \mathbf{a}_{-i}); \omega) \geq \mathbf{Exp}(g_i(b_i, \mathbf{a}_{-i}); \omega)$ for every b_i in A_i.[2] Let R_i denote the set of all the states at which player i is rational, and $R = \bigcap_{j \in N} R_j$.

Protocol. We assume that the players in N communicate by sending *messages*. Let T be the time horizontal line $\{0, 1, 2, \cdots t, \cdots\}$. A *protocol* among N is a mapping Pr of the set of non-negative integers into the Cartesian product $N \times N$ that assigns to each t a pair of players $(s(t), r(t))$ such that $s(t) \neq r(t)$. Here t stands for *time* and $s(t)$ and $r(t)$ are, respectively, the *sender* and the *receiver* of the communication which takes place at time t. We consider the protocol as the directed graph whose vertices are the set of all players M and such that there is an edge (or an arc) from i to j if and only if there are infinitely many t such that $s(t) = i$ and $r(t) = j$.

A protocol is said to be *fair* if the graph is strongly-connected; in words, every player in this protocol communicates directly or indirectly with every other player infinitely often. It is said to contain a *cycle* if there are players $i_1, i_2, \ldots, i_k$ with $k \geq 3$ such that for all $m < k$, i_m communicates directly with i_{m+1}, and such that i_k communicates directly with i_1. The *period* of the protocol is the minimal number of all the natural number m such that $\Pr(t + m) = \Pr(t)$ for every t.

Pre-play communication. By this we intuitively mean the learning process such that each player communicates privately his/her conjecture about the other players' actions through messages according to a protocol, and he/she updates his/her conjecture according to the message received. In addition, at every stage each player communicates privately not only his/her conjecture about the others' actions but also his/her rationality as messages, the receivers update their private information and revise their conjecture. When a player communicates with another, the other players are not informed about the contents of the message. Formally,

[2]We denote $\mathbf{Exp}(g_i(b_i, \mathbf{a}_{-i}); \omega) := \sum_{a_{-i} \in A_{-i}} g_i(b_i, a_{-i})\ \mathbf{q}_i(a_{-i}, \omega)$.

Definition 1. A *pre-play communication-process* according to a protocol among N for a game G with revisions of players' conjectures is a tuple $\langle \mathrm{Pr}, (P_i^t)_{i\in N}, (\varphi_i^t)_{i\in N} \mid t \in T\rangle$ with the following structures: the players have a common-prior μ on Ω, the protocol Pr among N, $\mathrm{Pr}(t) = (s(t), r(t))$, is fair and it satisfies the conditions that $r(t) = s(t+1)$ for every t and that the communications proceed in *rounds*[3]. The information structure P_i^t at time t is the mapping of Ω into 2^Ω for player i that is defined inductively as follows. If $i = s(t)$ is a sender at t, he/she sends the message W_i^t defined as below to $j = r(t)$ at t.

- Set $P_i^0(\omega) = P_i(\omega)$.
- Assume that P_i^t is defined. It yields the overall conjecture $\mathbf{q}_i^t(a_{-i},\omega) = \mu([\mathbf{a}_{-i} = a_{-i}] | P_i^t(\omega))$; whence
  - R_i^t denotes the set of all the state ω at which i is *rational* according to his conjecture $\mathbf{q}_i^t(\omega)$[4];
  - $\mathcal{Q}_i^t$ denotes the partition induced by $\mathbf{q}_i^t$ on Ω, which is decomposed into the components $\mathcal{Q}_i^t(\omega)$ consisting of all the states ξ such that $\mathbf{q}_i^t(\xi) = \mathbf{q}_i^t(\omega)$;
  - $\mathcal{G}_i$ denotes the partition $\{[\mathbf{g}_i = g_i],\ \Omega\setminus[\mathbf{g}_i = g_i]\}$ of Ω, and $\mathcal{R}_i^t$ the partition $\{R_i^t, \Omega \setminus R_i^t\}$;
  - W_i^t denotes the join $\mathcal{G}_i \vee \mathcal{Q}_i^t \vee \mathcal{R}_i^t$ the partition of Ω generated by $\mathcal{G}_i$, $\mathcal{Q}_i^t$ and $\mathcal{R}_i^t$[5].
- Then P_i^{t+1} is defined as follows:
  - If i is a receiver of a message at time $t+1$ then $P_i^{t+1}(\omega) = P_i^t(\omega) \cap W_{s(t)}^t(\omega)$.
  - If not, $P_i^{t+1}(\omega) = P_i^t(\omega)$. □

[3] That is, there exists a time m such that $\mathrm{Pr}(t) = \mathrm{Pr}(t+m)$ for all t.

[4] That is, each i's actual action a_i maximizes the expectation of his pay-off function g_i being actually played at ω when the other players actions are distributed according to his conjecture $\mathbf{q}_i^t(\omega)$ at time t. Formally, letting $g_i = \mathbf{g}_i(\omega)$, $a_i = \mathbf{a}_i(\omega)$, the expectation at time t, $\mathbf{Exp}^t$, is defined by $\mathbf{Exp}^t(g_i(b_i,\mathbf{a}_{-i});\omega) := \sum_{a_{-i}\in A_{-i}} g_i(b_i,a_{-i})\ \mathbf{q}_i^t(\omega)(a_{-i})$. Player i is said to be rational according to his conjecture $\mathbf{q}_i^t(\omega)$ at ω if for all b_i in A_i, $\mathbf{Exp}^t(g_i(a_i,\mathbf{a}_{-i});\omega) \geq \mathbf{Exp}^t(g_i(b_i,\mathbf{a}_{-i});\omega)$.

[5] Therefore the component $W_i^t(\omega) = [g_i] \cap [\varphi_i^t] \cap R_i^t$ if $\omega \in [g_i] \cap [\varphi_i^t] \cap R_i^t$.

It is of worth noting that $(P_i^t)_{i\in N}$ is an RT-information structure for every $t \in T$. We require that the pre-play communication-process satisfies the following two conditions. Let K_i^t be the knowledge operator corresponding to P_i^t; [6]

A-1 For each $i \in N$ and every $t \in T$, $[\varphi_i^t] \subseteq K_i^t([\varphi_i^t])$ and $R_i^t \subseteq K_i^t(R_i^t)$;

A-2 For every $t \in T$, $\bigcap_{i\in N} K_i^t([g_i] \cap [\varphi_i^t] \cap R_i^t) \neq \emptyset$.

The specification of **A-1** is that each player's conjecture and his/her rationality are truism, and the specification of **A-2** is that each player knows his/her payoff, rationality and conjecture at every time t. For sufficiently large $t \geq \tau$ we denote τ by ∞. Hence we can write $\mathbf{q}_i^\tau$ by $\mathbf{q}_i^\infty$ and φ_i^τ by φ_i^∞

Proof of Main theorem. We now state the main theorem in Introduction as below:

Theorem 1. *Suppose that the players in a strategic form game have a common-prior. In a pre-play communication process according to a protocol among all players in the game with revisions of their conjectures $\{(\varphi_i^t)_{i\in N} \,|\, t = 0, 1, 2, \dots\}$, there exists a time τ such that for each $t \geq \tau$, the n-tuple $(\varphi_i^t)_{i\in N}$ induces a mixed strategy Nash equilibrium of the game.*

Proof of Theorem 1. Follows from the below lemma and proposition. A non-empty event H is said to be P_i- *invariant* if for every ξ of H, $P_i(\xi)$ is contained in H.

Fundamental lemma. *Let $(P_i)_{i\in N}$ be an RT-information structure with μ a common-prior. Let X be an event and q_i be an i's posterior of X; that is, $q_i = \mu(X|P_i(\omega))$. If there is an event H such that the following two conditions* (a), (b) *are true:*

(a) *H is non-empty P_i-invariant, and* (b) *H is contained in $[q_i] := \{\omega \in \Omega \,|\, \mu(X|P_i(\omega)) = q_i\}$,*

then we obtain that $\mu(X|H) = q_i$. □

Let $\omega_\infty \in \bigcap\limits_{i\in N} K_i^\infty([g_i] \cap [\varphi_i^\infty] \cap R_i^\infty) \subseteq \bigcap\limits_{i\in N}([g_i] \cap [\varphi_i^\infty] \cap R_i^\infty)$. The following result is the another key to proving Theorem 1.

[6] That is, $K_i^t E = \{\omega \in \Omega | P_i^t(\omega) \subseteq E\}$.

Proposition 1. *In a pre-play communication-process among all the players in a game with revisions of their conjectures* $\{(\varphi_i^t)_{i\in N} | t = 0,1,2,\dots\}$, *both the marginals of the conjectures* φ_i^∞ *and* φ_j^∞ *on* A_{-i-j} *must coincide for all* $i,j \in N$; *that is,* $\varphi_i^\infty(a_{-i-j}) = \varphi_j^\infty(a_{-i-j})$ *for all* $a \in A$.

Proof. It suffices to verify that $\mathbf{q}_i^\infty(a;\omega) = \mathbf{q}_j^\infty(a;\omega)$ for all $(a;\omega) \in A \times \Omega$. Let us first consider the case that $(i,j) = (s(\infty), t(\infty))$. We denote by Π the partition on Ω with which each component is defined by $\Pi_i^\infty(\omega) = \{\xi \in \Omega \mid P_i^\infty(\xi) = P_i^\infty(\omega)\}$. In view of the construction of $\{P_i^t\}_{t\in T}$ we can observe that $P_i^\infty(\omega)$ is P_j^∞-invariant; i.e., $P_j^\infty(\xi) \subseteq P_i^\infty(\omega)$ for all $\xi \in P_i^\infty(\omega)$. It immediately follows that $P_i^\infty(\omega)$ is decomposed into a disjoint union of components $\Pi_j^\infty(\xi)$ for $\xi \in P_i^\infty(\omega)$; $P_i^\infty(\omega) = \bigcup_{k=1,2,\dots,m} \Pi_j^\infty(\xi_k)$ where $\xi_k \in P_i^\infty(\omega)$. It can be observed that $\mu([\mathbf{a} = a] \mid P_i^\infty(\omega)) = \sum_{k=1}^m \lambda_k \mu([\mathbf{a} = a] \mid \Pi_j^\infty(\xi_k))$ for some $\lambda_k > 0$ with $\sum_{k=1}^m \lambda_k = 1$[7]. By Fundamental lemma we note that $\mu([\mathbf{a} = a] \mid \Pi_j^\infty(\xi_k)) = \mathbf{q}_i^\infty(a;\xi_k)$ and thus by the above equation it can be observed that for all $\omega \in \Omega$ there is some $\xi_\omega \in P_i^\infty(\omega)$ such that $\mathbf{q}_i^\infty(a;\omega) \leq \mathbf{q}_j^\infty(a;\xi_\omega)$. Continuing this process according to the *fair* protocol the below facts can be plainly verified: for each $\omega \in \Omega$

(i) For every $i \neq j$, $\mathbf{q}_i^\infty(a;\omega) \leq \mathbf{q}_j^\infty(a;\xi)$ for some $\xi \in \Omega$; and

(ii) $\mathbf{q}_i^\infty(a;\omega) \leq \mathbf{q}_i^\infty(a;\xi) \leq \mathbf{q}_i^\infty(a;\zeta) \leq \cdots$ for some $\xi, \zeta, \dots \in \Omega$.

Since Ω is finite the equation $\mathbf{q}_i^\infty(a;\omega) = \mathbf{q}_j^\infty(a;\omega)$ can be obtained for every a, ω and for all i, j, in completing the proof. □

Concluding remarks. Our real concern is with what learning process leads to a mixed strategy Nash equilibrium of a finite strategic form game from the epistemic point view. As we have observed, in the pre-play communication process with revisions of players' conjectures about the other actions, their predictions induces a Nash equilibrium of the game in the long run. Where the players privately communicate each other through message according to any non-acyclic graph, and they are required neither to have the common-knowledge assumption about their conjectures nor to have a partition information structure.

The communication process treated in this article will give a new aspect of the algorithms converging to Nash equilibrium from the

[7] This property is called *convexity* in Parikh and Krasucki [6].

epistemic point of view. This issue needs to be sorted out at a more fundamental level, and it has not been discussed at all. There is a research agenda of potential interest which we hope to pursue further.

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Eigenvectors in Systems with Operations max and $\otimes$

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Several problems in operations research and economics lead to analogues of linear operator Ax, where addition is changed for max and multiplication is changed for a binary operation $\otimes$. We enlarge the set of such examples and consider the examples from a uniform point of view based on introducing operations on characteristic pairs of paths.

Two examples are usually being considered in the literature.

Example 1. Scheme of dynamic programming. Here $\otimes = +, X = R \cup -\infty$; there is a zero element $0 = -\infty$. Among operations research problems leading to the example are the choice of optimal schedule and the optimal forestry management. The matrix A can be easily modified to ensure the existence of eigenvalue defined as below.

Example 2. Fuzzy algebra or bottleneck algebra. Here $\otimes = \min$. We assume that $X = R_+, \mathbf{0} = \mathbf{0}$.

We will consider two more examples.

Example 3. $g \otimes h = g + \beta h$ where $\beta \in (0,1)$ is a given number (discount factor), $g, h \in X$, $X = R \cup -\infty$, $\mathbf{0} = -\infty$. This example appears in economics (a version of the Ramsey model), in finance, and in operations research.

Example 4. $g \otimes h = \min(g, \delta h)$ where δ is a given number, $X = R_+, \mathbf{0} = \mathbf{0}$. The example generalizes example 2 and may serve as a model of a situation where the goal is to choose a path with the maximin " income" (i.e. to avoid a period of low incomes) but where later failures are considered as less essential than earlier ones.

Some rather general mathematical constructions were proposed which generalize examples 1 and 2 (see [1, 2] but they can not be applied directly to examples 3 and 4.

As usually, an oriented graph corresponding to the square matrix A will be considered. Note that a path is characterized by two numbers: its weight and its number of arcs.

We take into consideration pairs (a,k), $a \in X$, $k \in N$ which we call characteristic pairs. The operation $\otimes$ does not always provide associativity, but using the characteristic pairs allows to attain associativity.

Characteristic pair of a path $s = i_1, ..., i_t, i_{t+1}$ is defined as (c_s, t), where $c_s = (a_{i_1,i_2}, 1) \otimes ... \otimes (a_{i_{t-1},i_t}, 1) \otimes (a_{i_t,i_{t+1}}, 1)$. The number c_s is called the weight of the path.

Let us introduce the operation of multiplication of a characteristic pair to a number. This operation puts in correspondence to a characteristic pair (a,k) and an elment $(b \in X)$ some element $(a,k) \otimes b \in X$ in such way that $(a,1) \otimes b = a \otimes b$. Specifically in Examples 1 and 2, $(a,k) \otimes b = a \otimes b$; in Example 3, $(a,k) \otimes b = a + \beta^k b$; in Example 4, $(a,k) \otimes b = \min(a, \delta^k b)$.

We define the product of two characteristic pairs in the following way:

$$(a_1, k_1) \otimes (a_2, k_2) = ((a_1, k_1) \otimes a_2, k_1 + k_2).$$

Let us consider a contour $\sigma = (i_1, i_2, :, i_k, i_1)$ with the number of arcs k. Let $c_\sigma(i_1)$ be the weight of one circuit of the contour. The weight of the infinite circuit of the contour σ is defined as the limit

$$\nu\sigma(i_1) = \lim_{r\to\infty}(c_\sigma(i_1), k)^{\otimes r} \otimes c_\sigma(i_1).$$

Let us define class **K** as a set of paths including 1) paths consisting of an infinite circuit of a simple (without self-intersections) contour (with indication of the origin of the circuit), and 2) paths consisting of an elementary path and an infinite circuit of an elementary contour (with indication of the origin of the curcuit).

The class **K** contains a finite number of paths. For each node $i \in M$ a path with the maximum weight, $V(i)$, exists in this class.

Theorem. *The vector V with elements $V(i), i \in M$, is an eigenvector of the matrix A i.e. $V \otimes A = V$.*

To find the eigenvalue the recurrent formula $x_{t+1} = A \otimes x_0$ may be used. For examples 1 and 3 the author [3, 4] described conditions under which the sequence x_t converges to the eigenvalue independently on the initial x_0. In Examples 2 and 4 the vector composed of the maximum elements of the rows of A should be taken as initial. In this case the convergence arises from the following property which is true for all four examples.

Proposition. *Let initial vector x_0 be composed of maximum elements of rows of the matrix A. Then each vector $x_k, k = 1, 2, ...$ from this sequence is composed of maximum elements of rows of the matrix $A^{\otimes(k+1)}$.*

In Example 2 the eigenvalue calculated in such way is maximal in the set of eigenvalues. A method for constructing all eigenvalues in Example 2 is also found.

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A Fishery Game Model with Migration: Reserved Territory Approach

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key words: *bioresource management problem, game theory, Nash and Stakelberg equilibriums*

1. Introduction. A dynamic game model of a bioresource management problem (fisheries) is considered. The center (state) which determines the reserved portion of the reservoir (where fishing is prohibited), and the players (fishing farms) which harvest fish are the participants of the game. Each player is an independent decision maker, guided by the considerations of maximizing the profit from fish sale. We consider finite and infinite planning horizon. Pontryagin's maximal principle and Hamilton-Jacobi-Bellman equation were applied to determine Nash and Stakelberg equlibriums.

2. Game model. Let us divide the water area into two parts: S_1 and S_2, where fishing is prohibited and allowed, respectively. Denote by x_1 and x_2 the size of the population per unit area of S_1 and S_2, respectively. Then $s = S_1/S$ is the reserved area. There is a migratory exchange between the two parts of the reservoir with the exchange coefficient $\gamma = q/s$, where q is the exchange rate.

The dynamics of the fishery is described by the system of equations:

$$\begin{cases} x_1'(t) = \varepsilon x_1(t) + \gamma_1(x_2(t) - x_1(t))\,, \\ x_2'(t) = \varepsilon x_2(t) + \gamma_2(x_1(t) - x_2(t)) - u(t) - v(t)\,,\ x_i(0) = x_i^0\,, \end{cases} \tag{1}$$

where $x_1(t) \geq 0$ – size of the population at time t in the reserved area; $x_2(t) \geq 0$ – size of the population at time t in the area where fishing is allowed; ε – natural growth rate of the population; $u(t) \geq 0$ – first farm's fishing efforts at time t; $v(t) \geq 0$ – second farm's fishing efforts at time t; $s(t)$ – reserved portion of the reservoir and $\gamma_i = q/s$ – coefficients of the migratory exchange, $i = 1, 2$.

Then the payoffs of the two players over a fixed time period $[0, T]$

are

$$J_1 = \int_0^T e^{-rt}[m_1((x_1(t)-\bar{x}_1)^2 + (x_2(t)-\bar{x}_2)^2) + c_1 u(t)^2 - p_1 u(t)]dt,$$
$$J_2 = \int_0^T e^{-rt}[m_2((x_1(t)-\bar{x}_1)^2 + (x_2(t)-\bar{x}_2)^2) + c_2 v(t)^2 - p_2 v(t)]dt, \tag{2}$$

where $\bar{x}_i(t)$ – size of the population which is optimal for reproduction, m_i – penalty for deviation from the state $(\bar{x}_1, \bar{x}_2)$, c_i – catching costs of the i-th player and p_i – market price for each player, $i = 1, 2$.

2.1. Nash optimal solution. To find the Nash equilibrium we have to solve the following system:

$$\begin{cases} x_1'(t) = \varepsilon x_1(t) + \gamma_1(x_2(t) - x_1(t)), \\ x_2'(t) = \varepsilon x_2(t) + \gamma_2(x_1(t) - x_2(t)) - \frac{\bar{\lambda}_{12}(t)+p_1}{2c_1} - \frac{\bar{\lambda}_{22}(t)+p_2}{2c_2}, \\ \bar{\lambda}_{1i}'(t) = -2m_1(x_1(t) - \bar{x}_1) - \bar{\lambda}_{1i}(t)(\varepsilon - \gamma_1 - r) - \bar{\lambda}_{1j}(t)\gamma_2, \\ \bar{\lambda}_{2i}'(t) = -2m_2(x_1(t) - \bar{x}_1) - \bar{\lambda}_{2i}(t)(\varepsilon - \gamma_1 - r) - \bar{\lambda}_{2j}(t)\gamma_2, \\ i, j = 1, 2\,,\ i \neq j\,,\ \bar{\lambda}_{i1}(T) = \bar{\lambda}_{i2}(T) = 0\,,\ x_i(0) = x_i^0\,. \end{cases} \tag{3}$$

Theorem 1.

$$u^*(t) = \frac{\bar{\lambda}_{12}(t) + p_1}{2c_1} \quad and \quad v^*(t) = \frac{\bar{\lambda}_{22}(t) + p_2}{2c_2}\,,$$

with $\bar{\lambda}_{i2}$, $i = 1, 2$ satisfing (3), form the Nash optimal solution of the problem (1)-(2).

2.2. Stackelberg optimal solution. To find the Stackelberg equilibrium we have to solve the following system:

$$\begin{cases} x_1'(t) = \varepsilon x_1(t) + \gamma_1(x_2(t) - x_1(t)), \\ x_2'(t) = \varepsilon x_2(t) + \gamma_2(x_1(t) - x_2(t)) - \frac{\bar{\lambda}_{12}(t)+p_1}{2c_1} - \frac{\bar{\lambda}_{22}(t)+p_2}{2c_2}, \\ \bar{\lambda}_{1i}'(t) = -2m_1(x_1(t) - \bar{x}_1) - \bar{\lambda}_{1i}(t)(\varepsilon - \gamma_1 - r) - \bar{\lambda}_{1j}(t)\gamma_2 + 2m_2\mu_i(t), \\ \bar{\lambda}_{2i}'(t) = -2m_2(x_1(t) - \bar{x}_1) - \bar{\lambda}_{2i}(t)(\varepsilon - \gamma_1 - r) - \bar{\lambda}_{2j}(t)\gamma_2, \\ \mu_1'(t) = \mu_1(t)(\varepsilon - \gamma_1) + \mu_2(t)\gamma_1, \\ \mu_2'(t) = \frac{\bar{\lambda}_{12}(t)}{2c_2} + \mu_2(t)(\varepsilon - \gamma_2) + \mu_1(t)\gamma_2, \\ i, j = 1, 2\,,\ i \neq j\,,\ \bar{\lambda}_{i1}(T) = \bar{\lambda}_{i2}(T) = 0\,,\ x_i(0) = x_i^0\,,\ \mu_i(0) = 0. \end{cases} \tag{4}$$

Theorem 2. *The strategies*

$$u^*(t) = \frac{\bar{\lambda}_{12}(t) + p_1}{2c_1} \quad and \quad v^*(t) = \frac{\bar{\lambda}_{22}(t) + p_2}{2c_2}\,,$$

with $\bar{\lambda}_{i2}$, $i = 1, 2$ satisfing (4), form the Stackelberg optimal solution of the problem (1)-(2).

Let's compare the players' payoffs when we use different optimality principles.

Profit of player I, which corresponds to different sizes of the reserved area $s(t)$, are shown in Table 1, and profit of player II – in Table 2.

Table 1. Player I profit

$s(t)$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Nash	11591	6670	3177	1017	103	353	1689	4040	7341
Stakelberg	11240	6329	2837	673	-253	-26	1277	3582	6820

Table 2. Player II profit

$s(t)$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Nash	11591	6670	3177	1017	103	353	1689	4040	7341
Stakelberg	12426	7498	4002	1846	943	1210	2571	4955	8294

In the case of the Nash equilibrium both players are in the same conditions, so their strategies and profits are equal.

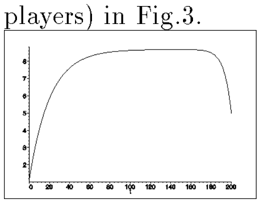
In the case of the Stackelberg equilibrium player I is the leader and, as Tables 1, 2 show, this equilibrium is better for player I, but worse for player II. This solution gives the gain to player I, whereas player II carries all the expenses of maintaining a stable population development.

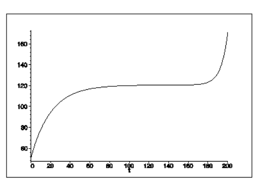
Example.

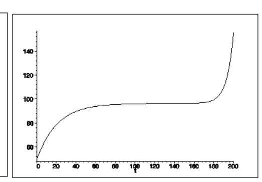
Modelling was carried out for the following values: $q = 0.2$, $\gamma_1 = \gamma_2 = q/s$, $\varepsilon = 0.08$, $m_1 = m_2 = 0.09$, $c_1 = c_2 = 10$, $p_1 = p_2 = 100$, $T = 200$, $r = 0.1$.

Let the initial size of the population be $x_1(0) = 50$, $x_2(0) = 50$. And the optimal for reproduction population sizes are $\bar{x}_1 = 100$ and $\bar{x}_2 = 100$.

You can see the optimal values of $u^*(t)$ and $v^*(t)$ (equal for both players) in Fig.3.







Values of $x_1^*(t)$ Values of $x_2^*(t)$ Values of $u^*(t)$

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The Specific Measures of the State Regulation on the Transitive Securities Markets

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The specificity of state regulation of the formed securities markets in the transitive economic systems is discussed. The brief review of arising here problems is made. The special attention is given to "traps" and myths, trapping the society in the regulation of developing markets. The cautions concerning not critical borrowing of samples of foreign experience from the well-developed countries are expressed.

The features of the formed markets as an object of the state regulation. A securities market is one of the pivotal mechanisms of attracting financial resources to promote investment and economy modernization in order to stimulate production growth. At the same time, as demonstrated by many decades of experience, global securities markets can be the source of considerable financial instabilities, macroeconomic risks and social disturbances. Emerging markets are especially likely to pose problems. The Russian market was one of the riskiest capital markets in the world.

Recreated in the beginning of the nineties, after many years' non-existence, the Russian market did not fulfill its main objective during the past decade. It did not redistribute financial resources into the real sector to cover the needs for investments and it did not create a meaningful capability for market business valuation. Instead, there formed a very speculative market with a large share of foreign short-term investors, marked by super-concentration in Moscow and encompassing a limited number of securities. Free movement of capital (liberalization of foreign investments in government securities and predomination of off-shore settlements in stock trading) coincident with artificially high returns on ruble-nominated financial assets and practically fixed exchange rate of RUR contributed to vast opportunities for a speculative warm-up of the market with subsequent large falls. The oligopolistic structure of the market, its information asymmetries, the highest concentration of trading activities, of securities to deal with and of financial institutions determined the inevitability of price manipulation. The corporate securities market was fully separated from public investors. Its price movements and liquidity were fully determined by dynamics of external markets and speculative foreign investors' demand.

The financial crisis of 1997-1998 became the consequence of the aforementioned problems along other deficiencies of the Russian capital market. The crisis has virtually ruined the securities industry and has raised the question about a new and more fruitful strategy of securities market development in Russia.

At the same time, in 1999-2001 the government and the larger part of the professional community made only limited attempts to understand lessons of the crisis, evaluate its fundamental causes, create policy, which would enable positive financial sector development (first of all the development of the securities market). The Russian capital market was restored to its former nature — speculative, hardly connected to investment in the real sector. We underline a few, possible key ideas:

- Future of the capital market should be programmed by the government and financial community ("policy guides", "programs of restructuring", "strategies of development"). Without this, it quickly transforms into the set of market myths and functions distant from its real purpose - attracting investments to modernize economy. In Russia such programming is non-existent;
- Competitiveness of the market, its ability to attract trading activ-

ity, long-term financial resources and new issues, intermediaries and investors in competition with other emerging markets should be accentuated especially in development;

• Model and structure of the Russian securities market, and in a broader sense economy's financial structure will inevitably be determined by the "model" of the market economy itself ("stakeholder" or "shareholder" capitalism). The "stakeholder" model arises not only in Russia, it prevails in all the countries with developing or transitional economies and in some parts of the industrial world. The model of the securities market is influenced by traditional and religious values of population, by the degree to which socio-political and economic system of the country is liberalized and by degree to which it is characterized as "open" or "closed" economy. Taking this into account, the future of the Russian securities industry is fundamentally different from the "American model", which was imposed on the Russian market in nineties (besides the "American model" itself does not exist anymore in the original form);

• There must be a new role for the state in securities market, the new "interventionism", expressed by the decrease of the excessive regulatory burden in securities industry, by promotion of new issuers listings and IPO's, by the direct participation in creating new financial instruments and markets, by strengthening the government's stimulatory role, first of all in establishing tax incentives, allowing investments to be distributed in the real sector through the securities market;

• Restructuring of the securities industry in Russia is inevitable. Charges must include redesigning its architecture, directing it towards improving the operational capability, decreasing volatility and risks inherent to the domestic securities market;

The formed markets as object of state regulation have a number of features in comparison with the advanced markets:

They are in an initial stage of development, low liquidity and saturation by securities, the product structure is simplified, the demand for the products is narrowed at higher risks in comparison with the markets of the well-developed countries;

The superconcentration in structure of the property, in distribution of money resources, in structure of the market (superhigh weight of the shares of the several emitters and narrow group of the brokers/dealers) are characteristic for these markets; these markets are fragmentary;

The markets differ by a smaller transparency and degree of efficiency in comparison with the markets of the well-developed countries, greater scales of insiders trade, manipulations and others disfunctions; the investors are to a lesser degree protected, the mass retail investment is undeveloped; the role of the foreign investors is great;

The forming markets, their development or, on the contrary, degradation are under the influence of the changeable factors, as a choice of socio economic model of the device of a society, political structure, social stability, degree of an openness of a society, religious factors;

The influence of the state is higher , than in the advanced markets of the well-developed countries, loading is high-regulative, the influence on the perspective and current macroecon0omic policy (money, currency, budget, tax, percentage, investment policy, is higher in the field of the accounts of the capital, condition of management of the state debt etc.).

In the base document of the International Organization of Securities Commissions(IOSCO) [1] are formulated the three purposes of regulation in the specified sphere: a) protection of the investors (against manipulating and insidering); b) achievement of validity, efficiency and transparency of the markets (equality of the participants in the market equal and wide access to the information, prevention of dishonest trade practice); c) reduction of system risk (effective management of risks in the branch of securities).

"An interventionism" of the state in the developing markets. In transitive and developing economic systems the state has the large resources, than private the business. "Self-financing" of the market which has been not maintained by the state, as the practice shows, is connected to scale losses and crises, slow rates of development, mass infringements of the rights of the investors. The interventionism of the state can be shown in the following forms:

– The accelerated creation of the regulative infrastructure and the transition to the market of the best foreign customs, the structure of financial products, forms and technologies of professional activity in the market, rules of organization of share business, management of the conflicts of interests and risks etc. The state should carry out functions of support or even the initiation of financial innovations;

– Definition of strategy and programming of development of the market, direct participation in construction of the new markets and

change of architecture of branch in those elements, which create the system risks; rendering of financial support in creation of an infrastructure of the market (trade, listing, information, settlement, depositary and educational systems); reasonable protectionism allowing, on the one hand, to create domestic the branch of securities, and with another - to use operational ability of foreign investment institutes and share markets; the formation of a system of economic stimuli, creating the interest of all classes of the investors to an investment of means in securities, the stimulus to develop the professional activity of the dealer companies and the operational ability of the markets;
– Demonstration of the effectiveness of supervision behind honesty and validity of the price-formation mechanism, inevitability of the sanctions for infringements of the rights of the investors and, on this basis, assistance in introduction of business customs and formation of the standard market culture and ethics. The experts of the scientific institutes and research divisions have prepared the project "The programs of development of the market of securities in Russia till 2010". It prescribes, in particular, the concrete measures on expansion of a constructive role of the state in construction of the share market and optimization of state intervention in its functioning.

Traps and myths about the regulation of the formed market. The myth of "self-financing", above mentioned. The experience of 1990 years in Russia has shown, that the concept of "self-financing", which adhered the Government, results in huge meanings of market risk, to deformation of organization of the market, to default by the market of the base tasks.

A serious trap can appear with the incorrectly chosen purpose for a regulator of the share market. For example, one frequently accepts the model of the US shares-market. The potential choice is wide: the American model, "Rhine" capitalism of the German sample, the Japanese model, the Swedish model, closed Islamic systems of economy, which belong to market economies. Their roots lay in national characters, in ability of the population to accept risks and to work on an individual basis, in the economic doctrines of religions, in a history and in financial sources of industrialization of 19th century. As to transitive economies, the research, executed in USA [2], states, that the property tends to be extremely concentrated in the transitive economies. Accordingly, limited and slowed down in the development are the markets of securities, first of all the markets of shares. Both in Czechia, and in Poland the markets of the liabilities surpass the markets of the shares, the structure of the property interferes with formation of the mass and retail market of securities. Concerning Russia it is known, that the structure of the property in the country is wholesale (it is estimated, that not less than 60 - 70 % of the joint-stock capitals are assembled in large or control packages). As the consequence, Russia is an example of debt economy, in which the financing of the economy is based on the liabilities. Therefore attempts to impart in the pure American model are artificial.

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Investigation of Optimum Regimes of Information Reception with Help of Nonantagonistic Dynamical Games

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The time is right to create the information ecology. In particular this new science would be able to investigate the optimum and permissible regimes of the information reception for people, the social-economical systems and the wide class of the technical systems. The game theorists are able to make a contribution to the development of this science. Optimum regimes of the information reception are investigated broadly in the antagonistic theory of game. Let mention at least the papers of L.A. Petrosjan, F.L. Chernous'ko, A.A. Melikyan.

The first paper in the theory of nonantagonistic game devoted to the investigation of this question was the paper of A.F. Kononenko. Further E.Z. Mokhonko worked in this field. She suggested the following scheme of investigation of the dynamics of the information reception in the nonantagonistic dynamic games.

1. To clarify whether all moments of the information reception are important. May be it is possible to manage without some information.

2. In some class of games, moments of the information reception are such that the received information is inexact and late, but the lateness and inexactitude do not influence negatively on the control decisions. To clarify the permissible extent of lateness and inexactitude and such moments of time.

3. Let the systems of the information reception of the participants of the nonantagonistic conflict have some peculiarities in their work. To define the regimes of the information reception which are optimum and admissible in some sense in this case.

4. To impose some restrictions on the ability to receive information. It is possible to lead to the success end some types of conflicts as if these restrictions were absent. To define these type of games.

5. Some deviations from the agreement are real. They connect with the unfavourable external conditions which do not depend on the participants of nonantagonistic conflicts. To clarify the character of the influence of such deviations on the optimum regimes of the information reception.

6. To estimate the utility of the received information for the participants of the nonantagonistic conflict in the different moments of time. To define factors which influence on the utility.

7. To investigate the optimum regimes of the reception of information of different quality. Namely, to consider more complex information flows. Let consider the following example. Information is the result of the definite influence on the individual and the individual's key of decoding the influence. The key of decoding is the model of world. The world is changing and the individual has to change his model. It is necessary to define the moments of changing the model, that is the key of decoding. In this case, the individual works with two information flows. The first flow is the information influence associated with the course of conflict going. The second flow is information about the change of the world.

8. To investigate the phenomenon of information, its nature.

The investigations on such scheme lead to the following results.

1. Mathematical apparatus was elaborated for the solution of the problems of the dynamics of the information reception in nonantagonistic conflicts in which the peculiarities of the player's system of the information reception take into account.

2. New statements of the problems was introduced which had not been considered earlier in the game theory. The problems were solved for the wide class of cases. By this an application field of game theory is enlarged. Namely, the following problems were considered.

a) The problem of defining the solution of dynamic nonantagonistic game, in which the gain of the game is made more exact in the process of the game going;

b) the problem of determining the pragmatic value of the received information;

c) the problem of expressing by the formula the boundary of the set of solutions nonantagonistic dynamic game under constrains on the possibility of the player's system of the information reception;

d) the problem of determining the degree of stability of some function. The function characterizes the optimum information reception in the concrete game;

e) the problem of the influence of the additional payment on the qualitative character of the optimum information reception;

f) the problem of determining the optimum information regimes when the players are situated on the different hierarchical levels.

3. New types of strategies, namely r-strategies, rs-strategies, Mr - strategies were introduced. They permit to model new possibilities and restrictions in the information reception of the players and thereby they give a new possibility to describe the nonantagonistic conflict more adequately to the reality.

The Optimal Level Stopping Rule for American Options*

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key words: *american option, lower bound, integral representation*

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We develop new lower bound for price of standard American Option on a dividend-paying asset. The existence of optimal constant level decision rule is proved. We use integral representation formula and deduce from it equation for optimal level.

Consider an asset at time t, with pri Consider asset at time t, with price S_t, that follow the geometrical Brownian motion equation $dS_t = S_t(\alpha dt + \sigma dz(t))$, where z is standard Wiener process with $z(0) = 0$. Here $\delta = r - \alpha$ is the dividend rate, r is the constant rate of interest, and σ is the volatility of the price of the asset.

In the absence of arbitrage, the price of the American option at time t with strike price K and expiration date T is the supremum, over all stopping times T' with values belonging the segment $[t, T]$, of

$$E[e^{-r(T'-t)}(S_{T'} - K)_+], \tag{1}$$

where $a_+ = \max(a, 0)$. We denote the supremum of (1) by $F(S_t, \tau)$, $\tau = T - t$. Denote $C(S_t, \tau)$ the value of the corresponding European option.

The stopping time $T^* = \inf\{0 \le t \le T : F(S_t, \tau) = (S_t - K)_+\}$ is an optimal option-exercise strategy (see [1]). So, $\mathcal{E}_t = \{S_t : F(S_t, \tau) = (S_t - K)_+\}$ is an immediate exercise region. Let $B_t = \min\{S_t : S_t \in \mathcal{E}_t\}$. Then $\mathcal{E}_t = \{S_t : S_t \ge B_t\}$. In [2] the following integral representation was provided: for all $t \in [0, T]$,

$$F(S_t, \tau) = V(S_t, B(\cdot), \tau) \stackrel{def}{=} C(S_t, \tau) +$$

$$\int_t^T (e^{-\delta s}\delta S_t N(d_1(S_t, B_s, s-t)) - e^{-rs} rKN(d_2(S_t, B_s, s-t)))ds,$$

where $N(x)$ – cumulative standard normal distribution function and $d_{1,2}(S, K, \tau) = (\ln(S/K) + (\alpha \pm \sigma^2/2)\tau)/(\sigma\sqrt{\tau})$. Equation

$$B_t - K = V(B_t, B(\cdot), \tau), \ \ \forall\, t \in \mathcal{T} \subset [0, T], \tag{2}$$

is nessecery, but not sufficient, condition for $B(\cdot)$. If some function G_t satisfies (2) then, for $S_t \le G_t$, $V(S_t, G(\cdot), \tau)$ is a lower bound for $F(S_t, \tau)$. We use this condition for a constant $G_t \equiv L$ with $\mathcal{T} = \{0\}$.

Denote $\tilde{\alpha} = \alpha - \dfrac{\sigma^2}{2}$,

$$\xi = \sqrt{\tilde{\alpha}^2 + 2r\sigma^2}, \ \theta_{1,2} = \frac{-\tilde{\alpha} \pm \xi}{\sigma^2}, \ b_{1,2}(L, t) = \frac{\ln(S/L) \pm \xi t}{\sigma\sqrt{t}}.$$

Proposition. There exists unique constant $L^* > K$, satisfying the equation $L - K = V(L, L, T)$, which gives the lower bound for $F(S, T)$

$$V(S, L^*, T) = C(S,T) + Ke^{-rT}N(d_2(L^*, K, T)) - Se^{-\delta T}N(d_1(L^*, K, T)) +$$

$$\frac{1}{2\xi}\sum_{i=1}^{2}\left(\frac{S}{L^*}\right)^{\theta_i} N(b_i(L^*, T))[L^*(\xi + (-1)^{i+1}(\tilde{\alpha} + \sigma^2)) - K(\xi + (-1)^{i+1}\tilde{\alpha})],$$

asymptotically optimal when $T \to \infty$.

Other approach to optimal level strategy based on direct calculation of mean option holder's gain see in [3].

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Mathematical Modeling of Interaction Beetwen Power Indusrty and Energy Consuming Branches in Russian Economy*

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key words: *mathematical modeling, economy, power industry, balanced economic growth*

Actually the economic situation in Russia is largely determined by the situation in the fuel and power industry. Export of fuel gives the basic part of exchange earnings and the big part of the budget income. But the considerable proportion of export profit is removed

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from the country and only small part is returned as capital investment. At the same time the wastage of the fixed capital and potential depletion of developed fields are the most actual problems of fuel and power complex. The renewal of fixed capital and new fields developing (usually, in hard-to-reach areas) needs large-scale investments. For solving this problem, there are different programs of investment to fuel and power complex considered in the government. In countries with market economy, the household savings form the basic source of investments. In Russia, after breakdown of 90th, the confidence of people to the banking system is on the low level. The government strategy of economic development (G. Gref's program) does not make provision of any efforts to attract the household saving in investment process, but is based on improvment of conditions for the export profit reinvestment. At the same time the price growth in power industry is extensively considered. This is the way to make the additional investment in the fixed capital. In fact, the domestic prices for energy recourses in Russia are of the essence less than the world prices (domestic prices for oil are two times off than world prices, for gas — 10 times). However, the growth of domestic prices will lead to the change of price structure for power industry and energy consumers branches of economy. By that, the economic situation in energy consuming part of the industry will notedly decline. It can lead back to the non-monetary accounts, barter etc.

Thus, the macroeconomic consequences of reforms in the fuel and power industry are multivalued and additional investigation is requested. To make this investigation, we develope the mathematical model of Russian economy. This model allows to analiyze the macroeconomic characteristics of power industry and economic situation in various scenarios according to different variants of state economic policy regarding fuel and energy complex.

The model has two sectors: the first one includes all non - power branches of economy, the second one includes only fuel and energy complex (FEC). The following economic agents are advertised in the model: two production sectors, households, public bodies, the central bank, exporters and importers. The agents are connected through three markets: two-commodity markets and currency exchange market. The structure of the model is determined by balance equations corresponding to production and distribution of commodities (in real terms) and balance equations of emission, distribution and transfers of money. Details of model description are given in [1-3].

With the help of the constructed model and comparative static methods, macroeconomic parameters of the economy (growth rate, prices structure, the part of FEC added value in GDP etc.) are investigated in the balanced economic growth mode. We have found the relations between these parameters and indicators of government policy, money circulation and industry functioning ([1-3]). Despite the branches of FEC are aggregated into one sector, the model gives the possiblity to make a priori analysis of macroeconomic consequences of reforming particular FEC branches. The situation when parameters of the particular FEC branch change and all others thing are constant we model by the proportional change of the appropriate FEC-parameter taking into account the branch part in the whole FEC. Such investigation was made to analyze consequences of probable gas industry reforms.

One of the main problems in gas industry is the decreasing of gas digging on main deposits in west Siberian instead of nessesary growth, according to Russian energy strategy calculation. Increasing of gas digging is possible for account of new deposits, which are usually in hard-to-reach regions. Reclamation of these deposits needs large investments. Lack of investments in gas industry is connected basically with the low domestic gas prices. The external gas price are nearly ten times more than domestic price. Rates of domestic prices growth are clamped by the government in connection with apprehension of essential negative action on inflation rates and GDP growth rates. At the same time the part of gas production in the total production volume (in internal prices) is near 2,5 %, the part of gas industry in tax revenues (including export) is 2,2% and the part of gas cost in total Russian production costs (including power industry) is near 2,9%.

The analysis of macroeconomic consequences of domestic gas prices changing and tax filling changing is topical. The investigation of appropriate scenarios was made on the basis of the constructed model with the help of the following estimation: the gas industry part in total FEC added value is near 10%.

Scenario 1. Lowering of gas industry tax filling. Deceleration of price growth rates in gas industry can be balanced with exclusive lowering of gas industry tax filling. In the model, FEC tax revenues are described by the part n_{22} of total FEC production added value. To find out possible macroeconomic consequences of earned income credits in gas industry, we consider the scenario, in which we assume the full tax remission for gas industry. In the model, it means that parameter n_{22} is decreasing to 10the basic value of $n_{22} = 0.16$. Calculations show that immunity of gas industry from taxes does not have essential influence on the macroeconomic indicators of Russian economy. The decreasing of FEC tax revenues connected with the gas industry tax immunity leads to insignificant growth of inflation rate, from 14% in basic variant to 15% in the scenario calculation. GDP growth rate does not changing (near 7% per year). Decreasing of gas industry tax revenues does not affect on the part of household consumption in GDP, FEC part of GDP and price structure π between FEC and other industry. The part of FEC investments in total investments does not change. So the influence of tax revenues decreasing from gas industry on the macroeconomic characteristics is inessential.

Scenario 2. Advance of internal gas prices. In this scenario calculation, we analyze the consequences of increasing of domestic gas price in two times in conditions of all others thing constant. In the model, it corresponds to the decreasing of price ratio π on 10%. (The price ratio π on the FEC and others economy production is one of the variables but not parameter.) The required price ratio π change was modeled by decreasing of the part of distributed profit in the total non-power industry added value (parameter n_{11}) from 31% in basic variant to 26% in scenario calculation.

Calculations show that two times gas prices increasing can lead to the inflation rate growth from 14% (in base variant) to 17,3% together with the increasing of GDP growth rate from 7% to 8% per year. In this case, the part of household consumption in GDP may decrease on 2% and the FEC part in GDP may increase on 2%. The part of FEC investments in total investments does not change. So the change of macroeconomic parameters in the case of two times gas price growth turns up to be significant but does not call crisis events.

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Mathematical foundations of the expenditure taxes advantages

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The comparison of the income taxation and the expenditure taxation is carried out. Expenditure taxes and income taxes are compared on the regulative action efficiency. With the help of the classical theory of automatic control it is found out, that the accuracy of regulation for the expenditure taxes is higher, then for the income taxes.

In working and established tax systems, the value of a tax depends on values of incomes and expenditures of a tax bearer. In the present work, we consider the generalized taxes, which depend not only on functions of incomes and expenditures but also on their integrals and derivative with respect to time.

Traditional political economy classifies the taxes to "direct" and "indirect". The name "indirect" is explained by the fact that the payer of the tax is the seller. Rearrangement of the tax burden from the seller to the buyer and vica versa is well investigated by the mathematical economists (see, for example, the book of J.E. Stiglitz [1]). The inadequate and archaic terminology creates mess at the unsophisticated tax bearers and students. Therefore henceforth we shall name "indirect" taxes as expenditure taxes. (By the way, the property taxes depend on integrals of the current expenditures of the tax bearers, and property taxes can be interpreted as a specific version of the expenditure taxes).

From the government point of view, taxes carry out fiscal function and regulating function. The fiscal function with the help of expenditure taxes is carried out much more effectively and with smaller costs, than with the income taxation. It has convincingly been proved by the founder of the taxation theory William Petty in 1662 [2]. A number of authoritative economists prefer to return to the expenditure taxation instead of the income taxation. Here it is necessary to note the activity of Nicolas Kaldor (see [3] and the consequent editions of his works).

In the question on fiscal function of the expenditure taxes, there is a complete clearness. Further, there is a question on comparative efficiency of income taxes and of expenditure taxes in performance of regulative functions by them. The author has undertaken in the work [4] an attempt to investigate a question of tax regulation with the help of the standard theory of automatic control. The elementary statistics testifies that the overwhelming majority of persons plans their current charges depending on current, previous and forthcoming (assumed) incomes. Such behavior is natural for a man and is named by "income formation of the charges". The behavior is described by the equation:

$$y(t) = \hat{A}x(t),$$

where $x(t)$ and $y(t)$ are functions of time and designate the current incomes and current expenditures of the tax bearers, and $\hat{A}$ — is an operator. Generally, we have here a system of integro-differential equations with deviating arguments, which in special cases can degenerate differential, difference equations and the properly integral equations. These equations have already been investigated within the framework of the mathematical theory of automatic control. In the language of the theory, $y(t)$ refers to an adjustable value and $x(t)$ — to perturbative influence. In practice of automatic control of technical devices, there are the two principles of constructing the regulators:

1. principle of regulation on deviations or error-closing control (J. Watt, 1784);
2. principle of regulation on perturbations or disturbance compensating control (J.Poncelet, 1830).

Accordingly, the regulative control signal looks like a mathematical function:

$$u = u\left(y(t), y(t-\tau), \ldots, \dot{y}(t), \dot{y}(t-\tau), \ldots, \int y(t)dt\right) \quad \text{(Watt)}$$

and

$$u = u\left(x(t), x(t-\tau), \ldots, \dot{x}(t), \dot{x}(t-\tau), \ldots, \int x(t)dt\right) \text{ (Poncelet).}$$

The Watt-type regulators provide essentially more accuracy of regulation in comparison with regulators of Poncelet (compensators) because of discrepancy and incompleteness of the information about perturbative influences. At the tax regulation of expenditures, a regulator of Poncelet corresponds to an income tax, and a regulator of Watt corresponds to an expenditure tax. Thus, the accuracy of regulation at the expenditure taxation is much higher than at the income taxation.

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Analysis of the U.S. Drug Market *

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key words: *pharmaceutical industry, drug market, mathematical model, economic agents*

To define market share for a given drug company it is necessary to construct a model of U.S. pharmaceutical industry. To construct the model, it is necessary to take into account its features.

Pharmaceutical industry has two main specific cost conditions: (1) very large sunk costs include the costs of bringing a product to

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market (doing basic research, winning patent approval, engaging in development, performing clinical trials, obtaining final approval from FDA); (2) the marginal costs of manufacturing for most traditional drugs are very small.

These cost conditions have implication for pricing. Patent protection gives firms the ability to influence to the price. One would expect price and marginal-cost conditions (P and MC) to approximate by Lerner markup relation as

$$(P - MC)/P = -1/\varepsilon_P,$$

where ε_P is the demand price elasticity.

The pharmaceutical industry consists of a large number of firms that produce many different (and mainly nonsubstitutable) drug products, ethical and over-the-counter, branded and generic. Production capacity for assembling active and inert ingredients into pills or capsules is largely fungible. Thus, although actual competitors for a given drug or therapy may be few, potential entrants are numerous.

Drug markets are divided into markets for prescription drug products (RX), markets for over-the-counter drug products (OTC), and markets for discontinued drug products (DISCN).

Marketing information stocks positively affect sales. The sales elasticity is the largest for detailing, followed by journal pages of advertising, and is the smallest for direct-to-consumer advertising.

Less than two decades ago competition among drug companies was focused on gaining the allegiance of prescribing physicians. More recently the doctor's prescription under the influence of information technology has become just the starting point in determining what drug the pharmacist dispenses.

To construct a model it is necessary to select a set of agents[1]. A model of US drug industry includes the following agents:

1. drug companies that charge different prices to different groups of buyers,

2. health maintenance organizations (HMOs),

3. pharmacy benefit managers (PBMs),

4. physicians (doctors),

5. pharmacists,

6. groups of buyers that choose among alternative drug treatments (e.g., hospitals),

7. patients.

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Premium Computation Models in Automobile Insurance

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key words: *insurance, net premium, policy with constraints, decision making problems, optimal strategy, stopping rule*

The paper is devoted to the research of different insurance policies types for a particular mathematical model of losses occurrence random process. Insurance products with certain constraints of loss transfer from insured to insurer such as policy limit, total claims number limitation etc. are considered.

For policies with constraints of the mentioned type, the problem of determining the optimal strategy of insured for transferring the loss to insurer is solved. In this problem, it is required to find the function $D(t)$ which is the critical value of loss at the time t: the insured decides to file a claim to the insurance company if and only if the loss occurred at the time t exceeds the critical value $D(t)$.

Occurrence of losses is simulated by the Poisson process with the parameter μ on the interval $[0,1]$. The discount factor with the continuous interest rate δ is taken into account. It is supposed that the loss random variable does not depend on the time of loss occurrence and has the probability density function $f(x)$. The value of insurance limit for each accident is denoted by T.

The main result is the optimal strategy $D(t)$ for the policy with one-time claim filing. The problem of obtaining this strategy is solved for the discrete case first, and then for the continuous one. In the continuous case the Bellman equation

$$\int_{D(t)}^{\infty} (D(t)-x)f(x)\,dx = \frac{D'(t)-\delta D(t)}{\mu} - R(T)$$

holds.

Similar equations appear in the decision making problems with stochastic uncertainty conditions (see, for example, [3]), but in this case the equation has the analytical solution

$$D(t) = F_3^{-1}(t),$$

where

$$F_3^{-1} \text{ is the inverse function for } F_3; \quad F_3(x) = \int_0^x \frac{1}{F_2(x)}\, dx;$$

$$F_2(x) = \int_0^x (\mu F_1(x) + \delta)\, dx + \mu(R(T) - R(0));$$

$$R(T) = \int_0^\infty x f(x+T)\, dx; \quad F_1(x) = 1 - \int_0^x f(x)\, dx.$$

In addition to the one-time claim filing, other settings of problems are considered. Namely, a policy with multiple claim filing with insurance limit for total policy payout is also considered. For this setting, the Bellman equation has the following form:

$$P_t'(t,C) = \mu(R(C) - R(0)) + \mu P(t,C) - \mu \int_0^C P(t, C-x) f(x)\, dx,$$

where $P(t,C)$ is the expectation of the insurer's loss as the result of coverage from the time t to the time of policy expiration with the condition that the remaining aggregate coverage value equals C. The solution of the equation can be found using the Laplace transform.

The net premium (expectation value of insurance company loss made by servicing the policy) is calculated for each considered insurance policy. A numerical solution of the problem for one of the policies in the case of gamma distribution is given in the addendum. The obtained program of numerical calculations allows to apply the algorithm for different values of parameters and quite wide class of probability density functions.

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New Value for Dynamic Coalitional Games

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Game-theoretic models considered in the modern game theory can be divided into two classes: strategic and cooperative games. In strategic games, players choosing their strategies try to get maximal payoffs, in cooperative games it is assumed that players try to maximize the sum of their payoffs and the problem consists in the allocation of this maximal total payoff between players. There is a number of papers where the intermediate case is considered: the case when the cooperation is not full, and players form coalitions choosing strategies with the intention to maximize the payoff of the coalition to which they belong.

In what follows as basic model we shall consider the game in extensive form with perfect information, which includes the possibility of changing coalitional partitions in some fixed vertices of the game tree.

Definition 1. A game tree is a finite oriented treelike graph K with the root x_0.

We shall use the following notation. Let x be a vertex (position). We denote by $K(x)$ a subtree of the tree K with the root in x. We denote by $Z(x)$ the set of vertices directly following after x. The vertice y directly following after x is called alternatives in x ($y \in Z(x)$). The player who makes decision in x (who selects the next alternative position in x), is denoted by $i(x)$. The choice of player $i(x)$ in position x will be denoted by $\overline{x} \in Z(x)$.

Let $N = \{1, \ldots, n\}$ be the set of all players in the game. Under partition of the set N we understand the family of subsets $\Delta_k = \{S_j\}_{j=1}^{|\Delta_k|}$ such that $S_j \cap S_i = \emptyset$; $j \neq i$; $\bigcup S_j = N$. The set of all admissible partitions of N is denoted by Δ.

Now we shall give the strict definition of the game with perfect information and changing coalitional partition.

Definition 2. A game in extensive form with perfect information and changing coalitional partition , (x_0) is a game tree K with the following additional properties :

- (I) The set of vertices (positions) is divided into $n+2$ subsets $P_1, P_2, \ldots, P_n, P_{n+1}, P_{n+2}$, which form a partition of the set of all vertices of the graph tree K. The vertices (positions) $x \in P_i$ are called players' i personal positions, $i = 1, \ldots, n$; vertices (positions) $x \in P_{n+1}$ are called positions of the chance and vertices (positions) $x \in P_{n+2}$ are called terminal positions.

- (II) For each x the partition $\Delta(x) \in \Delta$ of the player's set N is given such that

$$\begin{cases} \Delta(x) \equiv \Delta(y), & \forall y \in Z(x), \qquad \text{if } x \notin P_{n+1}, \\ \exists y \in Z(x): & \Delta(x) \not\equiv \Delta(y), \quad \text{if } x \in P_{n+1}. \end{cases}$$

- (III) For each $y \in Z(x) = \{y_1, \ldots, y_r\}$, $x \in P_{n+1}$ the probability distribution $p(y_1), \ldots, p(y_r)$, $\sum\limits_{y \in Z(x)} p(y) = 1$, $r = |Z(x)|$ is defined.

- (IV) In each final vertex (position) the system of real numbers $h(w) = (h_1(w), \ldots, h_n(w))$, $w \in P_{n+2}$, $h_i(w) \geq 0$, $i = 1, \ldots, n$, is defined. Here $h_i(w)$ is the payoff of player i at the final vertex (position).

Definition 3. A strategy of player i is mapping $U_i(\cdot)$, which associates each position $x \in P_i$ with a unique alternative $y \in Z(x)$.

The set of all strategies of player i is denoted by $\mathbf{U_i}$.

In this setting, we suppose that the player $i \in N$ in his position set $y \in P_i$ is playing in the interests of the coalition S_j, ($i \in S_j$, $S_j \in \Delta(x_j)$) to which he belongs, trying to maximize the payoff of this coalition. Suppose the players choose the strategy profile $U_1(\cdot), \ldots, U_n(\cdot)$. Then the game develops in the following way. The game starts at $x_0 \in P_{n+1}$ in which the coalitional partition $\Delta(x_0) \in \Delta$ is given. In this position x_0 the chance, according to the probability distribution defined in (III), is selecting the alternative $\overline{x}_1 \in Z(x_0)$, in which a coalitional partition $\Delta(\overline{x}_1) \in \Delta$ is defined. Suppose $\overline{x}_1 \in P_{i(\overline{x}_1)}$, then in position $\overline{x}_1$ the player $i(\overline{x}_1)$ makes a move choosing the alternative $\overline{x}_2 = U_{i(\overline{x}_1)}(\overline{x}_1) \in Z(\overline{x}_1)$, and acting in the interests of the coalition to which he belongs and which was realized after the chance move makes a choice at the initial position of the game $x_0 \in P_{n+1}$. Suppose that in $\overline{x}_2$ the coalitional partitions $\Delta(\overline{x}_2)$ and $\Delta(\overline{x}_1)$ coincide ($\Delta(\overline{x}_2) \equiv \Delta(\overline{x}_1)$). If $\overline{x}_2 \notin P_{n+1} \cup P_{n+2}$, then in position $\overline{x}_2$ player $i(\overline{x}_2)$ makes a move and chooses an alternative $\overline{x}_3 = U_{i(\overline{x}_2)}(\overline{x}_2) \in Z(\overline{x}_2)$ according to the interests of the coalition to which he belongs. If $\overline{x}_2 \in P_{n+1}$, then in position $\overline{x}_2$ the chance according to the probability distribution defined by (III) from definition 2 for position $\overline{x}_2$ is selecting the alternative $\overline{x}_3$. In the randomly selected position $\overline{x}_3 \in Z(\overline{x}_2)$, the coalitional partition $\Delta(\overline{x}_3) \in \Delta$ is defined which may or may not coincide with coalitional partition in $\overline{x}_2$. If on the stage k, $x_k \notin P_{n+1} \cup P_{n+2}$, then in x_k the player $i(\overline{x}_k)$ chooses an alternative $\overline{x}_{k+1} = U_{i(\overline{x}_k)}(\overline{x}_k) \in Z(\overline{x}_k)$, acting according to the interests of the coalition to which he belongs. If on the stage k the game is in the position of the chance, i.e $\overline{x}_k \in P_{n+1}$, then in the position $\overline{x}_k$ an alternative $\overline{x}_{k+1}$ is chosen according to the probability distribution defined for this position $\overline{x}_k$ according to the condition (III) of the definition 2. In the chosen position $\overline{x}_{k+1} \in Z(\overline{x}_k)$ the coalitional partition $\Delta(\overline{x}_{k+1}) \in \Delta$ is defined, which may or may not coincide with coalitional partition defined for the position $\overline{x}_k$. The game ends after reaching the final position $\omega \in P_{n+2}$. This happens after a finite number of steps, since the game tree is finite. In the position ω, according to the condition (IV) of the definition 2, the payoffs of the players $(h_1(\omega),\ ...,\ h_n(\omega))$ are defined. Thus every strategy profile $U(\cdot) = (U_1(\cdot),\ ...,\ U_n(\cdot))$ defines the probability distribution over the set of final positions (because of the chance moves in $x \in P_{n+1}$), and thus the payoff $h_i(\omega)$ of the player i is a random variable. Consequently to each strategy profile $U(\cdot) = (U_1(\cdot),\ ...,\ U_n(\cdot))$ uniquely corresponds the vector of mathematical expectations of players payoffs $E_i(U_1(\cdot),\ ...,\ U_n(\cdot)) = E[h_i(\omega)]$, $i \in N$.

Thus we can formulate our game as game in normal form

$$< N; U_1, ..., U_n; E_1, ..., E_n > .$$

But the normal form of the game is not very useful for the investigation of our game, since it does not take not into account the belonging of players to different coalitions and the possibility of the transformation of the coalitions. This makes the use of classical solution concepts for the construction of optimal behavior practically impossible .

In what follows we shall propose a new approach to the construction of the solution, which takes in account the existing coalitional structure of the game and the dynamics of its evolution, and the new value (the so called PMS-value) for such games.

Disproportionality Indices for Proportional Representation Systems

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key words: *disproportionality, proportional representation*

The reasons why any electoral system distort the voters' preferences are discussed. Several well-known disproportionality indices for proportional representation systems are analyzed. Two new disproportionality indices are introduced and calculated for parliaments in several countries.

The idea of proportional representation (PR) systems is that in the elections the voters vote for parties or blocks, which run for the certain number of seats in any representative body, and get seats proportionally to the received votes. An example of PR system is the elections to a parliament under party lists. In this case, parties reflecting interests of various groups of the voters receive seats in the parliament according to the size of these groups: more popular parties receive more seats, less popular parties receive less seats. Thus, one can say that the purpose of proportional representation is to enable maximal number of the voters to receive the representatives in the parliament.

Let us assume that if the voter come on the poll and vote for certain party, then this party reflects his political preferences, and, vice versa, if the voter does not come this means that there is no party which fits his interests. So, the votes received by parties can be considered as “true” preferences of the voters. If for any party the percentage of the received votes is not equal to the percentage of the seats in the parliament, one can say that the voting procedure distorts preferences of the voters. There are some reasons of such disproportionality, such as electoral “threshold”, voters that ignore the polls or vote “against all parties” and impossibility to allocate seats strictly proportionally to the received votes.

There is a question that frequently arises after any elections: in what degree the elected parliament reflects the interests of various groups of the voters, or, in other words, to which extent it is representative? To answer this question the indices are introduced evaluating the degree of proportionality of an electoral system, named "disproportionality indices". Disproportionality indices are based on the comparison between the quota of votes and quota of seats each party obtains. Since the end of nineteenth century many of such indices were introduced, e.g. Rae Index, Grofman Index, Loosemore-Hanby Index, Gallagher Index, Equal Proportion Index, d'Hondt index, etc.

We introduce two more disproportionality indices. The index of Relative Representation is appropriate to use in case of some parties get no seats in parliament. The Representation Index Taking into Account Absence of the Voters takes into account the absense of the voters and possibility of voting "against all" and shows how many voters are represented in parliament according to their interests.

These indices are calculated for parliamentary elections in several countries.

Application of Variety Analysis Methods for Estimation of Effectiveness of Optimization Algorithms

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key words: *optimization algorithms*

Analyzing the work of optimization algorithms, it is important to choose characteristics which allow to estimate the effectiveness of algorithms and to compare different algorithms. Different statistical indicators like sums and means are used as such characteristics, but they are not convenient always. For example, statistic characteristics are not adequate in the cases of deterministic chaos, when even small disturbance in initial conditions of completely deterministic algorithm can lead to much change of the results of its work. These situations are typical for application of local optimization methods to multiextremal functions.

In the paper, we consider a new approach of determining characteristics for optimization algorithms. This approach is based on methods of variety analysis. In this case, we can represent characteristics not in the form of separated indicators but with the help of type and range distributions. They are the most stable in time and determined on the basis of minimax optimality criterion.

Variety needed for forming type and rank distributions can be introduced artificially, for example, by the consideration of families of functions depending on the set of random parameters.

On a Continous Method of the Minimization of a Maximum Function*

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Consider the function

$$f(x) = \max_{i \in I} f_i(x).$$

where $f_i(x),\ i \in I = \{1, \dots, s\}$, are twice continuously differentiable strongly convex functions on R^n. Let $m_i > 0, i \in I$, be their moduli. That is

$$f_i(\lambda x_1 + (1-\lambda)x_2) \leq \lambda f_i(x_1) + (1-\lambda) f_i(x_2) - m_i \lambda (1-\lambda) ||x_1 - x_2||^2,$$

$$\forall \lambda \in [0,1], \quad \forall x_1, x_2 \in R^n, \quad i \in I.$$

Under these assumptions the function f is also a strongly convex function with the modulus $m = \min_{i \in I} m_i > 0$. Consider the following optimization problem: find

$$\min_{x \in R^n} f(x). \tag{1}$$

The solution of the problem (1) exists and a minimizer of the function f on R^n is unique. Let us assume that there exists a constant $M \geq 1$ such that the following inequality

$$\langle f_i''(x) w, w \rangle \leq M ||w||^2 \quad \forall x \in R^n, \quad \forall w \in R^n, \quad \forall i \in I,$$

holds. Here $f_i''(x)$ is the matrix of the second derivatives of the functions f_i at the point x.

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The optimization problem

$$\min_{w\in R^n}\max_{i\in I}\left\{(f_i(x)-f(x))M+\langle f_i'(x),w\rangle+\frac{1}{2}||w||^2\right\}= \tag{2}$$

$$=\max_{i\in I}\left\{(f_i(x)-f(x))M+\langle f_i'(x),w(x)\rangle+\frac{1}{2}||w(x)||^2\right\}=\nu(x).$$

is connected with each point $x\in R^n$. A minimizer of this problem $w(x)\in R^n$ is also unique.

It follows from the necessary conditions for the minimum of the problem (2) that there exist coefficients $\lambda_i(x)\geq 0$, $i\in I$, $\sum\limits_{i=1}^{s}\lambda_i(x)=1$, such that the equalities

$$w(x)=-\sum_{i=1}^{s}\lambda_i(x)f_i'(x),$$

$$\nu(x)=\sum_{i=1}^{s}\lambda_i(x)(f_i(x)-f(x))M-\frac{1}{2}||w(x)||^2\leq 0$$

are valid.

Lemma 1.

1) *The function* $\nu(x)$ *is equal to zero iff the point* x *is the minimizer of the function* f *on* R^n.

2) *The vector* $w(x)=0_n$ *iff* $\nu(x)=0$.

Thus if a point $x\in R^n$ is a minimizer of the function f on R^n then $w(x)=0_n$. If a point x is not a minimizer of f on R^n then the direction $w(x)$ is a direction of the descent of the function f at x.

Describe a method for minimizing the function f on R^n .

Choose any point $x_0\in R^n$. If the point x_0 is a minimizer of the function f on R^n then the process terminates. Let the point $x_k\in R^n$ be already found. If the point x_k is a minimizer of the function f on R^n then the process terminates. Assume that the point x_k is not a minimizer of the function f on R^n. Find the direction $w(x_k)$, that is, solve the optimization problem (2) at $x=x_k$. Then there are multipliers $\lambda_i(x_k)=\lambda_i^k\geq 0$, $i\in I$, $\sum\limits_{i=1}^{s}\lambda_i^k=1$, such that the relations

$$w(x_k)=-\sum_{i=1}^{s}\lambda_i^k f_i'(x_k)\neq 0_n,$$

$$\nu(x_k) = \sum_{i=1}^{s} \lambda_i(x_k)(f_i(x_k) - f(x_k))M - \frac{1}{2}||w(x_k)||^2 < 0$$

are satisfied. Assume $\bar{\alpha} = \frac{1}{M}$ and $x_{k+1} = x_k + \bar{\alpha} w(x_k)$.

Lemma 2. *At each point x_k the inequality*

$$f(x_{k+1}) - f(x_k) \leq \frac{1}{M}\nu(x_k) \tag{3}$$

is valid.

Inequality (3) shows, that the process is relaxational, hence, the sequence $\{f(x_k)\}$ is converging. If the sequence x_k is infinite, then $\nu(x_k) \to 0$ at $k \to +\infty$.

As for any strongly convex function, the set $\mathcal{L}(x_0) = \{x \in R^n \mid f(x) \leq f(x_0)\}$ is a compact set then the sequence $\{x_k\}$ generated by this method lies in the set $\mathcal{L}(x_0)$.

Lemma 3. *For the given method the inequalities*

$$\nu(x_k) \leq m\,(f(x^*) - f(x_k))\,, \quad f(x_{k+1}) - f(x_k) \leq \frac{m}{M}(f(x^*) - f(x_k))$$

are satisfied at each point x_k.

Hence the sequence $\{x_k\}$ is converging and its cluster point x^* is the minimizer of the function f on R^n.

Theorem 1. *In the given method, at any point x_0 the sequence $\{x_k\}$ converges to the minimizer x^* of the function f with the rate of a geometric progression:*

$$f(x_{k+1}) - f(x^*) \leq q(f(x_k) - f(x^*)),$$

where $q = 1 - \dfrac{m}{M}$, and there exists a positive number Q, such that the inequality

$$||x_k - x^*|| \leq Q(\sqrt{q})^k$$

is true.

Consider in more detail the computing aspect of the solution of the problem (2). To find the direction $w(x_k)$, it is necessary to solve the quadratic programming problem

$$\min_{\lambda \in \Lambda} \left\{ \langle G(x_k)\lambda, \lambda\rangle + M\langle p(x_k), \lambda\rangle \right\},$$

where $\Lambda = \{\lambda = (\lambda_1, \ldots, \lambda_m) \in R^m \mid \sum\limits_{i=1}^{m} \lambda_i = 1,\ \lambda_i \geq 0,\ i \in I\}$, and $G(x_k)$ is the Gram matrix of the vectors $f_i'(x_k)$ that is

$$G(x_k) = \begin{pmatrix} \langle f_1'(x_k), f_1'(x_k)\rangle \ldots . \langle f_1'(x_k), f_m'(x_k)\rangle \\ \cdots \\ \langle f_m'(x_k), f_1'(x_k)\rangle \ldots . \langle f_m'(x_k), f_m'(x_k)\rangle \end{pmatrix}$$

and

$$p(x_k) = (p_1(x_k), \ldots, p_m(x_k)) \in R^m,\ p_i(x_k) = f(x_k) - f_i(x_k),\ i \in I.$$

Thus it is necessary to determine the multiplies $\lambda_i, i \in I$, and to express the vector $w(x_k)$ in terms of λ_i's by the formula $w(x_k) = -\sum\limits_{i=1}^{s} \lambda_i^k f_i'(x_k)$.

Optimal Two-model Quadratures for Numerical Integration

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key words: *numerical integration, two-model quadrature formulas*

The effectiveness of the so-called two-model quadrature formulas is analyzed. Such formulas combine two integrand-evaluating models, which differ in accuracy and complexity. The problem of finding optimal two-model quadratures is stated and solved in the class of Lipschitz integrands.

The Approach to Modelling of Educational Process

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key words: *bimatrix game, modelling, teaching process, extraproximal approach*

1. Statement of the problem. We consider a situation with two participants: one is a teacher, the other is group of the students.

The teacher represents knowledge which is output of work, for example, in the form of series of the lectures in certain knowledge area. The knowledges differ by a level of complexity of stated ideas or concepts. It means that knowledges, for example, can be very simple, available for majority of people; rather complex, available for people with some intellectual background and very complex requiring sizeable intellectual training and, alpha and omega, significant strong-willed efforts to possess by these knowledges. The other participant of the situation is a group of students who wishes to gain certain knowledges from some technological, economic or scientific areas. The collision of situation consists in that if the group of students has not sufficient intellectual and strong-willed training, then the certain complex knowledges cannot be adopted by the student in a sufficient measure. Thus, level of intellectual and strong-willed training of the student group should be matched to complexity of knowledges, that is in learning process the student group could effectively acquire knowledges from new area for them. To describe the situation formally we introduce two payment matrixes P and S, which describe the gains of players for teacher and student group

$$P = \begin{pmatrix} p_{11} & p_{12} & \dots & p_{1n} \\ p_{21} & p_{22} & \dots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{n1} & p_{n2} & \dots & p_{nn} \end{pmatrix}, \quad S = \begin{pmatrix} s_{11} & s_{12} & \dots & s_{1n} \\ s_{21} & s_{22} & \dots & s_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ s_{n1} & s_{n2} & \dots & s_{nn} \end{pmatrix},$$

and two deontological (V.Lefebvre, 2003) matrixes D and B, which determines strong-willed and ethical relations of the teacher and student group to learning process

$$D = \begin{pmatrix} d_{11} & d_{12} & \dots & d_{1n} \\ d_{21} & d_{22} & \dots & d_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ d_{n1} & d_{n2} & \dots & d_{nn} \end{pmatrix}, \quad B = \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & b_{22} & \dots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nn} \end{pmatrix}.$$

Here, the rows $s_{ij}+b_{i,i}$, $i = 1, 2, ..., n$, of matrices $S+B$ are the strategies, which describe some distribution of students on the levels of intellectual and ethic training. The columns $p_{ij} + d_{j,j}$, $j = (1, 2, ..., n)$ of matrices $B + D$ are strategies of the teacher, reflecting his intellectual and ethic level of training. If the strategy (i, j) is chosen, then $p_{i,j} + d_{j,j}$ means the lecture complication of jth level of teacher for the students with ith level of training and $s_{i,j} + b_{i,i}$ means the efficiency of mastering of the jth theacher's lecture for the student subgroup of ith level. Pursuant to the rational behaviour principle both players try to reach a maximum level of the efficiency. In the model, these strategies are denoted by (i^*, j^*):

$$(I^*, j^*) \subset \operatorname{Arg}\max_i (s_{i,j^*} + b_{i,i}), \quad i = (1, 2, ..., n),$$

$$(I^*, j^*) \subset \operatorname{Arg}\max_j (p_{i^*,j} + d_{j,j}), \ j = (1, 2, ..., n). \qquad (1)$$

Here, it is required to find such game strategies that the one of them provide a maximal efficiency in obtaining knowledges and the other presents the maximal complication of delivered knowledges. The computing of Nash equilibrium for large scale bimatrix games requires the development of special methods. The other problem connected with bimatrix games consists of that the Nash equilibrium can not exist, then reasonablly to pass to problems in the mixed strategies.

2. Matrix games in the mixed strategies. Formally the statement of mixed strategies matrix games looks like

$$x^* \in \operatorname{Argmax}\{\langle x, Sy^* \rangle + \langle Bx, x \rangle \mid \ \langle e, x \rangle = 1, \ x \geq 0\},$$

$$y^* \in \operatorname{Argmax}\{\langle Px^*, y \rangle + \langle Dy, y \rangle \mid \ \langle e, y \rangle = 1, \ y \geq 0\}, \qquad (2)$$

where $e = (1, 1, ..., 1)$. The dimension of e is equal to dimension x or y. Constraints are the simplexes $\langle e, x \rangle = \sum_1^n x_j = 1$, where $x_j \geq 0$ and $\langle e, y \rangle = \sum_1^n y_i = 1$, where $y_i \geq 0$. The objective functions of the players represent sums of bilinear and square functions $\langle x, Sy \rangle + \langle Bx, x \rangle$, $\langle Px, y \rangle + \langle Dy, y \rangle$, where S, P and $B \geq 0$, $D \geq 0$ are matrices. The bilinear components of these functions reflect interrelation or effect of one player to other, if one of them has chose his strategy. Square components pursuant to the concept of the Decision Making theory describe of preference for each of the players under selection alternatives on own strategic sets. The solution of game x^*, y^* is a fixed point or Nash equilibrium. For the solution of game (2) it is naturally to apply the simple iteration method:

$$x^{n+1} \in \operatorname{Argmax}\{\langle x, Sy^n \rangle + \langle Bx, x \rangle \mid \ \ \langle e, x \rangle = 1, \ x \geq 0\},$$

$$y^{n+1} \in \operatorname{Argmax}\{\langle Px^n, y \rangle + \langle Dy, y \rangle \mid \ \ \langle e, y \rangle = 1, \ y \geq 0\}. \qquad (3)$$

If the mapping of this system $(\operatorname{Argmax}\{...\}, \operatorname{Argmax}\{...\})$ is strongly contracting, then, according to the contraction mapping principle, the process (3) converges to (x^*, y^*). However in a common case the game mapping is not strong contract and, therefore, process (3) does not converge to Nash equilibrium. In this case we offer to use the extraproximal approach, which is founded on two ideas: an incompressibility of game mapping generated by regularization and splitting of proximal step into two half-steps: the first half-step is

$$\bar{x}^n = \text{argmax}\{(1/2)|x - x^n|^2 + \alpha(\langle x, Sy^n\rangle + \langle Bx, x\rangle) \mid \langle e, x\rangle = 1,\ x \geq 0\},$$

$$\bar{y}^n = \text{argmax}\{(1/2)|y - y^n|^2 + \alpha(\langle Px^n, y\rangle + \langle Dy, y\rangle) \mid \langle e, y\rangle = 1,\ y \geq 0\},$$

and the second half-step is

$$x^{n+1} = \text{argmax}\{(1/2)|x - x^n|^2 + \alpha(\langle x, S\bar{y}^n\rangle + \langle Bx, x\rangle) \mid \langle e, x\rangle = 1,\ x \geq 0\},$$

$$y^{n+1} = \text{argmax}\{(1/2)|y - y^n|^2 + \alpha(\langle P\bar{x}^n, y\rangle + \langle Dy, y\rangle) \mid \langle e, y\rangle = 1,\ y \geq 0\}.$$

The initial state of the teacher-student system is determined by an initial level of training for players to learning process. Level of lecture complexity of the teacher and level of training of the student group are described by vectors of knowledges x^n and y^n for the teacher and students accordingly. On each step of learning process the teacher acts as the processor of knowledges, which transforms a vector of old knowledges of the students y^n to a vector of new knowledges x^{n+1} for mastering by the students. This process assumes that the teacher reflexive reflects in his thinking a level of knowledges of the students. On the other hand, in the same learning process the students on the basis of old knowledges (vector x^n) submitted in the teacher lectures gain a vector of a new knowledges y^{n+1}. The sequence of the lectures determines multistep process, which forms a sequence of vectors of knowledges of the students y^n obtained by them from the teacher lectures x^n. The sequence of vectors of knowledge (x^n, y^n) should be completed by assimilation of objective domain of knowledge with maximum efficiency. We formulate the statement about convergence of process [1]. We introduce system of matrixes C_1 and C_2, which determines a behaviour of the players as of the unified system

$$C_1 = \begin{pmatrix} 0 & S \\ P & 0 \end{pmatrix}, \quad C_2 = \begin{pmatrix} D & 0 \\ 0 & B \end{pmatrix}.$$

If $C_1 + C_2 \geq 0$, the length of a step $\alpha < (1/\sqrt{2}|C_1 + C_2|)$, then the proximal method converges to a Nash equilibrium, i.e. $(x^n, y^n) \to (x^*, y^*)$ as $n \to \infty$. The objective functions for each of the players reache a maximal values in an own variables for each component of the vector (x^*, y^*). The author expresses steep thanks to A.S.Antipin for fruitful arguing for the game model.

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About Polyhedral Approximation of Edgeworth-Pareto Convex Hull for the Special Class of Integer Multicriteria Optimization Problems*

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key words: *multicriteria integer optimization*

Most of multicriteria methods are based on interaction with decision-maker and searching alternatives, which are most suitable for the decision-maker from undominated (Pareto) frontier, i.e. set of all undominated alternatives and presenting these alternative the decision-maker.

This work is devoted to an integer multicriteria optimization problem for one special class of the goal functions.

In this work, the algorithm for a solution of such problem is proposed. This algorithm uses ideas developed within the framework of solving single-criterion integer optimization problems and concept of optimal polyhedral approximating methods of multidimensional convex compacts.

The mathematical simulation is a conventional tool for searching effective solutions of complex problems. Multicriteria optimization methods play key role in the decision-making process [3], because they allow to take into account different demands conflicting with

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one another. The usual formulation of the multicriteria problem is given in the following form:

$$f : X \to \mathbb{R}^d,$$
$$f(X) = Y \subset \mathbb{R}^d,$$
$$f(x) \to \max_{x \in \mathbb{R}^d}.$$

Most of multicriteria methods are based on interaction with decision-maker and searching alternatives most suitable for the decision-maker from undominated (Pareto) frontier $P(Y)$, i.e. set of all undominated alternatives

$$P(Y) = \{y \in Y \,|\, \{y' \in Y | y' \leqslant y\} = \emptyset\}, \tag{1}$$

and presenting these alternatives to the decision-maker.

This work is devoted to an integer multicriteria optimization problem for one special class of the goal functions. Consider the following problem:

$$[c, f] \to \min_{x \in X}, \tag{2}$$

where X is discrete set,

$$X = \left\{x \in X_0 | g_k(x) \leqslant 0, k = \overline{1, N_g}\right\}, X_0 = \{0, \ldots K\}^n,$$

and components of vector functions c, f and g are monotonous:

$$\begin{aligned} c_i &: X_0 \to \mathbb{R},\ c_i(x') \geqslant c_i(x''), x' \geqslant x'',\ i = \overline{1, N_c}; \\ f_j &: X_0 \to \mathbb{R}, f_j(x') \leqslant f_j(x''), x' \geqslant x'', j = \overline{1, N_f}; \\ g_k &: X_0 \to \mathbb{R}, g_k(x') \leqslant g_k(x''), x' \geqslant x'', k = \overline{1, N_g}. \end{aligned} \tag{3}$$

Such problems often arise, for example, in environmental problems, where criteria are divided in two groups. First group characterizes the cost of environmental project and the second one is connected with environmental quality. It is natural for environmental problems that the cost of a project is increasing when more expensive technologies are used.

It is suggested to solve this problems using multicriteria visualization-based technique of the Interactive Decision Maps (the IDMS) [2]. The technique is based on approximation of the Edgworth-Pareto convex hull that is the set with the same nondominated frontier as conv(Y). Formally,

$$Y_P^C = \mathrm{conv}(Y) + \mathbb{R}_+^d. \tag{4}$$

Then, visualization of Pareto frontier of this set is used. After completion of the analysis of undominated frontier (including comprehension boundaries of feasible values of criteria and interrelation between criteria values) decision-maker can specify a goal point (the reasonable goals method)[2]. Further, some feasible solutions closed to the goal point are presented to decision-maker.

The most complicated mathematical problem of the approach is the approximation of the Edgworth-Pareto convex hull in the case of essential number of criteria (from three up to eight). The algorithm for a solution of such problem is presented in this paper. This algorithm uses ideas developed within the framework of solving integer optimization problems with one criterion [4] and concept of optimal polyhedral approximating methods of multidimensional convex compacts [1].

Problems (2) in the case of one criterion have been widely studied in discrete programming and common method of solving such problems is the branch-and-bound method. In this work, the ideas of the branch-and-bound method are adapted for the multicriteria case. It allows constructing algorithm that builds iterative sequence of polyhedrons, approximating Edgworth-Pareto convex hull in the integer multicriteria optimization problems for the special class of the goal functions.

It is proved that the developed algorithm has the following features.

Theorem. *The algorithm finishs its work for finite number of iterations N, not greater then 2^n. Moreover, the algorithm builds monotonous by inclusion sequences of polyhedrons $\{P_l\}$ and $\{Q_l\}$ such that*

$$P_0 \subset P_1 \subset \ldots \subset P_N = Y_P^C = Q_N \subset Q_{N-1} \subset \ldots \subset Q_0. \tag{5}$$

Sequences of polyhedrons $\{P_l\}$ and $\{Q_l\}$ are internal and external approximation of the Edgworth-Pareto convex hull correspondingly. These polyhedrons allow to represent nondominated frontier to decision-maker with an arbitrary chosen accuracy.

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Construction of a Stabilizing Control and Solution for a Problem of the Center and Focus for Differential Systems with a Polynomial Part on the Right Side

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key words: *differential systems of nth order, asymptotic stable systems, stabilizing control, domains of asymptotic stability*

The stationary differential systems with polynomial right parts are considered. The necessary and sufficient conditions are formulated when a given domain is a domain of asymptotic stability and the origin of coordinates is either focus or center. The problem of construction of stabilizing control in a form of polynomial is studied.

Introduction. In this paper, the differential systems with a right polynomial part f(.) of

$$x = (x_1, x_2, ..., x_n) \in \Re^n$$

are considered, i.e.

$$f(x) = (f_1(x), f_2(x), ..., f_n(x))^*,$$

where
$$f_p(x) = \sum_{l_1,l_2,\dots,l_n \in I_p} a^{(p)}_{l_1,l_2,\dots,l_n} x_1^{l_1} x_2^{l_2} \dots x_n^{l_n},$$

$p \in 1 : n, l_1, l_2, \dots l_n$ are non-negative integers and $*$ is the transposition sign, $a^{(p)}_{l_1,l_2,\dots,l_n}$ are real-valued numbers, I_p is a finite set of indexes of the polynomial $f_p(x)$.

To analyse the system

$$\dot{x} = f(x)$$

a method is suggested, which is different from Lyapunov's methods and based on a system transformation idea, so that we would be able to say something definite about stability.

Further the problem will be solved to construct a stabilizing control in any given region of the origin of coordinates for the system

$$\dot{x} = f(x, u)$$

where $x \in \Re^n$ is a phase vector , $u \in R^r$ is a control,

$$f(x, u) = (f_1(x, u), f_2(x, u), \dots, f_n(x, u))^*$$

is a vector polynomial of x and u, i.e. $f_p(x, u) =$

$$\sum_{l_1,l_2,\dots,l_n,m_1,m_2,\dots,m_r \in I_p} a^{(p)}_{l_1,l_2,\dots,l_n,m_1,m_2,\dots,m_r} x_1^{l_1} x_2^{l_2} \dots x_n^{l_n} u_1^{m_1} u_2^{m_2} \dots u_r^{m_r},$$

where $p \in 1 : n,\ \ l_1, l_2, \dots, l_n, m_1, m_2, \dots, m_r$ are non-negative integers, $a^{(p)}_{l_1,l_2,\dots,l_n,m_1,m_2,\dots,m_r}$ are real-valued numbers, I_p is a finite set of indexes of the polynomial $f_p(x, u)$. We will assume that the zero vector $0 = (0, 0, \dots, 0) \in \Re^n$ is a solution of the system, i.e. $f(0, 0) = 0$.

The main results. Let us consider the differential system

$$\dot{x} = f(x), \tag{1}$$

where $x = (x_1, x_2, \dots, x_n) \in \Re^n$ and

$$f(x) = (f_1(x), f_2(x), \dots, f_n(x))^*.$$

The vector-polynomials $f_p(.)$ and their coefficients $a^p_{l_1,l_2,\dots,l_n}$ satisfy the conditions written above in the Introduction.

We assume that $f(x) \neq 0$ for all $x \neq 0$ in some neighborhood of the origin.

We call the order $deg(f_p(.))$ of the polynomial $f_p(\cdot)$ the maximal degree of the polynomial $f_p(.)$ in the variables $x_j, j \in 1:n$, in totality, i.e.

$$deg(f_p(x)) = \max_{l_1,l_2,\ldots,l_n \in I_p} (l_1 + l_2 + \ldots + l_n).$$

So if for $n = 2$ and $l_{ij} \neq 0$

$$f_1(x) = l_{11}x_1^2 + l_{12}x_2^2 + l_{13}x_1x_2, \quad f_2(x) = l_{21}x_1^3 + l_{22}x_2 + l_{23}x_3$$

then the order of $f_1(x)$ is equal to two and the order of $f_2(x)$ is equal to three.

We call the order of the function $f(\cdot)$ the maximal degree of the polynomials $f_p(x), p \in 1:n$, regarding the variables $x_j, j \in 1:n$, i.e.

$$deg(f(x)) = \max_{p \in 1:n} deg(f_p(x)).$$

Consequently, the order of the function $f(\cdot)$, for example, written above is equal to $\max(2,3) = 3$.

Let us rewrite the system (1) in equivalent form:

$$\dot{x} = A(x)x \tag{2}$$

The elements $a_{ij}(x)$ of a matrix $A(x)[n \times n]$ are continuous polynomial functions of x. Conversion (1) to (2) is not unique. It can be done by an infinite number of methods. Thus

$$a_{ij}(x) = \sum_{l_1,l_2,\ldots,l_n \in I_i} \sum_{j} \alpha_{ij}(x)\, a^{(i)}_{l_1,l_2,\ldots,l_n} x_1^{l_1} x_2^{l_2} \ldots x_j^{l_j - 1} \ldots x_n^{l_n}$$

if $l_j \geq 1$. We have the following correlation for the coefficients $\alpha_{ij}(x)$, $i,\ j \in 1:n$, for all x from some region $D, 0 \in intD$,

$$\sum_{j \in 1:n} \alpha_{ij}(x) = 1, \ \forall\, i \in 1:n.$$

We consider all possible continuous matrices $A(x)$ whose elements are polynomial functions of x for that the system (1) is equivalent to the system (2) . We will denote the set of all such matrices by $\mathcal{A}$.

Theorem 1. *In order that the domain D consisting from the whole trajectories of the system (1), i.e. $x(\cdot, x_0, t_0) \in D, x_0 \in D$, for all $t > t_0$ were a region of asymptotic stability it is necessary and sufficient that there was such a matrix $A(\cdot) \in \mathcal{A}$ of the system (2) in the domain D whose eigenvalues have negative real parts at any point $x \in D, x \neq 0$.*

Now turn to the problem of the center and focus.

Theorem 2. *In order that the point $0 \in \Re^n$ is a focus of the system (1) with a right polynomial part $f(\cdot), f(x) \neq 0$ for $x \neq 0$ it is necessary and sufficient that there is such a continuous matrix $A(\cdot) \in \mathcal{A}$ of the system (2) that all its eigenvalues at any point $x, x \neq 0$, from some neighbourhood D of the origin, consisting from the whole trajectories, have negative real parts and non-zero imaginary parts.*

If, moreover, there is no a matrix $A(\cdot) \in \mathcal{A}$ of the system (2) with negative real-valued eigenvalues at all points $x \in D, x \neq 0$, then the trajectories turn around the origin infinitely often.

Theorem 3. *In order that the point $0 = (0, 0)$ be a center for a two-dimentional system (1) it is necessary and sufficient that there is a matrix $A(.) \in \mathcal{A}$ of the system (2) whose eigenvalues are non-zero imaginary numbers in a neighbourhood S of the origin, where $f(x) \neq 0$ for $x \neq 0, x \in S$.*

Let us turn to the question of finding a stabilizing control.

Consider the following differential system

$$\dot{x} = f(x, u), \tag{3}$$

where $x = (x_1, x_2, ..., x_n) \in \Re^n$ is a phase vector, $u = (u_1, u_2, ..., u_r) \in \Re^r$ is a control, $f(x, u) = (f_1(x, u), f_2(x, u), ..., f_n(x, u))^*$ is a vector-polinimial of x and u with constant real-valued coefficients, i.e.
$f_p(x, u) =$

$$\sum_{l_1, l_2, ..., l_n, m_1, m_2, ..., m_r \in I_p} a^{(p)}_{l_1, l_2, ..., l_n, m_1, m_2, ..., m_r} x_1^{l_1} x_2^{l_2} ... x_n^{l_n} u_1^{m_1} u_2^{m_2} ... u_r^{m_r},$$

$p \in 1 : n, \quad l_1, l_2, ..., l_n, m_1, m_2, ..., m_r$ are non-negative integers and

$$a^{(p)}_{l_1, l_2, ..., l_n, m_1, m_2, ..., m_r}$$

are real-valued numbers, I_p is a finite set of indexes of the polynomial $f_p(\cdot)$.

Let us assume that the zero-vector $0 = (0, 0, ..., 0) \in \Re^n$ is a solution of the system (3) for $u = 0 \in \Re^r$.

Instead of the equation (3) we consider the equation

$$\dot{x}= f(x, u) + \varphi(u), \tag{4}$$

where $\varphi(\cdot)$ is a vector-polynomial $\varphi(u) = (\varphi_1(u), \varphi_2(u), ..., \varphi_n(u))^*$ with the degree not greater than the degree of the function $f(z)$ as a function of $z = (x, u)$,

$$\varphi_p(u) = \sum_{i_1,i_2,\dots,i_r \in M_p} b^{(p)}_{i_1,i_2,\dots,i_r} u_1^{i_1} u_2^{i_2} ... u_r^{i_r},$$

where $p \in 1 : n$ and $b^{(p)}_{i_1,i_2,\dots,i_r}$ are constant real-valued numbers, $i_1, i_2, ..., i_r$ are non-negative integers, M_p is a finite set of indexes of the polynomial $\varphi_p(.)$.

We find a stabilizing control $u = u(x)$ in a form of a polynomial in x.

Theorem 4. *For any domain $D, 0 \in intD$, the vector-polynomials $u(\cdot)$ and $\varphi(\cdot)$ can be chosen such that*
1) D is a region of asymptotic stability for the differential system (4);
2) the degree of $u(x)$ does not exceed the degree of the vector-polynomial $f(x, u)$ as a function of x;
3) the degree of $\varphi(\cdot)$ is not greater than the degree of the function $f(\cdot)$ as a function of the variable $z = (x, u)$.

Applications of a More for Less Result to Labour Markets and to Auctions

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key words: *linear programming, bidding, distribution*

Introduction. In Ryan (2000a,b) I developed a general class of more for less results pertaining to economies of scale and scope and applied them respectively to regulation and merger related examples. Here I consider another class of more for less results and specialize them to labour markets and to auctions of ramp spaces at airports.

A class of more for less results Consider an assignment problem in which: a) x_{ij} represent assignments of inputs i to outputs j; b) $a_i > 0$ represent availabilities of inputs i and $b_j > 0$ represent target quantities of outputs j; c) c_{ij}represents the net cost or benefit associated with a transformation of input i into output j (e.g. via training $i \to j$); d) R_i^+, R_i^- and S_j^+, S_j^- measure potential shortages and surpluses of inputs and outputs relative to a_i and b_j.

Then, associating unit penalties d_i^+, d_i^- and e_j^+, e_j^- with potential shortages or surpluses respectively relative to available inputs a_i and to target outputs b_j, the minimum cost assignment can be found via (I):

$$\begin{array}{llll}
\text{Min} \sum_{ij} c_{ij}x_{ij} + \sum_i d_i^+ R_i^+ + \sum_i d_i^- R_i^- & & \text{Max} \sum_i \mu_i a_i + \sum_j \theta_i b_j & \\
+ \sum_j e_j^+ S_j^+ + \sum_j e_j^- S_j^- & & & \\
\text{st} \quad \sum_j x_{ij} + R_i^+ - R_i^- = a_i & (\mathrm{I}) & \text{st} \quad \mu_i + \theta_j \le c_{ij} & (\mathrm{I})' \\
\sum_i x_{ij} + S_j^+ - S_j^- = b_j & & -d_i^- \le \mu_i \le d_j^+ & \\
x_{ij}, R_i^+, R_i^-, S_j^+, S_j^- \ge 0 & & -e_j^- \le \theta_j \le e_j^+. &
\end{array}$$

Problem (I) is a linear program. Associating the dual variables μ_i, θ_j with its constraints its dual is (I)$'$. If $c_{ij} \ge 0$ all i, j and if all d_i^+, d_i^-, e_j^+, e_j^- are non negative and sufficiently large relative to c_{ij}, then (I) has a bounded optimal solution which, by the dual theorem, is equal in value to the optimum to (I)$'$.

Notice that the dual variables μ_i, θ_j are unrestricted in sign. In particular one or more may be negative. This suggests that overall cost might be reduced by reducing one or more inputs i and/or outputs j. More subtly, it follows that, if $\mu_i + \theta_j < 0$ for some non basic route i, j, it will be possible to reduce overall cost by increasing input of type i and output of type j. An example of that version of the phenomenon, which was first recorded in Charnes and Klingman (1971) and in Schwartz (1971), will be considered in the next Section. But the principal focus here is on a related but different class of results which potentially stems from changes in magnitudes of one or more of the quantities d_i^+, d_i^-, e_j^+, e_j^-. With that perspective consider Theorem 1:

Theorem 1. With notation as above and if: a) $c_{ij} \geq 0$ all i, j; b) all d_i^+, d_i^-, e_j^+, e_j^- are non negative and sufficiently large relative to c_{ij} to ensure that the solution to (II) is bounded and; c) $c'_{ij} \leq c_{ij}$, $d_i^{+\prime} \leq d_i^+$, $d_i^{-\prime} \leq d_i^-$, $e_j^{+\prime} \leq e_j^-$, $e'_j \leq e_j^-$ at least one c_{ij}, d_i^+, d_i^-, e_j^+, e_j^-, then:

$$\begin{aligned}
&\text{Min} \sum_{ij} c_{ij}x_{ij} + \sum_i d_i^+ R_i^+ + \sum_i d_i^- R_i^- \geq \text{Min} \sum_{ij} c'_{ij}x_{ij} + \sum_i d_i^{+\prime} R_i^+ \\
&+ \sum_j e_j^+ S_j^+ + \sum_j e_j^- S_j^- \qquad\quad + \sum_i d_i^{-\prime} R_i^- + \sum_j e_j^{+\prime} S_j^+ + \sum_j e_j^{-\prime} S_j^-
\end{aligned}$$

$$\begin{array}{lll}
\sum_j x_{ij} + R_i^+ - R_i^- = a_i \quad \text{(I)} & \qquad & \sum_j x_{ij} + R_i^+ - R_i^- = a_i \quad \text{(Ia)} \\
\sum_i x_{ij} + S_j^+ - S_j^- = b_j & & \sum_i x_{ij} + S_j^+ - S_j^- = b_j \\
x_{ij}, R_i^+, R_i^-, S_j^+, S_j^- \geq 0 & & x_{ij}, R_i^+, R_i^-, S_j^+, S_j^- \geq 0.
\end{array}$$

Proof. The proof is in two parts: First, a feasible solution exists to (I). (Consider $R_i = a_i$, $S_j = b_j$ all i, j). And, from the assumption that all d_i^+, d_i^-, e_j^+, e_j^- are non negative and sufficiently large relative to $c_{ij} \geq 0$ to ensure that the solution to (II) is bounded, together with the dual theorem, it follows that a bounded optimum exists to (I). Second; an optimal solution to (I) is a feasible but not necessarily an optimal solution to (Ia).

Since (Ia) is a linear program it generates a dual, (Ia)'. Since that dual is isomorphic with (I)$'$ for clarity dual variables $\underline{\mu}_i$, and $\underline{\theta}_i$ are associated with the constraints of (Ia). The dual theorem together with Theorem 1 then implies:

$$Max \sum_i \mu_i a_i + \sum_j \theta_i b_j \qquad \geq \qquad Max \sum_i \underline{\mu}_i a_i + \sum_j \underline{\theta}_i b_j$$

$$\begin{array}{llll}
\text{st} & \mu_i + \theta_j \leq c_{ij} \quad \text{(I)}' & \text{st} & \underline{\mu}_i + \underline{\theta}_j \leq c_{ij} \quad \text{(Ia)}' \\
& -d_i^- \leq \mu_i \leq d_j^+ & & -d_i^- \leq \underline{\mu}_i \leq d_j^+ \\
& -e_j^- \leq \theta_j \leq e_j^+ & & -e_j^- \leq \underline{\theta}_j \leq e_j^+.
\end{array}$$

A labour market application. Assume that a_i represent available workers with skills i, b_j represent required workers with skills j and $c_{ij} \geq 0$ represent unit costs (if any) of transforming i into j. Then (I) determines the minimum cost reskilling plan. In that context d_i^-, e_j^- would represent costs associated with hiring rather than reskilling additional workers with skills i, j and d_i^+, e_j^+ would represent the unit costs associated with shortages. An example will illustrate one kind of application of Theorem 1. Tableau 1 illustrates how information in (I) or (Ia) can be arranged into a Tableau format if optimally $R_i^+ = R_i^- = S_j^+ = S_j^- = 0$ all i, j (as will be assumed for this example). Tableau 2 illustrates an optimal solution to (I) for a case in which there are two labour markets, one with skills $i = 1, 2$, $j = 1, 2$, and the other with skills $i = 3$, $j = 3$ all i, j in effect separated by arbitrarily large values for c_{ij} except for transformations within those markets. (Such large weights might for example represent union induced demarcations.). Tableau 3 illustrates optimal solutions to (Ia) in which labour markets have been liberalised so that in principle workers of any of skills $i = 1, 2, 3$ may be retrained to any of skills $j = 1, 2, 3$:

c_{11} x_{11}	c_{12} x_{12}	c_{13} x_{13}	a_1
c_{21} x_{21}	c_{22} x_{22}	c_{23} x_{23}	a_2
c_{31} x_{31}	c_{32} x_{32}	c_{33} x_{33}	a_3
b_1	b_2	b_3	

Tableau 1

6 4	5 2	M	$a_1 = 6$
4	2 10	M	$a_2 = 10$
M	M	7 9	$a_3 = 9$
$b_1 = 4$	$b_2 = 12$	$b_3 = 9$	

Tableau 2

6	5 6	15	$a_1 = 6$
4	2 6	5 4	$a_2 = 10 + \delta$
1 4	6	7 5	$a_3 = 9$
$b_1 = 4 + \delta$	$b_2 = 12$	$b_3 = 9$	

Tableau 3

The reader can easily confirm that there is a cost reduction from a total of 117 to a total 101 in moving from Tableau 2 to Tableau 3, in line with the prediction of Theorem 1. The reader can also verify that, with reference to Tableau 3, values consistent with optimality of $(\mathrm{Ia})'$ are: $\mu_1 = 0$, $\mu_2 = -3$, $\mu_3 = -1$, $\theta_1 = 2$, $\theta_2 = 5$, $\theta_3 = 8$. [As promised Tableau 2 also illustrates the more for less paradox since $\mu_2 + \theta_1 = -1 < 0$. If supplies of skill 2 and demands for skill 1 are both increased by up to $\delta = 5$ units the reskilling cost could be reduced by -1δ. Whether or not the overall cost could or would be reduced would depend on the corresponding values of d_1^- and e_2^-.]

Notice, too, that, via successive applications of Theorem 1, successive embedding of labour markets within larger ones would potentially promise corresponding cuts in reskilling costs.

An auction related application. Assuming that $i = 1, 2, \ldots, n$ represent objects in an auction and $j = 1, 2, \ldots, m$ represent classes of bidders for those objects via individual bids v_{ij}, then the maximum proceeds from that auction may be represented as the solution to the following linear program:

$$\begin{array}{c} \text{Max} \sum_{ij} v_{ij} xij \\ \sum_j x_{ij} \leq a_i \\ \sum_i x_{ij} \leq b_j \\ x_{ij} \geq 0. \end{array} \qquad \text{(II)}$$

The structure of (II) is a variant of (I) for which: a) $c_{ij} \leq 0$ all i, j; b) $R_i^- = 0$, $S_j^- = 0$ all i, j as if via sufficiently large values of d_i^-, e_j^-. That in turn suggests consideration of circumstances in which a variant of Theorem 1 might yield useful auction related interpretations and predictions. With such possibilities in prospect consider Theorem 2:

Theorem 2. With notation as above and if $v_{ij} \geq 0$ all i, j, if all d_i^+, d_i^-, e_j^+, e_j^- are non negative and if also $c'_{ij} \leq c_{ij}$, $d_i^{+\prime} \leq d_i^+$, $d_i^{-\prime} \leq d_i^-$, $e_j^{+\prime} \leq e_j^-$, $e'_j \leq e_j^-$ at least one c_{ij} ,d_i^+, d_i^-, e_j^+, e_j^-, then:

$$\begin{array}{ll} \text{Max} \sum_{ij} v_{ij} x_{ij} - \sum_i d_i^+ R_i^+ - \sum_i d_i^- R_i^- \leq & \text{Max} \sum_{ij} v'_{ij} x_{ij} - \sum_i d_i^{+\prime} R_i^+ - \\ - \sum_j e_j^+ S_j^+ - \sum_j e_j^- S_j^- & \sum_i d_i^{-\prime} R_i^- - \sum_j e_j^{+\prime} S_j^+ - \sum_j e_j^{-\prime} S_j^- \\ \sum_j x_{ij} + R_i^+ - R_i^- = a_i \quad \text{(III)} & \sum_j x_{ij} + R_i^+ - R_i^- = a_i \quad \text{(IIIa)} \\ \sum_i x_{ij} + S_j^+ - S_j^- = b_j & \sum_i x_{ij} + S_j^+ - S_j^- = b_j \\ x_{ij}, R_i^+, R_i^-, S_j^+, S_j^- \geq 0 & x_{ij}, R_i^+, R_i^-, S_j^+, S_j^- \geq 0. \end{array}$$

Proof. Similar to Theorem 1

Remarks. As they stand (III) and/or (IIIa) might have unbounded solutions. Upper bounds could easily be imposed if necessary by adding constraints so that quantities R_i^-, S_j^- could not exceed exogenously designated levels which might for example correspond to available quantities of additional input, R_i^-, of type i.

Corollary. Theorem 2 implies that, *even if* $v'_{ij} = v_{ij}$ all i, j, the proceeds of an auction corresponding to an optimal solution to (III) might be increased as if via an optimal solution to (IIIa).

To illustrate this case consider a stylised interpretation with reference to the allocation of parking slots to aircraft at airports. For that purpose assume that, for a representative period, qualities of access to parking for aircraft are made up of various classes of ramp access $i = 1, 2 \ldots, n$, where x_{ij} is the number of users of class $j = 1, 2 \ldots, m$ allocated to ramp i plus apron parking, which is the residual, i.e. S_j^+ for class of user j. Assume further that parking fees v_{ij} are ramp and user contingent and that the airport is subject to nonnegative unit penalties d_i^+, d_i^-, e_j^+, e_j^- if under/over booking ramps or under/over admitting bidders for their use. In that case Theorem 2 predicts that in general, if any of these latter penalties is positive in (III), then a reduction or removal of it may lead to an increase in overall revenue to the airport as if via (IIIa). This result would hold a fortiori if that reduction was sufficient to ensure a negative value for one or more of quantities d_i^+, d_i^-, e_j^+, e_j^-. [For example the airport might move from free apron parking to charging a fee for apron parking.]

Conclusion. In this paper a class of more for less results has been presented with applications to labour markets and to auctions. I conclude by noting that analogous results may easily be generated for more inclusive classes of cases. Specifically: the labour market application might be extended to include cases where skills may optimally remain unemployed or jobs optimally remain unfilled (via correspondingly positive values for R_i^+, S_j^+) and/or in which additional jobs may be acquired and/or additional applicants acquired in accordance with optimally positive values for R_i^-, S_j^- in (III) and/or (IIIa).

With reference to auctions the example considered here corresponds to just one relatively simple class of auctions which can be represented via linear programs. But in principle any class of auction which can be represented by means of a linear program with weak inequality constraints — such as models P3 or P4 in Section 3 of Bikhchandani and Ostroy (2002), or specifically with reference to airport time slot allocation in Rassenti, Smith and Bulfin (1982) - can be embedded in a more comprehensive system analogous to (III) and (IIIa). In that way any such program is potentially open to a more for less result for that class of auctions analogous to that stemming from Theorem 2 and an auction related interpretation with reference to program (II). Finally attention has been confined here to linear programming representations. Analogous results can readily be obtained for nonlinear cases too.

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Asymptotic Long-time Behavior of Solutions of the Cauchy Problem for Some Quasilinear Parabolic Equation*

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key words: *quasilinear parabolic equation, Cauchy problem, asymptotic behavior, wave solutions, wave solution velocity*

In this paper, a quasilinear parabolic equation is studied. The behavior of solutions of the Cauchy problem for the equation reminds solutions' behavior of difference-differential analogue of the shock-wave equation and velocity formulas of wave solution for these cases are similar. O.A. Olejnik and A.M. Iljin studied different quasilinear parabolic equation with different viscous coefficient. Velocity formulas of wave solutions for these cases are different but asymptotic behavior is similar.

Formula of wave solutions velocity is found for the quasilinear parabolic equation. Theorem about necessary and sufficient condition of wave solution existence is proved for the equation. Estimates for wave solutions of the equation is found. Theorem about asymptotic long-term behavior of solutions of the Cauchy problem for the equation under bounded measurable initial function and some conditions is proved. The asymptotic long-term behavior of solutions of this Cauchy problem looks like a wave solution of the equation.

The shock wave equation

$$\frac{\partial u}{\partial t} + \varphi(u)\frac{\partial u}{\partial x} = 0 \tag{1}$$

and equation

$$\frac{\partial u}{\partial t} + \varphi(u)\frac{\partial u}{\partial x} = \varepsilon\frac{\partial^2 u}{\partial x^2}, \tag{2}$$

which is obtained from equation (1) by adding $\varepsilon\frac{\partial^2 u}{\partial x^2}$ to the right-hand side, were investigated in [1]. The addition was called artificial viscous.

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The velocity formula, the condition of wave solution existence and asymptotic long-term behavior of the Cauchy problem for equation (2) were found in [2].

The paper [3] is devoted to investigation of difference-differential equation

$$\frac{\partial u(x,t)}{\partial t} + \varphi(u(x,t))\frac{u(x,t) - u(x-\delta,t)}{\delta} = 0 \qquad (3)$$

with an arbitrary step δ . This equation is difference-differential analogue of (1). velocity c of a wave solution of equation (3) with overfall from u_- to u_+ satisfy the following equality:

$$\frac{1}{c} = \frac{1}{u_+ - u_-}\int_{u_-}^{u_+}\frac{du}{\varphi(u)}.$$

It was noted that at first sight the equation (1) seems to be similar to the equation (3), whose solutions behave quite differently.

In reality, the behavior of trajectories of the difference-differential equation (3) reminds one of the behavior of solution of the Burger's equation (2) but velocity formulas of wave solution for these cases were quite different.

A quasilinear parabolic equation

$$\frac{\partial u}{\partial t} + \varphi(u)\frac{\partial u}{\partial x} = \varepsilon\varphi(u)\frac{\partial^2 u}{\partial x^2}, \qquad \varepsilon > 0, \qquad (4)$$

is of interest. It is obtained from equation (2) by multiply viscous by $\varphi(u)$.

In the present paper, we investigate this equation and obtaine a wave solution velocity formula for it. The wave solution velocity formula looks like a wave solution velocity formula for the difference-differential equations system (3). Theorem about necessary and sufficient condition of wave solution existence is proved for the equation. Estimates for wave solutions of the equation are found. Theorem about asymptotic long-term behavior of solutions of the Cauchy problem for the equation under bounded measurable initial function and some conditions was proved. The asymptotic long-term behavior of solutions of this Cauchy problem looks like a wave solution of the equation.

Let us consider the Cauchy problem for quasilinear parabolic equation (4) under initial conditions

$$u|_{t=0} = u_0(x), \qquad (5)$$

where $u_0(x)$ is bounded and measurable function, one has $u_0(x) \to u_+$ as $x \to +\infty$ and $u_0(x) \to u_-$ as $x \to -\infty$.

A special solution of (4) is called a wave solution with overfall from u_- to u_+ if $u_\varepsilon(x,t) = \widetilde{u}_\varepsilon(s)$, $s = x - ct - d$, where velocity c of the wave depends on asymptotic values

$$\widetilde{u}_\varepsilon(s) \to u_+ \quad as \quad s \to +\infty \quad and \quad \widetilde{u}_\varepsilon(s) \to u_- \quad as \quad s \to -\infty. \qquad (6)$$

Assume that $\varphi(u)$ is continuous and is larger than a positive constant.

Let us introduce a function:

$$K(u) = \frac{1}{u - u_-} \int_{u_-}^{u} \frac{du}{\varphi(u)}.$$

Theorem 1. Let $u_- < u_+$. A wave solution $\widetilde{u}_\varepsilon(s)$ of equation (4) with overfall from u_- to u_+ exists if and only if $c = 1/K(u_+)$, $K(u) < K(u_+)$ for all $u \in (u_-; u_+)$. Any other wave solution of (4) with overfall from u_- to u_+ takes a form $\widetilde{u}_\varepsilon(s - d)$ for some constant d.

Moreover, if $1\backslash\varphi(u_-) < K(u_+) < 1\backslash\varphi(u_+)$ then $|\widetilde{u}_\varepsilon - u_-| \leq Me^{\alpha s}$ and $|\widetilde{u}_\varepsilon - u_+| \leq Me^{-\alpha s}$, where M and α are constants.

Theorem 2. Let $u_- < u_+$, $K(u) < K(u_+)$ for all $u \in (u_-; u_+)$, $1\backslash\varphi(u_-) < K(u_+) < 1\backslash\varphi(u_+)$, integrals

$$\int_{-\infty}^{0} (u_0 - u_-)dx \quad \text{and} \quad \int_{0}^{+\infty} (u_0 - u_+)dx$$

exist. A solution $u_\varepsilon(x,t)$ of the problem (4) under initial condition (5) uniformly by x converges to a wave solution $\widetilde{u}_\varepsilon(x - ct - d)$ as $t \to \infty$, where $c = 1/K(u_+)$ and d is a constant.

Remark. Let $u_- > u_+$. Statements of the theorems are fulfilled if $K(u) > K(u_+)$ for all $u \in (u_-; u_+)$, $1\backslash\varphi(u_+) < K(u_+) < 1\backslash\varphi(u_-)$ instead of $K(u) < K(u_+)$ for all $u \in (u_-; u_+)$, $1\backslash\varphi(u_-) < K(u_+) < 1\backslash\varphi(u_+)$.

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Application of Graph Theory to the Task of Estimation of Goal Importance Coefficients

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The task of finding coefficients of objects' importance based on the paired comparison in different settings has been widely discussed in national and international literature. However, till recent time only case of results of paired comparisons expressed as accurate numbers, i.e. "point" estimations, was investigated. The most popular and widely used method of such matrices processing is T. Saati method, which suggests to use adequate components of the eigenvector (corresponding the maximal eigenvalue) of a matrix composed of point results of paired comparison as objects' importance coefficients. From the methodological point of view, T. Saati model performs the transference of the Berge model for solution of "round tournament leader" task, which considers results of paired comparisons expressed using the ternary scale with gradation "better", "worse", "equal", to the case of results of every paired comparison are expressed by figure demonstrating "how many times one object is more important than other" [1]. The models used in these methods are phenomenological. Importance coefficients or "iterated powers" of objects mentioned in these models are rather assistants for ranging considered objects, than expressions of actual quantitative degree of superiority (importance) of objects. In our opinion, the approach described further, suitable both for interval and point results of objects' paired comparison, is more natural [2].

Statement of the task. We consider the finite aggregate of objects $Y_1, Y_2, \ldots, Y_n$, different by the degree of expression of characteristic C, in which we are interested. We use y_i to name the degree of expression of Y_i object's characteristic C. We assume that results of paired comparison of objects Y_i and Y_j related to the expression of characteristic C are presented by the interval numerical estimations:

$$a_{ji} \leq y_i y_j^{-1} \leq a_{ij}^{-1}; \quad a_{ij}, a_{ji} \geq 0; \quad i, j = \overline{1, n}; \quad i \neq j. \qquad (1)$$

It is needed to develop the method of finding points estimations

$$y_1^*, y_2^*, \ldots, y_n^*$$

of the degree of expression of characteristic C of objects $Y_1, Y_2, \ldots, Y_n$ based on inequalities (1). At the same time it is desirable to reduce to the reasonable minimum subjective elements introduced to the suggested method.

Method of task solution Matrix $||a_{ij}||$ by definition is called consistent, if system of inequalities (1) has at least one solution. The aggregate of solutions of system (1), if they exist, forms in the positive orthant $\mathbb{R}^n$ of n-dimensional space of variables $y_1, \ldots, y_n$ some cone L. Every ray, emerging from the point of origin and belonging to the cone L, is a solution for system (1). Let's assume that matrix $||a_{ij}||$ is consistent. In this case the problem is to select the ray y^* from the cone L. In this paper we suggest to accept "central" ray of cone L as a desired solution. We call central the ray which results from the limit of uniform approaches of all boundaries of the cone L. We suggest to execute this process through narrowing of intervals (1) (under conditions, that matrix $||a_{ij}||$ stays consistent) by uniform closing in of the ends of these intervals. If the initial matrix of interval estimations $||a_{ij}||$ is inconsistent, then we will start with the simultaneous uniform expansion of all intervals until matrix $||a'_{ij}||$ will become consistent. Then for the consistent matrix $||a'_{ij}||$ the central ray of thus acquired cone L' will be found, which is the desired solution y^* for the initial matrix. In practice finding central ray is added up to well-known standard task of graph theory: computing matrices of maximal weights of ways in the oriented weighted graphs.

Conclusion The suggested method has a number of advantages. We will describe some of them.

1. If the initial matrix of estimations $||a_{ij}||$, as it often happens in practice, is inconsistent, then the desirable solution y^* could exist only after some corrections of values of at least some elements of

this matrix. During the process of defining central ray in this case we automatically find both those absolute estimations that cause the matrix $||a_{ij}||$ inconsistence and the value of minimal needed correction. This advantage is estimated as rather significant characteristic during the work with experts, defining intervals (1) and permissibility of every correction.

2. Very often it is difficult for experts to describe the results of paired comparison with numerical scale. In this case normally the scale of ranked qualitative gradations of the degree of expression of studied characteristic. Then every gradation is given the quantitative value, i.e. we construct the quantitative scale used for defining corresponding point and interval estimations (1). Thus it is extremely important to define in every case the dependence of aggregate results produced by different methods on the used quantitative scale. It appears, that results of ranking objects according Berge and T. Saati methods are invariant only for the multiplication of the used quantitative scale by positive constant value, in case of ranking objects using the method of central ray is invariant for the wider spectrum of transformations — for every positive exponential transformation of initial quantitative scale.

In conclusion we should mention that the described method could be used in the case of point estimations without any changes and possesses the same advantages.

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Life Support Systems in Medicine and Biology*

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key words: *decision support systems, living systems, food contamination, rare endangered species*

An idealized conceptual model of Life Support System (LSS) is discussed. LSS is treated as Decision Support System (DSS) that supports living activities (survival) of population of organisms. Living conditions of the given population are assessed using integral indicators of Quality of Life (QoL) [1]. DSS is a management system, which can be treated as a control system with a feedback loop. Problem of QoL management is posed in these terms as control stable in sense of Lagrange-Poincaré [4].

Let us consider a living system (LS), or biosystem, as a dynamic system. Let the system be described at any instant of time t within an interval $a < t < b$ by means of p parameters $x^{(1)}, x^{(2)}, \dots, x^{(p)}$ that are time-dependent components of p-vector function $x = x(t)$. The function is the phase trajectory of the dynamic system within phase (state) space of the system.

Let $y^{(1)}, y^{(2)}, \dots, y^{(n)}$ be a set of time-dependent data obtained by means of observations of the LS due to monitoring process providing n-vector function $y = y(t)$. It is assumed that there exist a data generation model $y = g(x)$ so the data $y^{(1)}, y^{(2)}, \dots, y^{(n)}$ are dependent on $x^{(1)}, x^{(2)}, \dots, x^{(p)}$. Assuming the function $y = g(x)$ is known, and using the available set of data $(y = y(t), t \leq T)$, and the state parameters $x = x(t)$ for $t \leq T$, where $T \in (a, b)$ is a given time instant, can be assessed using a relevant assessment technique providing a pseudo-inverse $\hat{x} = g^{+}(y)$ for the data generation model $g(x)$.

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Let $u = u(t), a < t < b$, be an optional control issued by a control unit of the life support system, and let $x+ = f(x-, u+)$ be a forecast of the future states $x+$ of the living system under the optimally chosen optional control $u+$. Let $q(u+, x+)$ be a set of integral indicators of quality of life of the living system in the future under the chosen optional control $u+$.

A rather general conceptual model of a life support system (LSS) of a living system (LS) can be depicted as a feedback loop control system comprising four interconnected blocks.

The main block is a Living System (LS) and it is an object of management. Future states $x+$ of dynamics, or development, of the LS depend on the state x of the LS and the chosen control u. This dependence is described within dynamical model of the LS controlled (supported, managed) $x+ = f(x-, u)$. The model is based on a priori expert knowledge related to the LS.

Control Unit of the Life Supporting System has goals of control, or Quality of Life Criteria $q(u, x)$, that are being optimized over a set of optional controls (u) taking into account the available data y on the assessments $g^+(y)$ of the state x of the developing Living System, which is controlled (supported, managed).

Monitoring of the Living System maps development dynamics $x = x(t)$ of the LS into the available observed data y concerning the LS. This mapping is described by means of the data generation model, $y = g(x)$. It is based on a priori expert knowledge on the methods and techniques used to monitor (observe) the LS dynamics.

Development dynamics assessment is made using the available data y. The assessment technique is based on the data generation model as its generalized pseudo-inverse $x = g^+(y)$.

The four blocks are connected with arrows depicting the following four information flows: dynamics $x = x(t)$ of LS development, or LS evolution, available data $y = y(t)$, estimates x of dynamics $x = x(t)$ from the data observed $y = y(t)$, and chosen control $u = u(t)$.

The LS is being managed, or supported, in a sustainable way if trajectory of its development is remaining within its 'eco-niche'. This means that a relevant Life Support System, treated as a dynamical control system, provides for the living system stability. Here stability is meant in the sense of Lagrange as defined by Poincaré (1892-1899). The boundary of eco-niche is a set of the extreme conditions, or states of the biosystem in between life and death.

The ideas are illustrated with real examples of monitoring and management activities of Russian Federal bodies.

One example is related to social-hygienic monitoring of quality (contamination) of the main groups of food products in Russian regions [2] as maintained by Sanitary Epidemiological Surveillance of the RF Ministry of Public Health (SANEPIDNADZOR). Various methods were used to compute integral indicators to rank a set of main food products according to indicated concentrations of various contaminants in the food products. In particular, Pareto slicing technique and singular value decomposition of data matrix (contaminant vs. food product) were used. The obtained results were found to be in accordance with each other.

The other example is conservation and protection of rare (endangered, extinct) species of animals, plants and mushrooms [3] as maintained by the RF Ministry of Natural Resources (MINPRIRODA). A system of indices was developed to support decisions on conservation of rare and extinct species listed in the 'Red Book' of Russian Federation. The system is based on expert estimates of the species biological condition, social-economical significance of the species, cumulative factor of hazards, and a set of necessary and sufficient conditions for recreation of the species. A set of integral indicators developed using the system of indices allows to assess extent of hazards for each given species. The quantitative criteria support decision making process of risk assessment and risk management related to the 'Red Book' of Russian Federation.

The integral indicators developed in these applications can be used for related DSS in Russia on Federal and Regional levels.

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Games Theory Teaching Through Computer Games

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The present work is connected with the creation of a series of training computer games simplifying the process of teaching of the elementary games theory. It is supposed that the student at first gets acquainted with the computer game as an amusing adventure. He takes some actions targeted at controlling the objects of the game. He sends the beavers to work on allotments, directs the vessel to an unknown island through the storming sea, shoots at the hidden wolf and so on. Shortly he finds out that the computer wins the game with a considerable advantage, while the neighbor player who knows the theory of games, is capable of winning the game under the same conditions. That is how the interest and understanding of expediency of studying the theory and the opportunity of realizing theoretical provisions in practice occur. In such conditions not only a student but even a schoolboy is capable of understanding the mathematics of the games theory.

At present three games appropriate to the following sections of the theory of antagonistic matrix games have been created: one-step game with complete information ("Beavers"); multistep game with complete information ("Cats' mice"); multistep game with non-complete information ("Rescue of the Wolf").

The games can be created at two levels. Currently only the level at which the computer knows optimum strategy and exploits only this strategy, without reacting on the rival's actions has been realized. A more difficult level, on which the computer studies actions of the rival and changes its strategy optimally against the rival's real strategy has not been realized till now.

The programming of game is connected with splitting of mathematical accounts into two parts. That part of a task, which is necessary for creation of a payment matrix, is carried out with a mathematical package Maple. The constructed decision as a set of optimum strategy includes the list of active strategy and list of probabilities of application of this strategy. The found decisions are transferred in Flash program, which allows conveniently creating short videos and buttons. This program at realization of game works as follows. After the player chooses the game type, the computer stores the whole package of optimum strategy of the given game in operative memory. Then the computer plays, choosing active strategy with the help of random numbers.

Let us dwell on the actions of the computer in one-step game "Beavers" with the complete information. There are two participants, a player, sitting at the computer, and the computer. Each of them has in submission some objects. In the beginning of game the player arbitrary defines the number of objects for itself and for the computer, and also the number of rounds of game. The computer expects starting "capital" by multiplication of the game value for number of rounds. The values of all allowable games previously are designed and are stored in the memory. Each round occurs so. The player determines the number Beavers, sent by him to the first allotment. Others are directed to second. The computer distributes native Beavers to allotments. It causes the next random number and depending on it chooses strategy from among active. Further computer makes the analysis of a situation, deduces on a board result of a round and final sum of glasses. Gained game is considered at which the final number of glasses will appear more. The payment matrix of this game is close to a matrix of known game of the colonel Blotto, but differs from not by marks. In result, the game does not degenerate at the large number of objects. The meanings of the game value and probability of active strategy for 22 realized games are given in the table.

	Beavers ratio	game value	probability of active strategy 1 gamer	probability of active strategy 1 gamer
1	7:6	2	0-0-2-1-1-2-0-0	0-0-1-1-1-0-0
2	7:5	25/12	0-5-0-1-1-0-5-0	0-7-5-5-7-0
3	7:4	24/11	0-10-0-1-1-0-10-0	4-7-0-7-4
4	7:3	1.9	2-2-1-0-0-1-2-2	4-1-1-4
5	6:5	1.7	0-3-2-0-2-3-0	0-1-4-4-1-0
6	6:4	2	0-2-0-1-0-2-0	0-1-0-1-0
7	6:3	17/9	2-2-0-1-0-2-2	7-2-2-7
8	5:4	11/7	0-5-2-2-5-0	0-2-3-2-0
9	5:3	31/18	4-4-1-1-4-4	4-5-5-4
10	4:3	3/2	0-1-0-1-0	0-1-1-0

Let's dwell on the actions of the computer multistep-by-step matrix game with the complete information ("Cats - mice"— analogue of popular game "a stone - scissors - paper ". The player and computer form teams "CaE" — cat and elephants and "MoD" — mouse and dog with allowable number of objects up to 19 each types. The player has a team in structure $m1$ of mice and $m2$ of dogs, computer — team from $n1$ of the cats and $n2$ of the elephants (or on the contrary). On each step of game each of the gamers chooses the representative for a duel. As a result of this duel one of two chosen representatives is "eliminated", according to the following rules: a dog > cat > mouse > Elephant > dog. The player - fan (amateur) has in the team two additional objects — "cheburashka", which puts out of action any object of the opponent, but thus leaves also itself. The game proceeds so long as in submission of one of the players the objects only of one type will not stay. Each position is determined by set of four numbers $P = P(m1, m2, n1, n2)$. Let's designate value of game $V(m1, m2, n1, n2)$. The terminal positions have the following value: $V(m1, m2, n1, 0) = V(m1, m2, 0, n2) = 1$, $V(m1, 0, n2, n1) = V(0, m2, n1, n2) = -1$, where $m1$, $m2$, $n1$, $n2 > 0$, that is the value of game at a victory of the player is equal 1, and at a victory of the computer is equal (-1). Pure strategy of the player is the choice for a duel of the mouse or dog. Pure strategy of the computer is the choice for a duel the cat or elephant. The mixed strategy of the computer is pair $(p_k; 1 - p_k)$ probabilities of a choice for a duel on k-step the cat (p_k) and elephant (probability $1 - p_k$). The player collects the team, and the computer offers five variants of equivalent structure of the team. In memory of the computer there is for this purpose 4- dimension matrix of the values of game with number of elements 19^4, designed previously in the program Maple by recurrence relation: $V(m_1, m_2, n_1, n_2) =$

$$\Big(V(m_1-1, m_2, n_1, n_2)\cdot V(m_1, m_2-1, n_1, n_2)$$

$$-V(m_1, m_2, n_1-1, n_2)\cdot V(m_1, m_2, n_1, n_2-1)\Big)\times$$

$$\times\Big(V(m_1-1, m_2, n_1, n_2)+V(m_1, m_2-1, n_1, n_2)-$$

$$V(m_1, m_2, n_1-1, n_2)-V(m_1, m_2, n_1, n_2-1)\Big)^{-1}.$$

Actually, the computer on given $m1$, $m2$ solves the equation

$$V(m_1, m_2, n_1, n_2) = 0$$

and offers some decisions of this equation. Game further begins, and probability of application of the first active strategy p_k the computer determines by ratio, following from a Bellman principle because the strategy are completely mixed:

$$p_k = \frac{V(m_1, m_2, n_1, n_2) - V(m_1, m_2, n_1, n_2-1)}{V(m_1-1, m_2, n_1, n_2) - V(m_1, m_2, n_1, n_2-1)}.$$

Let's dwell on the actions of the computer multistep-by-step matrix game with the incomplete information "Rescue the Wolf". The player can play for any of following pair: Rabbit, armed by the automatic device with some number mount and Wolf with some number Chinese wire-armored flameproof clothing. The poor Wolf disappears behind a chain of stones, extreme of which is close to his house, in which the beautiful Wolfs Wife waits, being poured by tears. Rabbit finds out the Wolf after each shot on plaintive shouts and acting shivering tail, but while he overcharge a gun, Wolf can move on the stipulated number of stones (by default $n = 2$) in any direction. Rabbit can shoot at any stone or give a single shot. At each hit wire-armored clothing is fall to pieces Rabbit wins, if the Wolf has not reached houses, and at him wire-armored clothing has come to the end. The wolf wins, if Rabbit's mount come to the end or if the Wolf has reached houses. The matrix for each step of game has dimension $(2n+2)(2n+1)$ and its elements are determined by game in each position, in which the game after a shot can proceed. If up to a shot at Rabbit's number mount i, at the Wolf number wire-armored clothing j, and the Wolf was behind a stone with number k, after a shot battle the number of Rabbit's mount decreases on 1. If Rabbit shoots at that stone, behind which the Wolf disappears, the Wolf's number wire-armored clothing decreases on 1. Number of a stone, behind which the Wolf disappears, changes according to moving the Wolf. The functional equation of game is:

$V_{i,j,k} = Val$

$$\begin{bmatrix} V_{i-1,j-1,k-2} & V_{i-1,j,k-1} & V_{i-1,j,k} & V_{i-1,j,k+1} & V_{i-1,j,k+2} \\ V_{i-1,j-1,k-2} & V_{i-1,j-1,k-1} & V_{i-1,j,k} & V_{i-1,j,k+1} & V_{i-1,j,k+2} \\ V_{i-1,j-1,k-2} & V_{i-1,j,k-1} & V_{i-1,j-1,k} & V_{i-1,j,k+1} & V_{i-1,j,k+2} \\ V_{i-1,j-1,k-2} & V_{i-1,j,k-1} & V_{i-1,j,k} & V_{i-1,j-1,k+1} & V_{i-1,j,k+2} \\ V_{i-1,j-1,k-2} & V_{i-1,j,k-1} & V_{i-1,j,k} & V_{i-1,j,k+1} & V_{i-1,j-1,k+2} \\ V_{i,j,k-2} & V_{i,j,k-1} & V_{i,j,k} & V_{i,j,k+1} & V_{i,j,k+2} \end{bmatrix}$$

The mixed strategy of Rabbit includes a vector of probabilities of application of active strategy p of dimension $(2n+2)$, and Wolf — vector of probabilities of dimension $(2n+1)$. The terminal prize of Rabbit is equal 1, if $j = 0$ and (-1), if $i = 0$ or $k = 0$ (the Wolf has arrived home). For the decision of game, the maximal meaning of k is limited by K (differently number of the equations is beyond all bounds great, as the Wolf can escape in infinity and game basically infinite).

Two ways of reception of random numbers were investigated. In the beginning the standard way of generation of random numbers with the help of the built — in generator ActionScript was used. Thus the generation of a random number occurs in process of need that is at the moment, when the player already has made a course. The sequences of similar numbers were analyzed by Kolmogorov's criterion on conformity to their empirical function of distribution and function of uniform distribution. The confidential probability has appeared low. We used another way of the control of sequences of random numbers by results of a series of one-step-by-step games. For this purpose consistently all active strategy of the player got out. Then the modelling game about the large number of rounds (200 - 400) and use equally one active strategy of the player was carried out. Was typical $\sigma\sqrt{n}$ and critical $\sigma_{t(1-2\alpha)}\sqrt{n}$ Deviation from equilibrium value. Size σ received from distribution of probabilities designed for the given strategy on a payment matrix. And at this control of a deviation it appeared much more expected. The own generator therefore was created on the basis of the created beforehand sequence from thousand fixed random numbers, and the input in a sequence was carried out unitary at the moment of a beginning of game. The point of an entrance is determined by a random number certain generator ActionScript, which is connected to the moment of generation of number and is not predicted. The different ways of construction of a sequence were investigated. Best, by two specified criteria the result is received at use of random numbers constructed with the help of a Euler constant 0.5772 ... The value of this number from 10 000 marks after a point Paid off. The random number were determined as $0, nmk$, where n, m, k are three of consecutive figures of Euler constant. Further numbers were united in groups till hundred numbers, which "quality" was determined by Kolmogorov's criterion at a level of confidence more than 50 %. These hundreds numbers make more, than half of possible numbers constructed by a described way from Euler constant. For numbers "π"or "e" a share of such numbers is less than 10%. The carried out research testifies that the computer plays 'better', if random number to generate by the chosen way.

It is planned that the created and being created games will make an appendix to the textbook for the theory of games, however there has been found no book of the good kind in Russia. The prototype of the book is "The Theory of games" by L.A. Petrosyan and others. Probably, such a basic book will be found abroad or it will be written.

Evaluation of Power of Groups and Fractions in the Russian Parliament (1994-2003) for Constitutional Majority Decision Making Rule

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In this paper, the problem of evaluation and the analysis of distribution of power is considered in the State Duma of the Russian Federation for 1994-2003. At first glance, power of a party in a parliament directly depends on the number of its votes. To illustrate that it is not quite so consider an example. Let the parliament consisting of 99 seats be represented by 3 parties with votes of each party equal to 33. A rule of decision-making is the simple majority, i.e., 50 votes. In this case, winning coalitions (which can enforce the decision without votes of other parties) are A+B, A+C, B+C, A+B+C, i.e., any party makes winning any two-party coalitions. Obviously, in this example all parties have equal power. Now, if the distribution of seats in the parliament has changed and parties A and B have now 48 votes each, and a party C has only 3 votes, then the winning coalitions remain the same, and the party C, despite of sharp decrease of votes, makes winning the same number of coalitions as other parties. Hence, the opportunity of all parties to influence an outcome of voting is still equal. The above example shows that the number of votes cannot represent power of a party. Therefore, the power indices are introduced to measure a degree of power of a party in a parliament on the basis of number of coalitions which the party makes winning.

In this paper, Banzhaf power index (Banzhaf, 1965) is used to study the Russian parliament. It is based on calculation of the share of winning coalition, in which the party is pivotal, i.e., if this party leaves this coalition then this coalition becomes a losing one (which cannot enforce the decision without votes of other parties). The dependence in the changes of index values is compared with respect to political events during this period.

To track the dynamic of power change of each fraction during one parliamentary term deputy groups were formed on the sixteenth day of each month separately for each of three parliaments (1993, 1995, 1999). Further Banzhaf index was calculated using various hypotheses about coalitions formation (Aleskerov at all, 2003).

Three qualitative scenarios of coalitions formation are considered, one of which being considered as "real".

The results obtained fit well to the political events during this period that confirms adequacy of the suggested approach.

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Endogenous Formation of Political Structures and Their Stability

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This paper studies formation of political parties in the developing democracy. The most of theoretical issues devoted to the problem of political competition consider the political structure of the society (i.e. number of parties and their preferences) as given exogenously. However, the problem of formation of political structures is of special interest for countries in transition.

One relevant paper in this area is Gomberg F., Marhuenda I. (2000). In this work, authors consider a party as a unity of voters and its program as a bliss-point of its median member. The main difference of my work is the assumption that a payoff function of an agent depends not only on the party's proposed policy, but also on the party's size. Similar models were considered also in other areas of knowledge. In particular, it is possible to draw analogy of our model with models of horizontal product differentiation. Another similar paper is by Alesina A., Spolaore E. (1995) who study the model of forming of countries. But the conditions on stable structures (and therefore the results) in these models are essentially different from mine because of the substantial features of the researched problems.

In this work I consider a game-theoretical model of endogenous formation of parties. Players (agents) are voters distributed on the one-dimension policy space $X = [0,1]$. Every player has a unique bliss-point in this space $x \in X$, which corresponds to her policy preference. The set of voters on the whole is characterized by the continuous distribution function of bliss-points $F(x)$ with density function $f(x)$. The political parties are formed endogenously as the unities of voters. Every party i is characterized by its size r_i and political program p_i that equals the median of the distribution of bliss-points of agents belonging to this party. All voters choose "rationally", i.e. every agent belongs to the party that gives her the best payoff. The voter's payoff function $U(x, r, p) = r - \alpha V(|p - x|)$ increases proportionally to the size of the party and linearly $(V(t) = t)$ or quadratically $(V(t) = t^2)$ decreases in the distance between the agent's bliss-point and the party's policy.

The distribution of voters among parties is called Nash equilibrium if it is not profitable for any agent to change its party affiliation alone. I consider Nash equilibrium political structures. Any such structure is a partition of the policy space in not intersecting intervals $(0, C_1)$, (C_1, C_2) , ..., $(C_{n-1}, 1)$, where every party is associated with a unique interval.

For this model with the uniform distribution of voters, I study a problem of strong Nash equilibrium existence depending on the type of a payoff function. Recall that Nash equilibrium is strong if there is no coalition of agents (not necessary connected), such that it is strongly profitable for every member of the coalition to join and create a new party.

I obtain necessary conditions on the system parameters (including parameter α of the payoff function) for Nash equilibrium to be a strong equilibrium. I prove that if the payoff function linearly depends on the distance between the agent's bliss-point and the party's policy (i.e. the marginal rate of substation of party's size by distance between agent's bliss-point and party's policy is constant and equal to α), then every Nash equilibrium is a strong equilibrium for $\alpha = 2$. If $\alpha < 2$, then it is profitable for agents to unite two neighboring (i.e. with a common boundary agent) parties. If $\alpha > 2$, then it is profitable for agents with bliss-points close to boundary to separate into a small party where they would be closer to the median and the distance between their bliss-point and the party's policy would be smaller.

I also show that, if the payoff function is quadratic in the distance between the agent's bliss-point and the party's policy (i.e. the marginal rate of substitution is proportional), then a strong Nash equilibrium exists for every $\alpha \geq 8/3$. Any Nash equilibrium is strong, if $\frac{4}{3}n \leq \alpha \leq 4n$, where $n \geq 2$ is a number of parties in the Nash equilibrium. Thus, in this case, the permissible number of parties for the strong Nash equilibrium increases at parameter α that characterizes the rate of substitution.

Thus, I show that the type of dependence of the agent's payoff on the distance between the agent's bliss-point and the party's policy significantly affects to existence of the strong equilibrium political structures and the number of parties in the equilibrium.

Game Theoretic Modelling of Taxation Enterprises

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In the article for the chosen system of taxation enterprises the conceptual mathematical model as hierarchical dynamic game of government and enterprise is constructed and researched. The gain of the government (as player-leader) is analyzed in different variants of information exchange about strategies. Nash equilibrium points, Pareto optimal points, strong Nash equilibrium points are calculated.

1. Basic thesises of model.

1) The enterprise is engaged in some kind of business.

2) Each period the enterprise pays to the government single tax.

3) The rate of the tax is constant during all periods of time under consideration.

4) The object of the tax is value added of the enterprise.

5) The purpose of the enterprise is maximization of total value added for all periods of time with discounting by choice of appropriate strategy.

6) The purpose of the government is maximization of total tax receipts for all periods of time with discounting by choice of tax rate.

2. Mathematical statement of the problem. Criterion of the government :

$$F_N(\xi, \bar{u}) = \sum_{k=0}^{X-1} N(k, \xi, u(k)) * \left(\frac{1}{1+\delta}\right)^{k+1} \longrightarrow \max_{\xi \in [0,\beta]}, \qquad (1)$$

criterion of the enterprise:

$$F_R(\bar{u}) = \sum_{k=0}^{X-1} R(k, u(k)) * \left(\frac{1}{1+\delta}\right)^{k+1} \longrightarrow \max_{\bar{u} \in \bar{U}(\xi)}, \qquad (2)$$

where k is a number of period, X is an amount of periods, ξ is arate of the tax, $N(k, \xi, u(k))$ is sum of the tax, which the enterprise pays in kth period, $u(k)$ is a strategy of the enterprise in kth period, $\bar{u}$ is a vector of strategies of the enterprise, $\bar{u} = (u(0), ..., u(X-1))$, $\bar{U}(\xi)$ is a set of vectors of strategies for the enterprise, $\bar{U}(\xi) = U(0,\xi) \times ... \times U(X-1,\xi)$, $U(k,\xi)$ is a set of strategies of the enterprise in kth period, $R(k, u(k))$ is value added of the enterprise in kth period, β is a marginal tax rate, $\beta \leq 1$, $\bar{U}(\xi)$ is a empty set $\forall \xi > \beta$, δ is a rate of discounting, $N(k, \xi, u(k)) = \xi * R(k, u(k)), k = \overline{0, X-1}$, $F_N(\xi, \bar{u})$ is total tax receipts for all periods of time with discounting, $F_R(\bar{u})$ is total value added for all periods of time with discounting.

3. Method of solution the problem. Let's search the solution of the problem (1), (2) according to Stackelberg principle. In the constructed hierarchical game the leading party is the government; it sets for all periods of time under consideration a size of the tax rate, proceeding from which the enterprise plans its activity.

The search of optimum strategies of the government and the enterprise is carried out stage by stage. At first, the problem of optimal control of the enterprise in conditions of the fixed tax rate is solved. Then ξ^{opt} is calculated by maximization criterion function of the government with $\bar{u}^{opt}(\xi)$.

Stage 1.

Set of optimal vectors of strategies of the enterprise:

$$\bar{U}^{opt}(\xi) = \left\{ \bar{u}^{opt}(\xi) : F_R(\bar{u}^{opt}(\xi)) = \max_{\bar{u} \in \bar{U}(\xi)} F_R(\bar{u}) \right\}.$$

Stage 2.

Set of tax rates, ensuring maximal tax receipts:

$$\Xi^{opt} = \Bigg\{ \xi^{opt} :$$

$$F_N(\xi^{opt}, \bar{u}^{opt}(\xi^{opt})) = \max_{\xi \in [0,\beta]} F_N(\xi, \bar{u}^{opt}(\xi)), \forall \bar{u}^{opt}(\xi) \in \bar{U}^{opt}(\xi) \Bigg\}.$$

Let's consider the case of the government is indifferent, from the point of view of its criterion, with what tax rate to use, that is the capacity of set Ξ^{opt} is more than unit, it chooses tax rate, at which the enterprise creates more added value.

Therefore, set of optimal tax rates:

$$\Xi^{opt}_* = \Bigg\{ \xi^{opt}_* :$$

$$F_R(\bar{u}^{opt}(\xi^{opt}_*)) = \max_{\xi^{opt} \in \Xi^{opt}} F_R(\bar{u}^{opt}(\xi^{opt})), \forall \bar{u}^{opt}(\xi) \in \bar{U}^{opt}(\xi) \Bigg\}.$$

Thus, set of points

$$\{(\xi_*^{opt}, \bar{u}^{opt}(\xi_*^{opt})) : \xi_*^{opt} \in \Xi_*^{opt}, \bar{u}^{opt}(\xi_*^{opt}) \in \bar{U}^{opt}(\xi_*^{opt})\}$$

are solutions of hierarchical game, according to Stackelberg principle.

Theorem 1. For all points $(\xi, \bar{u})$, where $\xi \in [0, \beta]$ and $\bar{u} \in \bar{U}(\xi)$, it's true that $F_N(\xi_*^{opt}, \bar{u}^{opt}(\xi_*^{opt})) \geq F_N(\xi, \bar{u}))$, where $\xi_*^{opt} \in \Xi_*^{opt}, \bar{u}^{opt}(\xi_*^{opt}) \in \bar{U}^{opt}(\xi_*^{opt})$.

Theorem 1 shows that the optimal strategy of the government received according to Stackelberg principle, provides the greatest possible tax receipts. Hence, there is no necessity to enter information exchange about strategies in the constructed hierarchical game (1), (2), that is to consider all possible information expansions of initial game (approach, offered by Y.B. Germeier), as the gain of player - leader can not be improved.

Theorem 2. If $\xi_*^{opt} \in \Xi_*^{opt}, \bar{u}^{opt}(\xi_*^{opt}) \in \bar{U}^{opt}(\xi_*^{opt})$, then point $(\xi_*^{opt}, \bar{u}^{opt}(\xi_*^{opt}))$ is Nash equilibrium.

Theorem 3. If $\xi_*^{opt} \in \Xi_*^{opt}, \bar{u}^{opt}(\xi_*^{opt}) \in \bar{U}^{opt}(\xi_*^{opt})$, then point $(\xi_*^{opt}, \bar{u}^{opt}(\xi_*^{opt}))$ is Pareto optimum.

Theorem 4. If $\xi_*^{opt} \in \Xi_*^{opt}, \bar{u}^{opt}(\xi_*^{opt}) \in \bar{U}^{opt}(\xi_*^{opt})$, then point $(\xi_*^{opt}, \bar{u}^{opt}(\xi_*^{opt}))$ is strong Nash equilibrium.

In the case of the government is interested not only in maximization of tax receipts, but also in increase of added value of the enterprise (that is equivalent to increasing of gross domestic product on macro-level), it is necessary, in addition, to consider following two-criterion problem:

$$F_N(\xi, \bar{u}^{opt}(\xi)) \longrightarrow \max_{\xi \in [0,\beta]}, \tag{3}$$

$$F_R(\bar{u}^{opt}(\xi)) \longrightarrow \max_{\xi \in [0,\beta]}, \tag{4}$$

$\forall \bar{u}^{opt}(\xi) \in \bar{U}^{opt}(\xi)$.

The solution of the problem (3), (4) is set P_ξ of Pareto optimal tax rates.

Plans of Military Operation

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key words: *military operation plans*

On the basis of A.N. Kolmogorov's axioms [1, 2] we construct the discret fuzzy probability theory, which is applied to the planning of military operations. As an example we study the plan and realization of the operation "Iskra" on breaking the blokade of the Leningrad (12 - 18 January, 1943) by Leningrad's and Volhov's fronts. We conseder consequense of events $\Omega^{12}, ..., \Omega^{18}$, which were on the Leningrad's front (Ω^i is event of ith of January). Thus, we determine $\Omega^{12} = \Omega^{12}_{86} \times \Omega^{12}_{136} \times \Omega^{12}_{208} \times \Omega^{12}_{45}$, where certain events $\Omega^{12}_{86} = \{\omega_1, \omega_2\}$, $\Omega^{12}_{136} = \{\omega_3, \omega_4\}, \Omega^{12}_{286} = \{\omega_5, \omega_6\}, \Omega^{12}_{45} = \{\omega_7, \omega_8\}$ are the attacks of the V.A. Trubachev's 86th division, of the V.P. Simonyak's 136th division, of the Borshscov's 286th division, of the Krasnov's 45th guard division of the 67th army of the Leningrad's front on the line front of the 18th German army on the left side of the Neva river. (The attack of the 2nd strike army of the Volhov's front is formalized according).

Let $L = \{0, F, 1\}$ be linear ordering set with fuzzy element F ($0 < F < 1$, where F, possible, is not number).

From all fuzzy random events $A \in \sigma(\Omega^i_j)$, $\sigma(\Omega^i_j)$ ($i = 12$, $j = 86$, 136, 208, 45) with membership fuction $\mu_A : \Omega^i_j \to L$, in the first day there were realizations: $A^{12}_{136} = \{\omega_3\}$ - succesful attack of Simonyak's division, $A^{12}_{286} = \{(\omega_5, F)\}$ - party succesful attack of the Borshchov's division, $B^{12}_{86} = \{\omega_2\}$ - not succesful attack of the Trubachov's division, $B^{12}_{45} = \{\omega_8\}$ - not succesful attack of the Krasnov's division. Fuzzy random value $X^{12} : (\Omega^{12}, \sigma(\Omega^{12})) \to (R, \sigma(R))$, which describes the movment of the divisions on 12 January is given as followis [3]: 136 division 3 - 4 kilometers, 286 division 3 kilometrs, 86 and 45 divisions are not advanced. Here $(\Omega^{12}, \sigma(\Omega^{12}))$ is fuzzy measure of space, $(R, \sigma(R))$ is classic measure of space (Borel's line), $\sigma(\Omega_{12}) = \sigma(\Omega^{12}_{86}) \times \sigma(\Omega^{12}_{86}) \times \sigma(\Omega^{12}_{136}) \times \sigma(\Omega^{12}_{268}) \times \sigma(\Omega^{12}_{45})$ is a class of fuzzy sets. Volhov's front had moved toward the Leningrad's front on the 3 kilometers. The distance between fronts is equal to 14 kilometrs. Time of operation "Iskra" is 7 days. Middle velocity of movment of fronts is 1 km per day $= EX^{12} = ... = EX^{18} = EY^{12} = ... = EY^{18} =$ (Y^i - for Volhov's front). Variance for Simonyak's division is $4 < (X^{12} - EX^{12})^2 < 9$. This shows hight level of the military mastery.

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Robust Identification of Linear Models on Experimental Data

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key words: *parametric identification, robust estimation, least-modules method, surges, autoregression model, time series*

In parametric identification of models on experimental data, one of the problems is a choice of method of estimation. At present, the least-squares method, least-modules method, minimax method, L- and R-estimations are widespread. The choice of estimation is done depending on suggestions about nature of distribution function of the measurement error. We consider linear models of independent observations with presence of outliers in measurement error. We propose to use a generalized least-modules method:

$$(a_0^*, a_1^*, \ldots, a_m^*) =$$

$$\underset{a_0,a_1,\ldots,a_m}{\operatorname{argmin}} \sum_{i=1}^{N} |y_i - \hat{y}_i|^\lambda = \underset{a_0,a_1,\ldots,a_m}{\operatorname{argmin}} \sum_{i=1}^{N} \left| y_i - a_0 - \sum_{j=1}^{m} a_j x_{ji} \right|^\lambda, \quad (1)$$

where $a_0^*, a_1^*, \ldots, a_m^*$ are to be found; y_i $(i = 1, 2, \ldots, N)$ are observations of dependent variable; x_{ji} $(i = 1, 2, \ldots, N; j = 1, 2, \ldots, m)$ are observations explaining variables; $0 < \lambda < 1$. The given evaluation haves an advantage of the least-squares method, rank and sign methods of the evaluation, in the case when outliers of errors have different probabilities for different signs. The following statements are true.

Theorem 1. Let a sample of x_i, $i = 1, 2, \ldots, 2n-1$, and the function $f(x) = \sum_{i=1}^{2n-1} |x_i - x|^\lambda$, $0 < \lambda < 1$ be given. All of the xi are local minimums and there is a local maximum between neighboring xi and xi+1.

Proof. Let's determine the first two derivatives of the function f(x):

$$f'(x) = \sum_{i=1}^{2n-1} \lambda \left|x_i - x\right|^{\lambda-2} \operatorname{sign}(x_i - x),$$

$$f''(x) = \sum_{i=1}^{2n-1} \lambda(\lambda - 1)|x_i - x|^{\lambda-2}.$$

Since $\lambda < 1$, then $f''^{(x)<0}$. Consequently, for all values of x_i the derivative $f'(x)$ is monotonically decreasing from the left and is monotonically increasing from the right of x_i. That is why each of the $2n-1$ zeros of the derivative $f'(x)$ corresponds to a local maximum between x_i and x_{i+1}. Let's examine left and right limits of the first derivative. For any $1 \leq j \leq 2n - 1$

$$\lim_{x \to x_j+0} f'(x) = \lim_{x \to x_j+0} \sum_{i=1}^{2n-1} \lambda \left|x_i - x\right|^{\lambda-1} \operatorname{sign}(x_i - x) = +\infty,$$

$$\lim_{x \to x_j-0} f' = \lim_{x \to x_j-0} \sum_{i=1}^{2n-1} \lambda \left|x_i - x\right|^{\lambda-1} \operatorname{sign}(x_i - x) = -\infty.$$

Since the function $f(x)$ is continuous, then all points x_i of its first derivative discontinuity are local minimums, Q.E.D.

Theorem 2. Let a sample $(x_{1i}, \ldots, x_{mi}, y_i)$, $(i = 1, 2, \ldots, 2n-1)$ and the function $Q(a_0, a_1, \ldots, a_m) = \sum_{i=1}^{2n-1} \left|y_i - a_0 - \sum_{j=1}^{m} a_j x_{ji}\right|^{\lambda}$, $0 < \lambda < 1$ be given. All of the intersection points of hyperplanes

$$a_0 = y_i - \sum_{j=1}^{m} a_j x_{ij}, \quad i = 1, 2, \ldots, 2n - 1$$

are local minimums. This statement is derived from the previous one. Theorem 2 enables us to solve the problem (1). The search for the global minimum is reduced to the examining of hyperplanes intersection points. This method can be used for robust estimation of an autoregression model's parameters for time series. The problem has the following form:

$$(a_0^*, a_1^*, \ldots, a_m^*) = \underset{a_0, a_1, \ldots, a_m}{\operatorname{argmin}} \sum_{i=1}^{N} \left| y_i - \sum_{j=1}^{m} a_j y_{i-j} \right|^{\lambda}.$$

Also we introduce statistical properties of the generalized abstract least-modules method (1).

Simulation of the Russian Electricity Market

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According to the program of the Russian electricity market development, the deregulated market sector started operating since Autumn of 2003. In April 2004, the volume of the trade in this sector reached 8% of the total electric power production. Fifteen independent companies, besides RAO UES and its satellits, took part in the auctions.

In the present paper, we consider a model of the supply function auction for the Central Economic region of Russia. We regard the electricity market as local (that is, not accounting the network structure).

A typical producer on the market has several generators with limited production capacities and approximately constant marginal costs. According to the market rules, a strategy of any producer is a piece-wise constant non-decreasing supply function that determines the amount of the supplied electric power depending on the price. The market price is determined from the condition of the total supply and demand balance.

A.A Vasin and P.A. Vasina (2003) provided a theoretical framework for simulation of such market. They have showen that one of the Nash equilibrium points corresponds to the Cournot outcome. For the other equilibrium points, the cut prices lie between the Walrasian and the Cournot prices.Moreover, only the Cournot-type Nash equilibrium is stable with respect to some class of adaptive dynamics. Our computations for the data on the Central economic region of Russia show that, under a typical demand elasticity, the price for the oligopoly with 5 companies may be several times grater than the competitive equilibrium price.

A reasonable alternative to the standard auction is Vickrey auction with reserve prices. In such auction the payment to each company is made at reserve prices that are calculated on the base of the demand function and the bids of other firms.Our computations show that under typical values of electricity demand elasticity, the expected price for consumers in such auction would be essentially lower than at the Cournot equilibrium for the supply functions auction (see Table).

We designed a software for computation of the Cournot and Vickrey auction outcomes for any affine demand function and piece-wise constant supply functions. This software also permits to set the market structure by partitioning the set of electric generators into several generating companies.

The paper by Dyakova based on the data from the RAO UES provides the following values of marginal costs and production capacities of the generating companies in the Central economic region of Russia.

Mosenergo:

Generator	Marginal cost(Rub/mwth)	capacity(bln kwth per year)
G1	0	5
G2	75	10
G3	80	10
G4	85	25
G5	90	10
G6	100	5
G7	165	10

Rosenergoatom:

Generator	Marginal cost(Rub/mwth)	capacity(bln kwth per year)
	12.5	125.4

GC1:

Generator	Marginal cost(Rub/mwth)	capacity(bln kwth per year)
1	0	16
2	60	2
3	112	3
4	125	2
5	150	16
6	200	2
7	255	2
8	340	10

GC2:

Generator	Marginal cost(Rub/mwth)	capacity(bln kwth per year)
1	95	2.5
2	110	2.5
3	120	4
4	128	13
5	135	6
6	145	2
7	162	15

GC3:

Generator	Marginal cost(Rub/mwth)	capacity(bln kwth per year)
1	0	3.5
2	100	2.5
3	120	21
4	150	3.5
5	170	4.5
6	200	4.5
7	215	3

We consider several functions $D(p) = N - \gamma p$ corresponding to the consumption in 2000:

γ	0.1	0.2	0.4	0.6
N	279.9	316.6	388.4	460.7

We find the Cournot and Vickrey outcome for two variants of the market structure:

a) 5 independent companies;

b) 3 independent companies (Mosenergo, Rosenergoatom, and UGC including all the rest generators).

For each slope ratio γ ,we evaluate deviations of the NE prices from the Walrasian prices.

The table below shows the Walrasian, Cournot and Vickrey prices for the electricity market in the Central Economic Region of Russia.

γ	$\widetilde{p}$	$p_5^*/\widetilde{p}$	$p_3^*/\widetilde{p}$	$p_{V5}/\widetilde{p}$	$p_{V3}/\widetilde{p}$
0.1	135	4.24	5.65	1.59	2.19
0.2	150	2.45	3.10	1.49	1.92
0.4	172.5	1.56	1.87	1.49	1.76
0.6	219.67	1.15	1.34	1.30	1.46

$\widetilde{p}$ - Walrasian price;

p^* - Cournot price;

p_V - Vickrey price.

Thus, for practically interesting values $\gamma = 0.1 - 0.2$ Vickrey auction is essentially better with respect to consumers than a standard supply function auction.

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Supply Function Auctions and Electricity Markets*

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The paper provides new findings on existence and properties of the Nash equilibrium for the Cournot oligopoly, a model of competition via supply functions, Vickrey auction with reserve prices and its modification taking into account the common knowledge on producers' costs. In every case, the underlying market includes a fixed finite number of producers that are heterogeneous in production capacities and non-decreasing marginal costs of production. Consumers do not play any active role in the models. Their behavior is characterized by the demand function that is the common knowledge.

We start with investigation of the local market. We show that there exists a unique Nash equilibrium in the Cournot model for any non-increasing demand function with the non-decreasing demand elasticity under mild assumptions on the demand asymptotics as the price tends to infinity. We develop a descriptive method for computation of the Cournot outcome under any affine demand function and piece-wise constant marginal costs of producers. In the general case, we obtain an explicit upper estimate of the deviation of the Cournot outcome from the Walrasian outcome proceeding from the demand elasticity and the maximal share of one producer in the total supply at the Walrasian price.

Then we consider a model where the market price is determined from the balance of the demand and the actual supply of the sealed bid auction, where producers set arbitrary non-decreasing step supply functions as their strategies. Our constraint that permits only non-decreasing step functions is reasonable in context of studying electricity markets. The step structure of the supply function is typical for generating companies and corresponds to the actual rules and the projects of the markets in different countries

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(see Hogan, 1998). In our model the Cournot type equilibrium always exists under fixed production capacities since the agents set the production volumes as well as the reservation prices. We show that, besides the Cournot outcome, there exist other Nash equilibria. For any such equilibrium the cut price lies between the Walrasian price and the Cournot price. Vice versa, for any price between the Walrasian price and the Cournot price, there exists the corresponding equilibrium. However, we show that only the Nash equilibrium corresponding to the Cournot outcome is stable with respect to some adaptive dynamics of producers' strategies under general conditions. Our results differ from Klemperer and Meyer (1989) who study competition with arbitrary supply functions reported by producers. Under similar conditions, they obtain an infinite set of Nash equilibria corresponding to all prices above the Walrasian price.

The estimates of the Cournot outcome deviation from competitive equilibrium as well as the results of calculations for the concrete market show that market price in the supply function auction can essentially (3-5 times) exceed the Walrasian price under the current market organization. Thus, investigation of alternative variants of auction organization is of great theoretical and practical interest. Below we consider Vickrey auction with reserve prices. In such auction the cut-off price and production volumes are determined in the same way as in the standard supply function auction. However, the good obtained from a producer is paid at the reserve prices. The marginal price is a minimum of the marginal cost of the same output for other producers and the marginal reserve price of this output for consumers. The marginal cost is calculated on the basis of reported supply functions, but in this case reporting the actual costs and production capacities is a weakly dominating strategy. In absence of information on production costs the guaranteed value of total profit reaches its maximum at the corresponding Nash equilibrium.

Our results generalize the results of Ausubel and Cramton (1999) who studied Vickrey auction on trading a divisible good. In their model the players are consumers. Moreover, we show that the specified outcome corresponds to the so-called truthful equilibrium for the menu auction introduced in the paper by Bernheim and Whinston (1986), see also Bolle, 2004. At this equilibrium each producer obtains the profit equal to the increase of the total welfare of all participants of the auction due to his participation in the auction. However, the construction of this equilibrium in the specified papers needs the complete information on consumers' reserve prices (in our case, on production costs). In framework of the Vickrey auction, the equilibrium in dominant strategies realizes this outcome under any actual cost functions and private information of each participant on his function.

Our calculations for the Central Economic Region of Russia show that Vickrey auction price for consumers exceeds the Walrasian price only 1,5 times (to compare with 3,5-5 times for the standard auction). However, such increase seems to be also rather essential. Besides, there exists reasonable arguing that participants of the auction typically would not reveal their actual costs, that is, the specified equilibrium in dominant strategies is not realized (see Rothkopf et al., 1990). The main argument is that reporting the actual costs gives an advantage to the auctioneer (and also to other economic partners) in the further interactions with this producer.

The situation differs significantly if the marginal costs and the maximal capacity of each generator are a common knowledge, and the uncertainty relates to a decrease of capacities due to breakdowns and repairs. In this case current information on the working capacities is weakly correlated with the future state, and the specified argument against revealing the actual costs turns out to be invalid. Moreover, the common information may be used for redistribution of the total income in favor of consumers. We specify the rule for calculation of reserve prices with account of such information. This rule provides the maximal guaranteed value of the total profit of consumers. Under this rule, reporting of the actual producer's characteristics stays his dominant strategy, and the total welfare still reaches the maximum.

The second part of this study considers a simple network market - the market with two nodes. As above, each local market is characterized by the demand function and the finite set of producers with non-decreasing marginal costs. For every producer his strategy is a supply function that determines his supply of the good depending on the price. The markets are connected by the transmitting line with the fixed share of losses and transmission capacity. Under given strategies of producers, the network administrator first computes the cut prices for the separated markets. If the ratio of the prices is sufficiently close to one then transmission is unprofitable with account of the loss. In this case, the outcome is determined by the cut prices for isolated markets. Otherwise the network administrator sets the flow to the market with the higher cut price (for instance market 2). This flow reduces the supply and increases the cut price at the market 1. Simultaneously it increases the supply and reduces the cut price at the market 2. If the transmitted volume does not exceed the transmission capacity, the network administrator determines this volume so that the ratio of the final cut prices corresponds to the loss coefficient. Otherwise, the administrator sets the volume to be equal to the transmission capacity. Thus, he acts as if perfectly competitive intermediaries transmit the good from one market to the other. It is easy to show that such strategy maximizes the total welfare if the reported supply functions correspond to the actual costs.

First we consider the Cournot competition model for this market. Our study shows that there exist three possible types of Nash equilibrium: 1) an equilibrium with zero flow between the markets and the ratio of the prices close to 1; such equilibrium is determined as if there are two separated markets; 2) an equilibrium with a positive flow and the ratio of the prices corresponding to the loss coefficient; 3) an equilibrium with a positive flow equal to the transmission capacity and the ratio of the prices exceeding the loss coefficient.

Proceeding from the first order condition, we define local equilibria of each type and show how to compute them. Then we study under what conditions the local equilibrium is a real Nash equilibrium. For the market with constant marginal costs and affine demand functions, we determine the set of Nash equilibria depending on the parameters. One interesting finding is that, in the symmetric case with equal parameters of the local markets and a small loss coefficient, the local equilibrium corresponding to the isolated markets is not a Nash equilibrium, but there exist two asymmetric Nash equilibria with a positive flow of the good.

Then we consider a standard network auction of supply functions (the first price auction) and generalize the results obtained for the local auction: for any Nash equilibrium, the market prices lie between the Walrasian prices and Cournot prices. The inverse statement is also valid.

We also describe two variants of the network Vickrey auction with reserve prices: 1) without common information on costs and 2) with common information on marginal costs and maximal generation capacities. In each case we specify methods of computation of reserve prices and prove the optimality of the considered variant, that is: the supply function corresponding to the actual costs turns out to be a dominating strategy for each participant, the maximum of the total welfare and the maximal guaranteed value of consumers' welfare under uncertainty are realized at the corresponding Nash equilibrium.

Mathematical Models of Tax System Optimization*

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Creation of the efficient tax system is one of the most important tasks for countries in transition. The present paper aims to discuss the main components of this complicated problem. The first one is the choice of the tax structure and tax rates in order to get the desirable tax revenue. The second

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problem under consideration is determination of the optimal tax schedule and audit strategy for direct taxes. The paper also discusses several issues related to tax inspection organization and creation incentives for the efficient work of inspectors. Models of interaction between taxpayers and inspectors take into account corruption, audit costs and random mistakes of participants.

Creation of an efficient tax system includes solution of several interrelated problems. One is determination of the taxes and tax rates that should provide a desirable budget income. Several papers on this problem consider the welfare losses related to distortionary effects of different taxes [1, 2]. Their main conclusion is that profit tax is a preferable tool for solution of the social welfare optimization problem. However, economic data shows that for majority of the countries this tax plays a minor role with respect to indirect taxes in budget financing. Another important issue is the use of input taxes. Myles claims (p.231): "input taxes have been employed in many countries but they would be not form a part of an optimal tax system for a competitive economy". However, in practice some kind of an input tax - a fixed presumptive tax — is now widely applied in the taxation of small and medium businesses in several countries (see [6]). In this case some characteristics of the production capacity used by a firm or the number of employees determine the type of an agent. Depending on the characteristics used to evaluate the production capacity, this tax may depend on the natural resources employed (for instance, the size and the location of the land), or on the value of some observable input with a small elasticity of substitution (the square of a shop or a cafe, the number of employees and so on). In particular, Ukraine uses now a fixed tax dependent on the type of production and the number of employees, the trade permit for individuals providing sertain cervices and the market fee for every occupied trade place for selling agricultural products. Why is profit tax less popular than indirect taxes in spite of the standard theory prefers it? And under what conditions presumptive taxes dependent on production factors may be an efficient tool for budget financing? Paper [3] considers a welfare optimization problem for a one-good economy where each firm uses some natural resource for production, and its cost depends on the quality of this resource and are its private knowledge. The government knows the production capacity of each firm and the distribution of the whole resource over quality. The government can use sales tax, profit tax and a presumptive tax dependent on the production capacity for budget financing. The actual production volume and the profit of a firm may be revealed only by costly audit. Accounting tax evasion from profit and sales taxes differs our model from the previous works in this field. It permits to answer the given questions. For a model with Leontiev-type production functions, we find the optimal combination of profit tax and the presumptive tax. We characterize such types of product functions that the presumptive tax is more efficient than sales tax, and vice versa. In practice, formation of the tax system is complicated by corruption and inefficient work of the tax service. It is necessary to provide this service with sufficient resources and create incentives for its efficient work. Papers [4,5] consider appropriate mathematical model.

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On Formulation and Solution of Optimization Problems for Switching Discrete-time Processes

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In complex production systems, production processes are usually simulated as a succession of events that originate or terminate some partial processes (works) or change their conditions. On the contrary, in resource planning, the process is always treated as a multistage (discrete-time) process with stages fixed in the time. To make a planning model more adequate, one adopts the possibility of changing qualitative state of production system elements (subsystems) according to a roster given in advance (e.g., XPAC Autoscheduler [1], a planning subsystem of XPAC7, that is a software package for the mining industry).

Such an approach runs satisfactorily only for relatively simple systems when possible rosters are scarce and can be found easily. For more complex systems, however, it is necessary to work out a models of more general type.

They should include some conditions for production processes switching that can be applied formally.

The author puts forward such a generalized model (the model of a time-event controlled process) in [2], formally for the mining industry only, but really in a general form. In this paper the model [2] is generalized and discussed in more details.

If the qualitative state s of the production system in question does not change then the process may be represented either in the form of ordinary differential equation

$$\dot{x} = f(s, x, u, t)$$

or difference equation

$$x(t + \Delta t) = x(t) + F(s, x, u, t, t + \Delta t).$$

In fact, real processes on short time periods usually show their discrete nature, so both representations are only approximations. So it is realistic to regard the control u between two subsequent switches (during a kth stage) as a constant vector, and for a kth stage we apply an equation

$$y(k) = f(s(k), x(k), u(k), t(k), \Delta t(k)), \tag{1}$$

where $x(k)$ and $y(k)$ are state vectors for the beginning and the end of k-th stage, $t(k)$ is the time of the k-th stage beginning and $\Delta t(k)$ is its duration. It is reasonable to adjoin $t(k)$ to $x(k)$, $\Delta t(k)$ to $u(k)$ and $t(k) + \Delta t(k)$ to $y(k)$; corresponding vectors are denoted as $\bar{x}(k), \bar{u}(k), \bar{y}(k)$.

Let M be the number of types of switches. The universal form of a switch condition of the mth type is

$$g_{0m}(s(k), y(k)) = 0. \tag{2}$$

It is supposed that $g_{0m}(s(k), y)$ increases monotonously in time due to (1), i.e.

$$g_{0m}(s, f(s, x, u, t, \Delta t))$$

increases (to be more exact, does not decrease) monotonously with respect to Δt for arbitrary s, x, u, t.

Examples for such a condition are:
(a) for the event of the termination of a definite work run with a non-zero intensity

$$g_{0m}(s(k), y(k)) \equiv y_{j_m}(k) = y_{j_m T},$$

where $y_{j_m}(k)$ denotes the work quantity at the kth stage end and $y_{j_m T}$ denotes its total quantity and for arbitrary $s, x, u, t, \Delta t$ it is satisfied that $f(s, x, u, t, \Delta t) = 0$ when $u_{i_m} = 0$,

$$\partial f / \partial u_{i_m}(s, x, u, t, \Delta t) > 0, \quad \partial f / \partial \Delta t(s, x, u, t, \Delta t) > 0$$

for $u_{i_m} > 0, \Delta t > 0$;

(b) for the event of interchange of mode of usage (loading, unloading) for sections of a two-sectioned stock (in the case when a section is unloaded until it is empty and an opposite section is loaded then)

$$g_{0m}(s(k), y(k)) \equiv -y_{j_m}(k) = 0,$$

where $y_{j_m}(k)$ denotes the the material quantity in the section being unloaded at the k-th stage end;
(c) for the event of a fixed time interval termination

$$g_{0m}(s(k), y(k)) \equiv \bar{y}_T(k) = T_m.$$

The switch of mth type changes $s_l, l \in L_m, \;\; x_j, j \in J_m$. We suppose three forms of such changes: (a) independent of the control

$$s_l(k+1) = S_{lm}, \qquad x_j(k+1) = X_{jm}; \tag{3}$$

(b) depending on a discrete choice $p_m \in P_m(s_m)$

$$s_l(k+1) = S_{lm}(p_m), \qquad x_j(k+1) = X_{jm}(p_m); \tag{4}$$

(c) depending on a parameter $a_m \in [a_{low\ m}, a_{high\ m}]$

$$s_l(k+1) = S_{lm}, \qquad x_j(k+1) = X_{jm}(a_m), \qquad g_{1\ m}(s(k), y(k), a_m) \leq 0. \tag{5}$$

Formally, variables p_m can be adjoined to $\bar{s}(k)$ and a_m to $\bar{u}(k)$.

Other components of s and x do not change:

$$s_l(k+1) = s_l(k), l \notin L_m, \qquad x_j(k+1) = y_j(k), j \notin J_m.$$

The process termination (on the N-th stage) is treated as a final switch (of M-th type).

The process (1), (2), (3) (or (4), or (5)) runs under restrictions. We divide them into two types: technical or physical feasibility conditions in the form

$$g_j(s(k), x(k), u(k)) \leq 0, j \in J_1, \qquad g_j(s(k), x(k), u(k)) = 0, j \in J_2.$$

and target restrictions posed on m-th switch type

$$g_j(s(k), y(k)) \leq 0, j \in J_{1m}, \qquad g_j(s(k), y(k))) = 0, j \in J_{2m}.$$

After V.V. Velichenko [4] who regarded similar dynamical systems, we call the succession of $\bar{s}(k)$ a process scenario. Within a given scenario the switching process is a usual type of unstationary discrete-time process. For the given target functional $J(u(\cdot)) = F(y(N))$ we can find the optimal control (e.g., with a method [3]), but this is only the optimum within a scenario.

To compare two processes with different scenarios we determine formally the state and the control of the process in the continuous time with the formulas

$$\hat{u}(t) = u(k), \qquad \hat{s}(t) = s(k),$$
$$\hat{x}(t) = f(s(k), x(t(k)), u(k), t), \quad \hat{q}(t) = (\hat{s}(t), \hat{u}(t), \hat{x}(t)),$$

$t \in [t(k), t(k+1)), k = 1, \ldots, N$. So we treat two processes $q_1(\cdot)$ and $q_2(\cdot)$ as near-by if

$$|t_1(N_1+1) - t_2(N_2+1)| \leq \varepsilon, \qquad \int\limits_0^{\min(t_1(N_1+1),\ t_2(N_2+1))} \|\hat{q}_1(t) - \hat{q}_2(t)\| dt \leq \varepsilon.$$

The qualitative state $\hat{s}(t)$ of near-by processes may differ only on short time intervals with the total duration $\sim \varepsilon$. In the likely conditions their scenarios have the same number of events but some pair of events follow in the opposite order due to different scenarios.

According to this supposition the two-level method is constructed, in which the choice of the optimum scenario is fulfilled by the branch-and-bound scheme, and the optimum within a tested scenario is calculated with the decomposition method [3]. The latter enables the simple procedure of restrictions restoration when a new vertex is generated with a scenario that differs from the parent vertex scenario in the order of two subsequent adjacent events.

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Interior-point Method for the Linear Complementarity Problem*

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Many of optimization problems along with bimatrix games and linear problems of market equilibrium can be brought to finding the solution of the linear complementarity problem (LCP). Since its becoming an object of study in its own right in the mid 1960's, there were developed several approaches to solve LCP. The interior-point approach generalizing applicable approaches to solve linear programming problems is developing extensively over the last years. In this paper, we consider an interior point method which is a generalization of the barrier-projection method proposed earlier for linear and nonlinear programming problems [1, 2].

The LCP consists in finding two vectors, which satisfy the following systems of equalities and inequalities:

$$\begin{array}{rcl} x & \geq & 0_n, \quad y \geq 0_n, \\ y & = & Mx + q, \\ 0 & = & y^T x. \end{array} \tag{1}$$

In our further argumentation, we suppose that the matrix M is positive definite and the vector q is nonzero.

The LCP (1) has a unique solution [3], thus it is not difficult to show that it is equivalent to a nonlinear programming problem with bilinear objective function. To solve the latter we apply the stable barrier-projection method based on quadratic space transformation.

Let I_n be an $n \times n$ identity matrix, and let $D(z)$ be a diagonal matrix with a vector z on the diagonal. Let us denote

$$G(x,y) \; = \; MD(x)M^T \; + \; D(y).$$

The continuous version of the method is described by the following system of ordinary differential equations:

$$\begin{aligned} \frac{dx}{dt} &= -D(x)\left\{y + M^T G^{-1}(x,y)\left[(I_n - M)D(x)y + \tau(Mx - y + q)\right]\right\}, \\ \frac{dy}{dt} &= -D(y)\left\{x - G^{-1}(x,y)\left[(I_n - M)D(x)y + \tau(Mx - y + q)\right]\right\}. \end{aligned} \tag{2}$$

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If the pair $[x_*, y_*]$ is a nondegenerate solution of the LCP (1) and $\tau > 0$, then the system (2) is asymptotically stable at this pair.

The set $Z = \left\{[x,y] \in \mathbb{R}^{2n}: \; Mx - y + q = 0_n\right\}$ is positive invariant with respect to the system (2). It means that if an initial pair is in the set Z then the solution of (2) belongs to this set. For $\tau = 0$ we define system (2) as an feasible variant of the barrier-projection method (2):

$$\begin{aligned} \frac{dx}{dt} &= -D(x)\left[I_n + M^T G^{-1}(x,y)(I_n - M)D(x)\right] y, \\ \frac{dy}{dt} &= -D(y)\left[I_n - G^{-1}(x,y)(I_n - M)D(y)\right] x. \end{aligned} \tag{3}$$

Denote $Z_+ = Z \bigcap \mathbb{R}^{2n}_+$. Local convergence of (3) on Z_+ follows from local convergence of the underlying method (2).

The discrete version of (3) can be written in the form

$$\begin{aligned} x_{k+1} &= x_k - \alpha_k D(x_k)\left[I_n + M^T G^{-1}(x_k,y_k)(I_n - M)D(x_k)\right] y_k, \\ y_{k+1} &= y_k - \alpha_k D(y_k)\left[I_n - G^{-1}(x_k,y_k)(I_n - M)D(y_k)\right] x_k, \end{aligned} \tag{4}$$

where $\alpha_k > 0$ is the step length. The method (4) locally converges to the nondegenerate solution of (1) on Z_+ for any fixed sufficiently small α_k.

It is interesting to investigate non-local behavior of the method (4). Let J be an index set $\{1, ..., n\}$, and let $[x,y]$ be a feasible pair, i.e. $[x,y] \in Z_+$. We introduce subsets of J depending on this pair:

$$\begin{aligned} J_P(x,y) &= \left\{i \in J : x^i > 0, \; y^i > 0\right\}, \\ J_B(x,y) &= \left\{i \in J : x^i > 0, \; y^i = 0\right\}, \\ J_N(x,y) &= \left\{i \in J : x^i = 0, \; y^i > 0\right\}, \\ J_Z(x,y) &= \left\{i \in J : x^i = 0, \; y^i = 0\right\}. \end{aligned} \tag{5}$$

In accordance with (5) the matrix M is a square matrix having the block partitioned form:

$$M = \begin{bmatrix} M_{PP} & M_{PB} & M_{PN} & M_{PZ} \\ M_{BP} & M_{BB} & M_{BN} & M_{BZ} \\ M_{NP} & M_{NB} & M_{NN} & M_{NZ} \\ M_{ZP} & M_{ZB} & M_{ZN} & M_{ZZ} \end{bmatrix}.$$

Similarly one can partition vectors $x \in \mathbb{R}^n$ and $y \in \mathbb{R}^n_+$ into subvectors.

A feasible pair $[x,y]$ is called weak nondegenerate if a submatrix

$$\begin{bmatrix} M_{BP} & M_{BB} \\ M_{ZP} & M_{ZB} \end{bmatrix} \tag{6}$$

of the matrix M has full rank which is equal to the number of the rows of the submatrix (6). In the case of vacuous submatrix (6) the pair $[x,y]$ is weakly nondegenerate automatically. A feasible pair $[x,y]$ is called nondegenerate if it is weakly nondegenerate and a vector

$$p(x,y) = \begin{bmatrix} y^P + M_{PP}^T x^P \\ M_{PB}^T x^P \end{bmatrix}$$

is in the subspace of the rows of (6) if and only if $|J_Z(x,y)| = |J_P(x,y)|$. The problem is called nondegenerate if all feasible pairs are nondegenerate.

The method (4) is well defined for all weakly nondegenerate pairs $[x_k, y_k]$, i.e. the matrix $G(x_k, y_k)$ is nonsingular for these pairs. Moreover, under the nondegeneracy assumption of (1) the pair $[x_k, y_k]$ is a stationary point of the iterative process (4) if and only if this pair is an extreme point of the set Z_+.

It is possible to use the steepest descent approach for choosing the step length α_k. This approach is based on minimization of the function $V(x,y) = x^T y$ along the directions

$$\begin{array}{rcl} \Delta x_k & = & -D(x_k)\left[I_n + M^T G^{-1}(x_k,y_k)(I_n - M)D(x_k)\right] y_k, \\ \Delta y_k & = & -D(y_k)\left[I_n - G^{-1}(x_k,y_k)(I_n - M)D(y_k)\right] x_k. \end{array} \tag{7}$$

We have

$$V(x_k + \alpha\Delta x_k, y_k + \alpha\Delta y_k) = V(x_k, y_k) + c_1\alpha + c_2\alpha^2,$$

where the coefficients c_1 and c_2 are such that $c_1 \le 0$, $c_2 \ge 0$. Moreover these coefficients are equal to zero if and only if the pair $[x_k, y_k]$ is a stationary point of the iterative process (4).

Under the assumption that the pair $[x_k, y_k]$ is not a stationary point of (4) we take

$$\alpha_k = \min\{\alpha_k^{(1)}, \alpha_k^{(2)}\},$$

where $\alpha_k^{(1)} = -\dfrac{c_1}{2c_2}$ and

$$\alpha_k^{(2)} = \operatorname{argmax}\ \{\alpha \ge 0 : x_k + \alpha\Delta x_k \ge 0_n,\ y_k + \alpha\Delta y_k \ge 0_n\}.$$

It is shown also how the directions Δx_k and Δy_k can be modified in the case of the pair $[x_k, y_k]$ is a stationary point of (4). The finite convergence of the barrier-projection method (4) with the steepest descent is proven.

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Power Distribution in the European Council of Ministers in EU25

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key words: *power indices, European Union*

It is argued that enlargement challenges institutional balances and, in particular, relative powers of national actors within the European Union (EU). The paper emphasizes on the impact of the ongoing enlargement to 25-member Union on power distribution in the Council of Ministers based on the latest decisions taken. Due to the changes in decision-making regulations for the Council the measures of power are redefined to adjust the respective power index. The Banzhaf index is used to evaluate emerging power distributions in the EU institution. In the Council of Ministers the triple-majority rule is applied to the different voting configurations, causing the adjustment of the winning coalition definition for the calculation of the index. The adjusted definition makes the coalition to be winning, if the quota in members, the quota in votes and the threshold in population are passed, and to be loosing, if at least one of the three requirements is not met. Naturally, for these changes to be effective, each country is assigned two numbers: the number of votes in the Council and the population of the country. Namely, let S be some coalition of the member states. The coalition S is said to be winning if it can enforce the act to be passed no matter how the countries outside the coalition vote. So all the requirements: quota in votes, quota in members and quota in population should be met by the coalition S. The coalition S is said to be losing if at least one of the requirements is not met. Formally, let each member state i be assigned two numbers v_i — number of votes and w_i — population (in % of all EU population). The coalition S is said to be winning if all the following 1), 2) and 3) holds:

1) $\sum\limits_{i \in S} v_i \geq$ Quota in votes,

2) $|S| \geq$ Quota in members,

3) $\sum\limits_{i \in S} w_i \geq$ Population threshold.

The implications of this analysis are the following.

There is a considerable decline of power of all 'old' members since enlargement. The relative power of the former members, however, remains almost the same.

For the Council, with the new bill-passing regulations, the bigger states will be better off, relative to the smaller sates. Although the vote share of the big states is fallen significantly, the vote share of the small states is declined in a greater proportion, leading the big states to be comparatively better off. So, for the Council, the entrance of the new members into the European Union is very much compensated by the vote shares of the smaller states, which, in turn, creates a bias in favor of the big states in terms of power distribution.

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Political Advertising and Policy Polarization

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key words: *voting, Hotelling theorem, strategic behavior, negative advertising*

The author uses a game-theoretic model to analyze the reason why in a democratic election the candidates do not choose identical policy platforms, as they should do according to the well-known Hotelling theorem. It is assumed that after the political platforms are chosen, the candidates are capable of spending resources to increase (advertise) their political weight (valence). The candidates would not choose identical platforms because in that case their equilibrium cost of advertising will equal the expected benefit from winning the election. The author then analyzes how the distribution of population's policy preferences affects the policy difference between candidates and the amount advertising done by the candidates.

Equilibrium Concept and Decentralized Market Design for Network Double Auction

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key words: *network auction,equilibrium,clearing design*

Equilibrium study in network generalization of double auction put forth such critical features as limited network capacity and cost of transportation. So supply and demand balance for each bargaining spot should be accompanied with appropriate commodity flow pattern, which doesn't violate network constraints. We try to unify description of buyers, sellers and transportation agents by assigning each of them some relationship between price of commodity and it's flow. Then we are able to define equilibrium in terms of price and flow profiles and prove its existence and unicity under certain assumptions. We also show that equilibrium delivers maximum to surplus of the network auction. Concept of network equilibrium is complemented with specific market design based on induced orders, which makes it possible to decentralize clearing and localize behavior of each particular agent. Poisson based order making process together with real time clearing mechanism based on induced orders are proposed to conduct the set of experiments for which theoretical equilibrium can be calculated. Experimental prices and profit allocations are examined against theoretical outcome. Experimental price stabilization is shown to be consistent with equilibrium values. We also discuss relationship of suggested dynamic network auction price-making with alternative static concepts ("smart" auction) developed e.g. by V. Smith. Our mechanism can be regarded as limit case when the number of orders tends to grow infinitely. Market design under consideration provides much simplified way of network trading which reveals equilibrium prices without real-time solving of optimization problems as with alternative approaches.

Network double auction is thought to be natural generalization of ordinary "spot" double auction. Still its outcome is influenced not only by commodity demand and supply but also by possible network congestion and cost of transportation. Such auction can be represented by pointed graph, in which nodes correspond to buyers or sellers and edges correspond to agents wielding carrying capacity. Though we assume each edge working only in one direction it is always possible to arrange pair of opposite edges to model bilateral transport capacity. The bargaining object is uniform discrete commodity measured in units. Current state of each participant is determined by pair (P, λ), where P is price of the unit of commodity and λ is its flow in units per unit of time. Profiles pair $(\vec{P}, \vec{\lambda})$ is called state of the auction. Further each agent is assigned with relationship $P = P(\lambda)$ which determines at what price buyer wants to consume given commodity flow, seller — delivers given flow to the market and transportation agent — transmits given flow through his edge. This function is assumed to be monotonous increasing for sellers and transporters and monotonous decreasing for buyers. So each agent surely has reciprocal function $\lambda = \lambda(P)$, which is referenced further as capacity function of this agent. Also further price at node D is denoted by P_D, T_D represents array of agents located at D, $P_L^-$and P_L^+ correspond to price at start and end nodes of edge L respectively, Ω_D^+ and Ω_D^- designate set of incoming and outgoing edges respectively for node D and trigger δ is 0 for a seller or a transporter and 1 for a buyer.

Definition 1. *Equilibrium is the state of auction* $(\vec{P}, \vec{\lambda})$, *for which* $\lambda_i = \lambda_i(P_D)$ *for each* D *and each* $i \in T_D, \lambda_L = \lambda_L(P_L^+ - P_L^-)$ *for each edge* L, *and flows are balanced at each node:*

$$\Delta_D = \sum_{i \in T_D} (-1)^\delta \lambda_i(P_D) - \sum_{L \in \Omega_D^-} \lambda_L(P_L^+ - P_L^-) + \sum_{L \in \Omega_D^+} \lambda_L(P_L^+ - P_L^-) = 0.$$

In other words, equilibrium is coherent set of node prices and edge flows, which ensures that no agent experiences either excess of commodity/carrying capacity or their lack. Each buyer consumes exactly the same flow as network delivers to his node, each seller produces exactly the same flow as network takes away from his node and each edge transmits exactly the same commodity flow as price difference between its nodes imply given it's capacity function.

Corollary 1. If $(\vec{P}, \vec{\lambda})$*is equilibrium then* $\sum (-1)^\delta \lambda_i P_i = 0$.

Definition 2. *Function* $\lambda(P)$ *is permissible with the interval* $[A, B]$ *, if it is continuous, strictly monotonous within* $[A, B]$, *constant outside* $[A, B]$ *and one of the relationships is satisfied:* $\lambda(A) > \lambda(B) = 0$*or* $\lambda(B) > \lambda(A) = 0$.

Theorem 1. *In network auction, where buyers and sellers cannot be located at the same node, equilibrium exists and is unique if for some* $0 < P_{\min} < P_{\max}$*capacity functions of buyers and sellers are permissible with the interval*$[P_{\min}, P_{\max}]$ *and capacity functions of edges are permissible with the interval* $[0, P_{\max} - P_{\min}]$.

Consider a buyer B located at vertex D_B whose price at current state of the auction is P_B. Consider also seller S located at vertex D_S with current price P_S. Let the root M connect vertices D_B and D_S while sum of all its edge prices is P_M. We say that a buyer B induces a bid $P_B - P_M$ for a seller S, who in his turn induce ask $P_S + P_M$ for a buyer B. We also say that together they induce a bid $P_B - P_S$ for transportation along root M. Thus in equilibrium, all buyers whose bids can reach particular seller induce the same bid for him. Vice versa all sellers induce the same ask for particular buyer they can reach. Finally all induced transportation bids for particular edge are the same.

Definition 3. *Utility of particular agent in current state is*

$$(-1)^{\delta+1}\int_0^\lambda P(x)dx.$$

Sum of utilities S across the auction is surplus of the auction in current state.

Surplus of the auction depends on flow profile of the current state so we denote it by $S(\vec{\lambda})$. We can consider $P(\lambda)$ for buyer as redemption function. In this case utility is interpreted as redemption per unit of time that buyer receives for flow λ. We can also consider $P(\lambda)$ for seller and edge as cost function so sellers' utility is cost he incurs while supplying commodity flow λ.

Theorem 2. *Within bounds of Theorem 1 there exists unique* $\vec{\lambda}^*$ *solving the following problem:* $S(\vec{\lambda}) \to \max$ *given* $P_{\min} \leq P_i(\lambda_i) \leq P_{\max}$ *and* $\Delta_D = 0$ *for each* D *and* $i \in T_D$ *and* $0 \leq P_L(\lambda_L) \leq P_{\max} - P_{\min}$*for each edge* L*. Flow profile* $\vec{\lambda}^*$ *can be unambiguously complemented with price profile* $\vec{P}^*$*such that tuple* $(\vec{P}^*, \vec{\lambda}^*)$ *is equilibrium.*

Consider a buyer placing single unit orders at Poisson moments of time with intensity λ. Let his bids be random with cumulative distribution function $\xi(x)$. Producing bid higher then P has probability $(1 - \xi(P))\lambda dt$ during interval dt and hence is Poisson process with intensity $(1 - \xi(P))\lambda$. So average number of bids per unit of time with price better then P is $(1 - \xi(P))\lambda$. This relationship can be regarded as capacity function of this buyer in line with above definitions. Similarly $\xi(P)\lambda$ can be viewed as capacity function of seller or transporter. These capacity functions possess all necessary monotone properties and suffice to calculate equilibrium prices and flows. To conduct experiments order generated mechanism should be accompanied with clearing rules. In the environment of order generation and maintaining order queues we assume current price of the buyer to be the highest of his active bids and current price of the seller (or transporter) — the lowest of his active asks. With these prices we can calculate induced orders as it was described above. At each node and edge bilateral queue can be composed — active orders of local agent versus orders induced through the network.

Definition 4. *Price gap for particular agent is difference between his current price and price of the best induced order at the place of his localization. Origin agent of best induced order is his best current counteragent.*

Clearing starts when there are agents with positive gap. We propose a kind of auto crossing clearing where crossing orders are simultaneously satisfied at price of the eldest order.

Definition 5. *We say that positive price gap of particular agent is banned if price gap of correspondent best counteragent is greater than his. In this case this pair of agents are not mutually best counteragents for each other.*

Statement. *If currently there are positive price gaps then some of them are necessarily unbanned.*

Theorem 3. *Local implementation of auto crossing for agents with unbanned price gaps produces consistent results.*

Computational experiments with Poisson based agents enable comparison of tangible outcome with theoretical equilibrium. One can declare stabilization of trading prices for each agent with emerging of specific spread around his equilibrium. Both real and induced orders are accumulated at the bounds of this spread while orders inside are continuously cleared. Existence of the spread makes it possible to strike suboptimal deals and thus increase flows above equilibrium values at the cost of impairing total auction surplus. An alternative way of clearing order queues is to pursue surplus maximization via solving linear programming task. This approach, called "smart auction", determines exact number of orders for each agent to be satisfied but do not determine unambiguous price profile. If we specify price for each agent in "smart auction" as the price of his last accepted order then the following conclusion arise.

Theorem 4. *With Poisson based agents implementation of "smart auction" for order queues generated during time period T yields price and flow (number of accepted orders divided by T) profiles which tend to converge to equilibrium as $T \to \infty$.*

This statement is the consequence of surplus maximizing theorem. As real time trading roughly reveals equilibrium prices and flows it can be viewed as approximate decentralized method of solving surplus maximizing task.