The Andronov-Hopf's Bifurcation in Price Formation Models of Walras Type with Delay

N.K. OBROSOVA (Computing Center of RAS, Moscow)

Modeling of the rise of economic crises is a well-known problem in mathematical economy. Usually economic crises are explained by the loss of stability of the equilibrium prices resulting from the changes of macroeconomic parameters (see, e.g. [1]). Thus, an urgent problem in the field is to develop a price formation model such that the equilibrium looses stability when macro-economic parameters change. This price formation micro-economic model must be convenient for using as a block in a macro-economic model describing endogenous crises arising. The price formation model of Walras type with delays in consumer and producer reactions to price change as well as a micro-economic model is considered in [1]. In this model, the equilibrium price stability loss investigation (in the case of only one nonzero delay or two equal delays) could be reduced to the bifurcation analysis of the fixed solution $x(t) \equiv 0$ of the differential equation with delay

$$\frac{dx}{dt} = H(x(t), x(t-\tau)), \qquad (1)$$

where $\tau > 0$ is a constant delay. Function H(x, y) satisfies the following conditions:

1) H(x, y) is *n* time continuously differentiable $(n \ge 9)$; 2) the equation (1) under $x(t-\tau) = y(t)$ has a bounded solution on $[0, \tau]$ for any continuously differentiable function y(t) $(t \in [0, \tau])$; 3) $u = -\tau \partial H(x, y) / \partial x|_{x=0,y=0} \ge 0$, $v = -\tau \partial H(x, y) / \partial y|_{x=0,y=0} > 0$. In this paper, the following main result is proved.

Theorem. Let u > 0 and $u \neq 2\pi/(3\sqrt{3})$. Then a periodic orbit arises or dies in the system (1) if the fixed solution $x(t) \equiv 0$ of the equation (1) loses stability, i.e., if the parameter v increasing passes through the stability bound v_{qr} (the critical value v_{qr} is the solution of the equation

$$\sqrt{v_{gr}^2 - u^2} = \arccos(-u/v_{gr})).$$

The proof of the theorem contains three main steps. In the first place, the infinity-dimensional problem is reduced to the two-dimensional problem by the central manifold theorem. Secondly, the Hopf's theorem is applied to the two-dimensional manifold obtained. From the Hopf's theorem, emerging or dying of the one-dimensional invariant manifold is obtained in the origin problem when the bifurcation parameter v crosses through the critical value v_{gr} . Finally, it is proved that the periodic orbit of the equation (1) corresponds to the obtained one-dimensional invariant

manifold. This fact is proved with the help of the circle diffeomorphisms theory.

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A Simplified Model for the Simulation of Oligopolistic Markets

V.V. OKHRIMENKO (Computing Center of RAS, Moscow)

The economic system consisting of a finite number of partners is considered. The collective behavior of economic partners-oligopolists is modeled by means of the sequence of variational problems and inverse problems for ordinary differential equations. Each of the partners wants to reach the maximal value of the criterion function choosing the values of the variable he controls. The separate partners have not the complete information about the criterion functions of other partners. The system evolves at finite or semi-infinite interval of time. For example, in the aforesaid form, it is possible to represent the initial

For example, in the aforesaid form, it is possible to represent the initial purposes of the partners-oligopolists of the classical oligopolies, as it was shown by Edgeworth, Hicks, von Stackelberg, and von Neumann. The algorithm of the behavior can depend on derivatives of high orders. As the most simple example of differential algorithm it is possible to demonstrate the coordinate-wise gradient algorithm that simulates aspiration of the primitive partner at each moment to receive maximal increment of the current prize ("to snatch increment more and more"). The author's opinion is that the algorithms of such grade simulate a pursuit by the businessman behind profit not less adequately than the general equilibrium of Arrow-Debreu. Among the trajectories there are the trajectories of prime importance:

1. Periodic trajectories with asymptotic orbital stable limit cycles.

2. Rest points that are not stationary points in the sense of Nash-Roos-Walras-Isnard. (Those are, in particular, points of von Stackelberg).

3. Ergodic trajectories, filling a subset everywhere dense in itself, which has a finite measure in the space of controllable variables.

An Ecological Demographic Economic Model

N.N. OLENEV and N.V. BELOTELOV (Computing Center of RAS, Moscow)

A non-linear model of ecological, demographic, and economic processes is presented to give a useful tool for global analysis of international labor mobility and trade. In the model, a set of points (countries) on the plane is considered. Each of these points is determined by dynamics of its own variables, and they are interacted with each other.

The basis for this model is determined by the following hypotheses: 1) the population can migrate from one point to another to get the best level of living; 2) the population consists of two employment groups with skilled and unskilled labor; 3) the production function converts skilled and unskilled labor units into effective labor units.

The level of living at the specified point is determined by economic activity and quality of environment.

The research was partially supported by the grants "Scientific schools" N. 00-15-96118 and RFH N. 01-02-00019a.

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An Implementation of the Parallel Continuation Algorithm for Global Optimization

I.V. ORLYANSKAYA, Y.N. PERUNOVA, and S.K. ZAVRIEV (Moscow State University, OR Dept.)

The continuation method for global minimization of a smooth function $F(\cdot)$ on a k-dimensional box is considered [1]. This method is convenient for parallelization with a number of processors searching different quasioptimal trajectories. The parallel algorithm has been implemented and tested on 64-transputer (T800) parallel computer PARSYTEC in the Computing Center of Russian Academy of Sciences.

Suppose that p is a number of available transputers and s_i is a number of quasioptimal trajectories on *i*th step (i = 1, ..., k).

In the current implementation of the algorithm, the transputers are linked into a ring. Every transputer get its own $ID = 0, \ldots, p-1$.

Main procedures used in this implementation are as follows:

"Calculation" – calculation of a quasioptimal trajectory and search for a local minima x_{loc} on this trajectory;

"Send" – sending the surplus pairs (x_{loc}, i) further into the ring;

"End" - sending the notification that the calculation is finished;

"Transfer" – the transit transfer of surplus pairs (x_{loc}, i) further into the ring;

"Return Res" – return of an obtained result F_{min} . If the number of quasioptimal trajectories exceeds the number of transputers, several transputers begin to work in the time-sharing scheme.

The research is supported by grant "Scientific schools" N.00-15-96141.

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A Method of Computing the Best L₁ Approximations in Classes of Distributions V. P. PANTELEEV (Murmansk State Pedagogical Institute, Mathematical Analysis Dept.)

Computing the best L_1 approximations in a class of distributions is hampered by a number of reasons. This class is not the linear manifold. Further, the metric space L_1 is less perspective with respect to the solution properties, than, for instance, the space L_2 . The problem becomes more complicated if, besides, the continuity of functions has to be taken into account. At last, an accent is put here on constructive search for the best approximation by effective algorithms.

In this work, under the aforesaid requirements, the scheme of a computer search for the best L_1 approximations in classes of distributions is offered. The scheme was tested successfully.

Below we formulate certain essential facts. Let a class of distributions be given by the piecewise continuous density function f(x, a, b) of argument x. To prevent complicated designations we consider two parameters a and b and assume that the function f is continuous with regard to them. Let also the function p be given for which the best L_1 approximation f has to be found. In the case of non-negative functions f and p, we denote closures of sets $\{(x, y): 0 < y < f(x, a, b)\}$ and $\{(x, y): 0 < y < f(x, a, b)\}$ 0 < y < p(x) through F_{ab} and G respectively. Let S(M) denote the area

of a set M, then for normed distributions we have $\int_{-\infty}^{+\infty} |f(x, a, b) - p(x)| dx =$

 $S(G \triangle F_{ab}) = S(G) + S(F_{ab}) - 2S(G \cap F_{ab})$. From these equalities it follows that the minimum of the integral or, that is the same, the minimum of the area $S(G \triangle F_{ab})$ of symmetric difference of domains G and F_{ab} corresponds to the maximum of the area $S(G \cap F_{ab})$ of the common part of domains F_{ab} and G. If one of the domains F_{ab} and G is bounded and its boundary measure on the common part of boundaries ∂F_{ab} and ∂G is null, nothing hinders to search the best approximation f of the function p

by maximization of the area of the common part $G \cap F_{ab}$ of the domains. Significantly, that under conditions, mentioned above, the area $S(G \cap F_{ab})$ has the continuous gradient, which for the normal density functions f is equal to $g = \nabla S = \sum_{k} (-1)^{k} \{ \bar{\imath} y_{k} + \bar{\jmath} b^{-1} y_{k} (x_{k} - a) \}$, where $P(x_{k}, y_{k})$ runs all points, in which Gaussian curve f enters into domain G, if k is even, and leaves it, if k is odd. It opens practical possibility by means of iterations $r_{n+1} = r_n + h_n g_n$, $r_n = (a_n, b_n)$, at first to find functions f, which are the nearest to p in local sense, and then, by comparing, to select an element of the best approximation.

On Structure Preservations of Complex Control Systems

YU.N. PAVLOVSKY and G.I. SAVIN (Computing Center of RAS and Joint Supercomputer Center, Moscow)

The presentation is devoted to the illustration and analysis of the following statements. If observation of a complex control system is revealed a "structure" in this system, it means that there are mechanisms of the structure self-preservation. The notion "structure" means here that it may be seen sufficiently closed but in the same time interdependent parts of this system. The total sum of the parts and the character of its interdependence are the system structure. The realization of self-preservation mechanisms is a feedback in the most cases. In particular, the feedback brings into accordance the difficulty of finding the control problem solution and the information technology in-built in the system.

Aggregation of Multi-Attribute Objects in Multiset Metric Spaces

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The paper considers an approach for the aggregation of objects that are described with many qualitative attributes. These are, for instance, a collection of projects estimated by several experts using qualitative criteria with ordered or nominative scales, a file of textual documents whose substance can be expressed as a set of lexical units (descriptors, keywords), a sequence of recognized graphic symbols, and so on. All these objects can exist in several copies with various numbers of attributes.

In order to present and analyze the objects of these kinds, a theoretical model based on the concept of multisets is suggested. A multiset is a set with repeating elements. The different options for a multi-attribute objects' aggregation such as addition, union, intersection of multisets are

discussed. Proposed notions are used to build new methods of hierarchical and nonhierarchical cluster analysis in multiset metric spaces. Measures of difference/similarity between multisets are introduced. Some of them generalize the known expressions for new types of objects.

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Two-Model Quadrature Formulas in Numerical Integration

(Moscow State University, OR Dept.)

In the report, the approach for creation of quadrature formulas is considered. We call this approach as model decomposition in numerical integration.

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On Expansion of Exponential Complexity of Algorithms of Discrete Optimization into a Succession of Polynomials, Corresponding to Local Algorithms, Based on the Imbedded Partial Order Relations and Semi-Groups of Transformations S.F. Rogov (Moscow)

The title is a summary of the author's researches been conducted in 1981-1988 which are reflected in the following list of publications, theses of reports, and deposited materials. Here, we deal with the fact that besides of algorithms of monotone formation of successions (the branch-andbound method, the method of plan construction) based on the relation of partial order and estimation of precision of acceptable current solutions of the problem [2], a precise solution of any problem of discrete optimization of exponential complexity can be presented as follows.

Let us assume that $A_1, ..., A_n$ is a succession of local algorithms with imbedded systems of regions, based on imbedded partial order relations such that the local extremum for *n*th region is also the local extremum for (n-1)th region. The complexity of the algorithm A_n is a polynomial of degree *n* with respect to the problem dimension. Then the precise numerical solution of an exponentially hard problem can be presented as finding *n* local solutions $X_1, ..., X_n$ with the aid of algorithms $A_1, ..., A_n$, such that the initial admissible solution of the *n*th subproblem is the solution of the local (n-1)th subproblem.

There are examples of partial order relations for linear problem of discrete optimization in [1] and a transportation problem in [3]. In [3], there is an estimate of complexity for every local algorithm. The total number of local algorithms in the succession also depends on the scale of the problem. Obtaining the succession of such imbedded partial order relations together with a priori estimates of their complexity is an important part of the total research and solution of any problem of discrete optimization. Any obtained partial order relation can be used in the algorithm of formation of *F*-monotone successions with cutting off non-prospective subsets of solutions, both through estimates and orbits of some transformations forming semi-groups [2]. Here is the achievable estimate of complexity of the first algorithm from the hierarchy for a transportation problem: m(m + n + 1)(n - m) + (m + 1)(2m + 1)/3 - m(m + 1)/2. Here *n* is the number of suppliers and *m* is the number of consumers (see [4]).

You are welcome to the Internet site, which is devoted to mathematical researches and their prospects, http://www.sofia.aha.ru/math.html.

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Greedy Algorithms for Schedules Constructions with Minimization of Processor Number

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A construction of a greedy algorithm for solving a practically important class of problems arised during combined software and hardware design is discussed in this paper. That class of problems is scheduling and simultaneously minimizing a number of processors. For given program's behavior and time limit, it is necessary to find out a minimal number of processors in a computer system and to schedule the program on those processors. In this form it is an NP-complete problem.

A greedy algorithm means decomposition of a problem into a row of simpler problems. A decision is made on a principle of a local optimal solution for each problem in the row. A suggestion that those local optimal solutions lead to the global optimal solution is made.

An algorithm's scheme is described below.

1. Construct an initial schedule.

2. Choose two processors to merge their work interval sets into one. Compute local criteria. Check time limit.

3. Correct a number of processors and a schedule.

4. Stop if there is no pair of work interval sets to merge.

At the first step the algorithm constructs an initial schedule. A number of processors is set equal to a number of processes and all work intervals of every process are assigned to the same processor. At the next steps a pair of processors is chosen by means of exhaustive search. Local criteria are computed and time limit is checked for every pair of work interval sets and the best pair is chosen. The pair of sets is merged and a new schedule is constructed.

The investigation shows a high influence of local criteria on results. So it is important to tune the algorithm, to find dependencies between source data, criteria set, and results. It is also interesting to investigate an influence of random search at first steps. It may lower a computational cost but also lead to nonoptimal solution. Thus, it is needed to find out possibility to combine random search and local criteria search.

The results of computational investigations of the algorithm and its modifications will be presented.

Conditional Strategic Equivalence and Frame Restricted Choice with Applications to Regulatory Risk and Uncertainty: Some Further Results M. J. RYAN (Department of Economics, University of Hull)

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Theorem 1 (Framing and switching): With M arbitrarily large and if a feasible solution exists for (I), then:

$\operatorname{Max} \varphi(f((\theta \pi + c)x)) - M$	$Is^+ - Ms^-$	$\leq \operatorname{Max} \varphi(f((\theta \pi + c)x)) - h^+ s^+ - h^-$	s^{-}
st $g(x) + s^+ - s^- = b$	(I)	st $g(x) + s^+ - s^- = b$	(Ia)
$x, s^+, s^- \ge 0$		$x, s^+, s^- \ge 0$	

With $\varphi(f(\cdot)) \equiv f(\cdot), \theta = 1, c = 0$ this result is identical to a result in Ryan 1998a.

Definition 1 (Strong strategic equivalence):

If x^* is optimal in (II) then (II) and (III) are strongly strategically equivalent if x^* is also optimal in (III).

$$\begin{aligned} &\max \varphi(f((\theta \pi + c)x)) - Ms^{+} - Ms^{-} & \max f(x) - Ms^{+} - Ms^{-} \\ &\operatorname{st} g(x) + s^{+} - s^{-} = b & (\operatorname{II}) \\ & x, s^{+}, s^{-} \geq 0 & x, s^{+}, s^{-} \geq 0 \end{aligned}$$

 $[\varphi, \theta, c$ determine families of strongly strategic equivalent transformations of x.]

Using Theorem 1 and Definition 1 I will extend results draw in Ryan 1998a,b,c to generalize standard strategic equivalence results for the constant sum game to explicitly frame related and nonconstant sum cases. [Frame parameter dependent play: don't play decisions and complementary slackness related optimal response criteria will be key ideas here.]

Variants of Theorem 1 will be developed with reference to uncertainty and regulation. Emphasis will be on sufficient conditions for strategic *inequivalence* and uncertainty-reducing and innovation and thence learning increasing interpretations for strategic equivalence parameters θ and c_{ij} and for framing variables s_i^+, s_i^- .

A simple maximin crop planting example in which there are potentially six distinct varieties of uncertain outcomes ii)..iv) and vi)..viii) will illustrate salient ideas:

	Known crop	Unknown	Fallo w	Outsid e	
		стор		Oppor-	
				tunities	
Known regulatory state 1	$\theta \pi_{11} + c_{11}$ ia)	$\theta \pi_{12} + c_{12}$ ii)	$h^+ v$)	h^- vii)	
Known regulatory state 2	$\theta \pi 21 + c_{21}$ ib)	$\theta \pi_{22} + c_{22}$ ii)	h^+ v)	h^- vii)	
Unknown regulatory state 0	$\theta \pi_{31} + c_{31}$ iii)	$\theta \pi_{32} + c_{32}$ iv)	h^+ vi)	h^- viii)	
	1 1 1	• • • • • • • •		Ċ,	

[Since at least one crop and one state is initially unknown to the farmer the range of the crop distribution and of the states distribution is correspondingly initially unknown to the farmer.]

$$\begin{aligned} & \underset{i}{\operatorname{Max}}(\underset{j}{\operatorname{Min}}(\theta \pi_{ij} + c_{ij})p_{j}) - Mp^{+} - Mp^{-} = z \leq \\ & \text{st } \sum p_{j} + p^{+} - p^{-} = 1 \\ & p, p^{+}, p^{-} \geq 0 \end{aligned} \tag{IV}$$

$$\leq z' = \max_{i} (\min_{j} (\theta \pi_{ij} + c_{ij})p_{j}) - h^{+}p^{+} - h^{-}p^{-}$$

st $\sum_{j} p_{j} + p^{+} - p^{-} = 1$ (IVa)
 $p, p^{+}, p^{-} \geq 0$

1. If c_{i2} are sufficiently small and c_{3j} are sufficiently large to ensure that outcomes of types ii), iii), iv) will not be chosen, (IV) yields standard

maximin outcomes. 2. If $c_{ij} = c$ an optimum to (IV) will include at most outcomes ia) and ib) and will be consistent with standard strategic equivalence conditions for the constant sum case *iff* the other parameters remain such that outcomes ii)..iv), vi)..viii) are not chosen.

Conversely:

1. By announcing sufficiently large reductions in h^+, h^- a regulator may induce switches from strongly framed optima to (IV) to weakly framed optima to (IVa).

2. By announcing sufficiently large increases in c_{i2} or reductions in c_{3i} , h^- a regulator may induce choice of previously unknown crops, knowledge of previously unknown states and/or use of previously unknown additional land.

3. A regulator can alter choices by naming initially unknown states and detailing contingent payoffs to them and do so a fortiori by also conveying information concerning probabilities with which such states may be forthcoming.

In the paper and presentation these and other more general strategic equivalence and learning and regulation related cases will be developed. The latter cases will include uncertainty and learning related extensions both to the contestability based regulatory results in Ryan 1999a and to the landowner-landuser constrained game and bargaining results in Ryan 1999b.

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Fiducial Evaluation of the Mean Residual Discrete Useful Life of Furnishing Items of Flight Vehicles G.S. SADYKHOV, N.A. SEVERTSEV, and A.V. GERASIMOV

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Let the limiting condition of a furnishing item be determined by certain operations, after which the given item is not used as required; let ξ be the number of operations at which the failure of an item happens. Such items are the buttons, switches, alarm lamps, contactors, and other units in the structure of flight vehicles.

To extend the term service life of a flight vehicle we define the residual useful life of a furnishing item over k operations by the formula

$$\xi_k = \xi - \frac{k}{\xi} \mid \xi \ge k + 1.$$

The magnitude ξ_k is conditional and should not be confused with unconditional random variable equal to $\xi - k$.

Let's define a parameter "the mean residual discrete useful life" R(k) after k operations, according to [1], under the formula

$$R(k) = M(\xi_k) \tag{1}$$

where $M(\cdot)$ is mathematical expectation of the value included in brackets. If k = 0, from (1) follows

$$R(0)=R,$$

where $R = M(\cdot)$ is the mean operating time of failure.

Parameter "the mean residual discrete useful life" R(k) should not be confused with the parameter "the residual mean discrete useful life", equal to R - k, because they are related to each other as follows [1]:

$$R(k) \ge R - k$$

and the sign of equality is achieved if and only if the probability of a failure up to kth operation is equal to zero

$$\Pr(\xi \le k) = 0. \tag{2}$$

This implies that if the condition (2) is not fulfilled, the obtained evaluations for a parameter "the mean discrete useful life" cannot be good for "the mean residual discrete useful life" R(k), as the following ratio takes place

$$R(k) > R - k, \quad \Pr(\xi \le k) \ne 0, \tag{3}$$

e.g., if k is large, the evaluation for R(k) is degenerated as it is evident from (3). Therefore, there is a problem of the determining the fiducial evaluation for a parameter R(k). In this connection, we establish that in the case of monotone increase of the failure rate, defined by the formula of integral argument

$$\mu(n) = \frac{\Pr(\xi = n)}{\Pr(\xi \ge n)} \quad (n = 1, 2, 3, \ldots),$$

with the fiducial probability γ ($\gamma > 0.5$) the parameter "the mean residual discrete useful life" over k operations is estimated by the following value:

$$R(k) = \frac{R_n^{(m)}(k)}{1 + \frac{x_{\gamma}}{\sqrt{n-m}}}, \quad n > m.$$

Here

$$R_n^{(m)}(k) = rac{1}{(n-m)lpha(n,k)} \sum_{i=1}^{n-m} \zeta_k^{(i)},$$

n is the total number of items of the same type from the beginning of tests; *m* is the number of the items which failed during their operation before k (m < n), $\zeta_k^{(i)} = \zeta_i - k$, provided that $\zeta_i \ge k + 1$, where ζ_i is the number of operations before failure of the item *i*, $\alpha(n, k) = 1 - (1 - p^k)^n$

where p is the probability of non-failure of an item during one operation, x_{γ} is a value found from the ratio $\Phi(x) = \gamma - 0.5$ where

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_0^x e^{-t^2/2} dt$$

is Laplace integral.

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The System-Scenario Analysis of Global Financial Strategies

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The report deals with the problems of globalization in the world. The transformation of global financial flows and financial strategies are discussed.

The globalization is the key feature of a modern world development. The economy, external and internal policy, social life and culture of all countries put a pressure of this process. In essence, a new system of the international economic and political relations is reshaped. In this system, the modification in economics of one country results in modifications of other country economics. On the other hand, economic position of each country is resulted by state of the global economic system.

The modern world is a composite, uncertain, non-linear, non-stationary, and stochastic system characterized by the following properties [1]: any list of "entry conditions" is inexact principally; the same reason can generate unequal responses due to multi-variant events in points of a bifurcation (the rule of joker [2]). There are several approaches to study this process. The systems analysis supposes that environment will be developed uniformly and permanently. The systems analysis certainly reputes stability of a system as well, that is, the small modifications of input data result in the small modifications of performances of a system [3]. The methods of the scenario analysis [4], on the contrary, are concentrated on the impact of an environment. In [5], these two effective approaches are incorporated in a system-scenario method. The analysis of global financial flows on the basis of this method leads to the conclusion that the evolutions of the crises in Mexico (1994), in Asia (1997), in Russia and Brazil (1998), Turkey (end 2000 - beginning 2001) untwist on to the same script. The schemes of these financial strategies are discussed in detail.

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The Assessment of Ordering of the Multi-Dimentional Alternatives under the Condition of Limited Information

I.F. SHAKHNOV and S.P. MAKEEV (Computing Center of RAS, Moscow)

The present communication deals with an important applied problems of the operations research theory, the problem of the assessment of the ordering of multi-dimensional alternatives. The proposed method of the assessment of the ordering of multi-dimensional alternatives is demonstrated on the example of the problem to appraise the significance of purposes in the planning of the multipurpose operation. Let $P = [P_1, P_2, \dots, P_n]$ be a set of separate "simple" purposes. If P_i belongs to the subset P' of purposes then P_i is selected for realization, i.e., P_i is considered as an event that will certainly occur. The subset P' is called the selected entire "complex" purpose of the considered multipurpose operation. It is proposed in this study, that the purposes P_i are independent as follows: 1) realization of any purpose P_i neither facilitates nor prevents the achievement of any other purpose P_j and 2) if it complex purpose P' is more preferable than complex purpose P'' then the preference retains when the same subset P'''is added (or withdrew) to P' and to P''. It is assumed that the addition of supplementary simple purposes to a complex purpose will augment its preference.

The assumptions on the specifics of ordering of the complex purposes permit to use formal presentation of this preferences as the additive function of preference in many important cases of practical applications. During such presentation every simple purpose P_i is assigned with the value c_i called the coefficient of relative significance of the purpose P_i . The value of the preference function for a complex purpose P' is equal to the sum of coefficients of relative significance of the simple purposes included

in P'. A complex purpose P' is more preferable than a complex purpose P'' iff the value of the preference function for the purpose P' is greater than the value of this function for P''. The design of quantitative scales for the determination of quantitative values of the coefficients of relative significance is difficult especially at the initial phases of planning the operations. This is why a qualitative information is often used as a judgment of preference or equivalence of simple purposes and subsets of the set P of purposes. Formally this judgment is presented in the form of linear inequalities with respect to acceptable quantitative values of the coefficients c_i . Under such approach there will always be an uncertainty about concrete quantitative "pointwise" values of the coefficients c_i . In current publications this uncertainty is eliminated by accepting an arbitrary dot in the acceptable region or fixing a priory input functional dependence between different coefficients c_i (for example, linear, power, etc.).

In the present study, it is proposed to take the mathematical expectation of the coefficients c_i as pointwise assessment of their values, assuming the vector $c = (c_1, c_2, \ldots, c_n)$ be a random vector equidistributed in the acceptable region. A simple procedure of estimating the mathematical expectation of the coefficients c_i is described. It is based on the comparison of every purpose P_i with two next purposes P_{i+1} , P_{i+2} (the purposes are numbered in the order of the preference decrease). It is demonstrated, how the method could be applied for the case of such purposes P_i that are not only qualitatively but quantitatively determined. In conclusion it is necessary to notice that the suggested approach could be also used for the assessment of the relative significance of different limitations and for purposes' and limitations' combinations, e.g., fulfilling so called "system optimization".

Intermediation and the Poor Property Rights Protection D.V. SHAPOSHNIK (Computing Center of RAS, Moscow)

This paper studies the effect of poor property rights protection on a single product market with a set of middlemen competing by Cournot. Unlike the standard case, it will be shown that even with number of middlemen n tending to infinity there is undersupply at the market. Moreover, in some cases, regardless of number of the agents, they show the behavior as if there is a single monopolist on the market.

Let's $\Delta(Q) = D^{-1}(Q) - S^{-1}(Q)$ denote the difference between prices on wholesale and retail markets or the bid-ask spread, where $D^{-1}(\cdot)$ and $S^{-1}(\cdot)$ are the inverse demand and supply functions respectively.

The environment is taken to be characterized by a parameter α , which has the following meaning. If someone is ready to spend the amount A to force middleman *i* out of business, this middleman must pay αA in order to protect herself completely from the aggression. This money spent may be thought of as expenditures for procuring goods like guns, an alarm, etc. or as the payment to the protecting her mafia. Thus, it is easy to see that the case $\alpha = 0$ corresponds to the perfect property rights protection, and with the increase of the α , the agents become less protected. In other words, parameter α characterizes the technology of protection, available to the agents. The quality of this technology decreases with the increase of α .

of α . Now one has to determine the level of aggression an agent *i* can face with. For simplicity we assume that agents undertake no costs connected with transportation, storing, etc. Let's denote her supply on the market as q_i and the supply of the other agents as Q_{-i} . Then the gross profit of all the others will be $\Delta(Q)Q_{-i}$. If they cancel the agent *i* from the market, the total supply reduces, and their gross profit amounts to the $\Delta(Q_{-i})Q_{-i}$. By throwing the agent *i* out of the business, the others increase their profits by $Q_{-i} (\Delta(Q_{-i}) - \Delta(Q))$. So, it is namely the maximal amount of money they agree to spend on the aggression and the agent *i* has to spend $\alpha Q_{-i} (\Delta(Q_{-i}) - \Delta(Q))$ on self-protection. Therefore, her net profit will be

$$\pi(q_i) = \Delta(Q)q_i - \alpha Q_{-i} \left(\Delta(Q_{-i}) - \Delta(Q)\right).$$
(1)

Thus we have a game of n agents with target functions (1) and with q_i being the strategies.

At this stage I'd like to highlight several aspects. It would be more correct to consider a game with two-dimensional area of strategies: q_i and the defense expenditures of the agent *i*. One could also notice that after throwing an agent out of the market, the other middlemen should alter their defense expenditures in order to adjust them with a decreased number of agents. To resolve these difficulties, the model should be treated in the following way. There is an infinite number of the game periods; and if an agent is removed from the market, she returns back on the next period. The agents firstly set the quantities they supply on the market, then, once these quantities are known, they set defense expenditures. Every middleman is assumed to be perfectly risk-averse, so it is postulated that she protects herself against the worst situation, when all agents act against her. Thus this speculation leads to the game described above.

It is easy to see that at the symmetric Nash equilibrium the following formula is true:

$$\Delta'(Q)Q + \Delta(Q)\frac{1}{\frac{1-\alpha}{n} + \alpha} = 0.$$

From here it is easy to obtain the following propositions.

Proposition 1. If $\alpha = 1$, whenever number of agents is taken, the total supply will be the same as if there is only one middleman on the market.

Proposition 2. With number of agents tending to infinity, the total supply will be the same as in the case with $1/\alpha$ agents on the market and perfect property rights protection.

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Cooperative Allocation of Risk Capital

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Risk capital allocation problem occurs from the diversification effect in risk measurement of financial portfolios. Our approach is to apply cooperative game theory methods to fair allocation of full risk capital of organization to separate subportfolios (by subdivision, financial instrument, desk, etc.). Hereby the required properties of risk measure are stated. It is shown that risk allocation for Value-at-Risk (VaR) measure can be obtained in analytical form and is in accordance with widely used concept of component VaR. Also it is shown how these results can be applied in risk-adjusted performance measurement tasks.

Orbital Modular Constructions: Evaluation and Visualization of Control Processes

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The "MIR" Software Package (further referenced as "MIR") is intended for use on IBM PC Windows platform and serves for evaluation and visualization of control processes of orbital modular constructions.

"MIR" provides functionality to compute a vast spectrum of parameters, which, in particular, include: inertia tensor and mass of modular constructions, quaternions and angular velocities during orientation and stabilization processes, orbital parameters. Two main objects in "MIR" are the module and the station. Modules

Two main objects in "MIR" are the module and the station. Modules are prepared using Module Editor application. Stations are assembled from modules in Station Editor application.

It is possible to attach so-called DLAMs (Dynamic Link Algorithm Modules) to Station Editor. DLAMs are Windows dynamic libraries, which should submit to some special requirements. Fulfilling them, any developer can independently prepare a DLAM and attach it to Station

Editor. Each DLAM performs some computations by means of a particular algorithm, using parameters requested from Station editor. Results of computation are output to a datafile, which can further be loaded in Station Editor.

Datafiles are text files submitted to particular format. They could be used for visualization of station motion, and also for interpreting algorithms' output in graphic or numeric form. A datafile can be obtained by means of an output of a DLAM, separate program or by any other way.

DLAM could, for instance, evaluate an optimal turn of the station by gradient method or solve the problems of orientation and stabilization of the station involving uncertain disturbances.

Here we consider a particular algorithm of orientation of orbital modular construction involving uncertain disturbances. The algorithm was implemented as a DLAM and now works in composition with "MIR".

The motion of the coordinate basis of the moving object relative to the stationary basis is described as

$$\begin{cases} 2\lambda_0(t) = -\lambda_1(t)\omega_1(t) - \lambda_2(t)\omega_2(t) - \lambda_3(t)\omega_3(t) \\ 2\dot{\lambda}_1(t) = \lambda_0(t)\omega_1(t) + \lambda_2(t)\omega_3(t) - \lambda_3(t)\omega_2(t) \\ 2\dot{\lambda}_2(t) = \lambda_0(t)\omega_2(t) + \lambda_3(t)\omega_1(t) - \lambda_1(t)\omega_3(t) \\ 2\dot{\lambda}_3(t) = \lambda_0(t)\omega_3(t) + \lambda_1(t)\omega_2(t) - \lambda_2(t)\omega_1(t) \end{cases}$$

where $\Lambda = (\lambda_0, \lambda_1, \lambda_2, \lambda_3)$ is the normalized quaternion of the object's position, $\omega = (\omega_1, \omega_2, \omega_3)$ is the angular velocity of the object.

Adding three Euler's equations

$$\begin{cases} J_1 \dot{\omega}_1(t) + (J_2 - J_3)\omega_2(t)\omega_3(t) = u_1(t) - v_1(t) \\ J_2 \dot{\omega}_2(t) + (J_3 - J_1)\omega_3(t)\omega_1(t) = u_2(t) - v_2(t) \\ J_3 \dot{\omega}_3(t) + (J_1 - J_2)\omega_1(t)\omega_2(t) = u_3(t) - v_3(t) \end{cases}$$

we get the entire control system. Here J_i (i = 1, 2, 3) are central inertia moments; $u = (u_1, u_2, u_3), u \in P = S_{\rho}(0) \subset \mathbb{R}^3$ express total moments of engines which we can control; $v = (v_1, v_2, v_3), v \in Q = S_{\sigma}(0) \subset \mathbb{R}^3$ stands for uncertain disturbance.

The control process is considered on time interval $t \in [0, T]$. Also, we need the following two sets of boundary conditions:

$$\Lambda(0) = \Lambda_0, \quad \Lambda(T) = \Lambda_T, \quad \omega(0) = \omega_0, \quad \omega(T) = \omega_T.$$

The first set responds for the object's position, the second one responds for the object's angular velocities.

The quality criterion is presented by the following functional, describing fuel consumption, which should be less than the positive constant γ :

$$I(u(\cdot)) = \int_0^T (u_1^2(t) + u_2^2(t) + u_3^2(t)) dt \le \gamma.$$

The control u has been searched within the class of positional strategies in the sense of N. Krasovsky. The u-stable bridges are built and extremal shift strategies are used to find the control u.

Full report will contain numeric results for some test examples.

Branch and Bound Method for a Problem of Optimal Allocation of Funds S.K. SKRIPOV (Moscow State University, OR Dept.)

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Consider a problem of optimal allocation of own and borrowing funds among investment projects $j = \overline{1, m}$ of production of a science-intensive technology. Project j is given as a sequence of costs $c_t^j \ge 0$ and returns $r_t^j \ge 0$, depending on time $t = \overline{0, n}$. Under discount rate v the present value (PV) of balance of the project at time t is $B_t^j = \sum_{k=0}^{\iota} r_k v^k - \sum_{k=0}^{\iota} c_k v^k$.

Let $x_j = -\min_{0 \le t \le n} B_t^j$ be minimal funds necessary for realization of the

project j. An extended project definition includes different possible variants of realization. For each variant we can determine a minimal funds value x_j . So we get a minimal funds segment $[x_j^{min}, x_j^{max}]$. On this segment we define a piecewise constant function $f_j(x_j)$ of project's PV depending on the value of essential funds. Jumps of this function correspond to the different variants of the project j.

A customer is given by a loan with a credit flow g_t , $t = \overline{0, n}$, coming on his account. PV of the credit is $K_2 = \sum_{k:g_k>0} g_k v^k$, $g^t = \sum_{k=0}^{\iota} g_k v^k$ is a current debt. Let x_{jk} , $k = \overline{0, n}$, be a flow of credit to the project j coming from customer's account, $x_j^t = \sum_{k=0}^t x_{jk} v^k$. Let K_1 be a value of the customer's own funds. Then, the problem is to maximize the function $\sum_{i=1} f_j(x_j)$ subject to constraints

$$\sum_{j=1}^{m} x_j \le K_1 + K_2, \ x_j = x^n, \ B_j^t + x_j^t \ge 0, \ g^t - \sum_{j=1}^{m} x_j^t \ge 0 \quad \forall t, j.$$

The problem is solved with a modified branch and bound method that uses decomposition principle of fragmentation of this optimization problem to upper and lower level problems.

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On Information Structure in Games with Separate Dynamics

N.M. SLOBOZHANIN (St. Petersburg State University)

Let $\overline{N} = N \cup \{0\}, \overline{\overline{N}} = N \cup \{0\} \cup \{+\infty\}$ where N is the set of natural numbers. Let $T_{\alpha} \in \overline{\overline{N}}$. Define $\overline{T}_{\alpha} = [0, T_{\alpha}] \cap \overline{N}$. **Definition.** An ordered collection

$$\Gamma = ((X_{\alpha}, x_{\alpha 0}, D_{\alpha}, T_{\alpha}, l_{\alpha}, \Phi_{\alpha}), \alpha \in A)$$

of objects

1) X_{α} (a set), $x_{\alpha 0} \in X_{\alpha}$, 2) $D_{\alpha} : \underline{X_{\alpha}} \to 2^{X_{\alpha}} \setminus \emptyset$, 3) $T_{\alpha} \in \overline{\overline{N}}$,

4) $l_{\alpha}: \overline{T}_{\alpha} \to \prod_{\beta \in A} 2^{\overline{T}_{\beta}}$ (the ordered collection $l = (l_{\alpha}, \alpha \in A)$ satisfies the condition of information consistency as well), and

5) $\Phi_{\alpha} : \prod_{\beta \in A} X_{\beta}^{T_{\beta}} \to R$, is called an extensive form of the game with separate dynamics as to the set A.

By definition $\forall k \in \overline{T}_{\alpha}, \ l_{\alpha}(k) = (l_{\alpha}(k)_{\beta}, \beta \in A)$, where $l_{\alpha}(k)_{\beta}$ is a subset of the set \overline{T}_{β} (in particular, it may be empty). The subset $l_{\alpha}(k)_{\beta}$ consists of the numbers of such moves of the player β that are necessary and sufficient to know for the player α in order to make his own (k+1)th move. It's clear that not every ordered collection of information functions

 $l = (l_{\alpha}, \alpha \in A)$ adequately corresponds to an information structure of the game. The necessary and sufficient condition for the collection l to support the correct development of the game is called the information consistency [1]. The definition of information consistency for l was given on the base of an information solvability. In the present work we suggest another definition.

Consider the set C of ordered pairs (k, α) where $\alpha \in A$, $k \in \overline{T}_{\alpha}$. On C, determine the operator L_1 such that if k = 0 then $L_1(k, \alpha) = \emptyset$ else $L_1(k, \alpha)$ is equal to the set of ordered pairs (p, β) where $\beta \in A$, $p \in l_{\alpha}(k-1)_{\beta}$ or $(p, \beta) \equiv (k-1, \alpha)$. Let $r \in \overline{N}$. Suppose $L_0(k, \alpha) = \{(k, \alpha)\}$,

$$L_r(k,\alpha) = \bigcup_{(p,\beta)\in L_{r-1}(k,\alpha)} L_1(p,\beta), \quad r \in N.$$

Definition. The value of $\sup\{r \in \overline{N} | L_r(k, \alpha) \neq \emptyset\}$ is called the length of the algorithm of *l*-prehistory of the *k*th move of the player α .

Definition. An ordered collection of information functions $l = (l_{\alpha}, \alpha \in A)$ is called information consistent one if the algorithm length of *l*-prehistory of the *k*th move of the player α is finite for any α from A, for any k from \overline{T}_{α} .

Theorem. If $|A| < +\infty$ then the given definitions of the information consistency are equivalent.

The simulation of the information structure of the game by means of vector-functions enables to propose an algorithm of solving antagonistic dynamic games with incomplete information [2].

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Mathematical Models and Optimal Algorithms of Dynamic Data Structure Control A.V. SOKOLOV (Petrozavodsk)

This report concerns with mathematical models and algorithms of optimal stack, queue, and heap control in single- and two-level memory [1-4]. These models were constructed as 1, 2, and 3-dimentional random walks. In the report it is assumed that there are n stacks divided into pairs of stacks growing towards each other.

The problem of finding optimal initial memory distribution is reduced to choosing an optimal prism and maximizing the average time of walk inside the prism until the absorption in its border, provided the process begins from the origin. In the case of two-level memory the problem is reduced to choosing optimal initial state of the 1, 2, or 3-dimensional

random walk inside the corresponding area until the absorption in its border [5-6].

It should be noted that the task of mathematical analysis of algorithm of memory distribution for two stacks growing to each other was set by Knuth and then it was considered in a number of papers [2,7-10]. In the case of queues, Knuth determined that operating two queues in a common memory is impossible [1, ex. 2.2.2, N 17]. However, he does not consider the possibility of the travel of two queues one after the other around a circle. The algorithms as well as computional programs in C++ for this problems are developed. In the report, the results of numerical experiments are analysed.

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Characterization of Competitive Resource Allocations through the Local Bargaining

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We characterize the competitive resource allocations in economic equilibrium models in terms of the local bargaining. To this end we linearize preferences of agents at a given point and then use the linear utilities for the relevant Nash bargaining problem. An allocation is competitive if its image in the linearized utility space is a Nash bargaining solution of the corresponding bargaining problem. This problem is formed by mapping the set of feasible allocations and initial endowments of agents to the linear utility space. The bargaining powers of agents are equal to their production shares if the equilibrium allocation is an interior point in consumption sets of agents. They deviate from the shares on the boundary because of the implicit rents going from agents to the society and increase of the relative influence of agents.

We begin with formulating an abstract choice problem generalizing equilibrium models (with a unique production) and the bargaining model. Let there be given a convex set Z of publicly feasible alternatives, a statusquo point $\omega \in Z$, preferences of agents, and nonnegative numbers β_i , interpreted as rights of agents. A concept of the abstract economic equilibrium is introduced with individual prices and shares β_i . The equilibrium points are characterized now in terms of the local bargaining. To this end given a point $z \in Z$ we define a supporting linear profile 1 to preferences of agents at z and the Nash bargaining solution of the problem $\mathbf{l}(Z)$ with status-quo point $\mathbf{l}(\omega)$ and rights β_i . If $\mathbf{l}(z)$ is just the Nash bargaining solution of the problem then we call it a Nash agreement point. In particular, if a continuous choice of the linear profile is possible in Z then the corresponding point is called a Nash bargaining point in Z. We establish the existence of such points under standard assumptions.

Having got all the necessary results in the abstract setting we proceed to the applications. The general results imply that the competitive resource allocations are Nash agreement or (in smooth case) Nash bargaining points in Z. We apply these results in a resource allocation model and Arrow-Debreu model.

A New Approach for Estimation of the Risk in the **Financial Engineering**

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In the paper, the "formal scenario" approach formed of a complete Stress Testing Risk Management Framework is discussed. Based on Petri net fuzzy logic extension, the "formal scenario" methodology is used to develop the Collaborative Decision Support system for risk evaluation of corporate financial projects. We propose a methodology for Back Testing and evaluation of Stress scenarios. To support the processes of cooper-ative multicriteria choice, decision making, and What-If simulation, the expert ("brainstorming") system is introduced along with documentation for experts distributed over the Internet. The illustrative and practical examples of solving problems concerning an estimation of risk and choice of behavior strategy in the financial markets are considered.

The stability (or, on the contrary, variability) of the economic situation is defined by huge number of factors. Their influence, as a rule, cannot be expressed in a numerical mode. Therefore decision making in the business usually reminds actions of the commander on a battlefield, when it is necessary to use expert estimations. Such estimations are formulated, for example, as follows: if the combination of the factors looks like X, it can result in a situation from a class Y (rule $X \to Y$). Combination of the factors X and, accordingly, class of "interesting" situations Y can differ depending on the concrete participant of business (investor or acceptor of investments). Hence, there is a problem of processing opinions of large number of experts and developing rules similar to $\langle X \rightarrow Y \rangle$ in the interests of the concrete investor. The investor, thus, should receive the proof of serviceability of such rules.

The Evidence-analysis (E-analysis), stated in the given report, is a version of the discrete factor analysis for a case of essentially contrast nature of the factors: "Increase of the price - reduction of the price", "are favorable - are not favorable for aggressive advertising", "the law is accepted - the law is not accepted", and so on. The factors and their importance for concrete business come to light from mass media, interrogations of the population (each individual is considered as an expert), and analytical reports. All revealed factors are processed by special means in systems of business investigation. The E-analysis has original technology based on special tools of visualization of results of processing expert estimations: the compact areas (spots) according to the leading factors are allocated in pictures.

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D.C. - Programming: Theory and Algorithms A. STREKALOVSKY

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We consider the D.C.- (difference of two convex functions) minimization and optimization problems with D.C.-constraints. Convex maximization problem and respectively reverse convex problems can be viewed as particular cases of the aforesaid problems. Our approach to solving the problems is based upon the theory of Global Optimality Conditions (GOC).

The GOC are connected with classical extremum theory. On the other hand, they use the classical idea of linearization, but the linearization is applied to basic nonconvexity of the problems of interest. Further, they possess so-called Algorithmic Property (AP). This means that, if GOC are broken down at a given point, then GOC allow to construct a feasible point, which is better than the point in study.

Developing GOC, we obtain Optimality Conditions for minimizing and maximizing sequences (following the case). Furthermore, we consider theoretical Global Optimization schemes and give the convergence proofs for them. As a final point of the theoretical investigation we propose Principal Global Optimization Algorithms (PGOA) (so-called R-strategy, following the case) and give the convergence proofs for them. In addition, we develop the Special Local Search Algorithm for each problem under consideration and give the corresponding convergence theorems.

The proposed approach to solving the D.C.-programming problems was tested for several well-known problems, such as quadratic optimization over a box or a polyhedron. Also, R-strategies were applied to the following combinatorial problems: knapsack problem, maximum clique problem, quadratic assignment problem, and others. All computational experiments were rather successful. Besides, the similar approach yielded the positive results for non-convex optimal control problems.

Thus, one can say, that although the solution theory is not fully completed, the ideology is arising in nonconvex optimization, such that it is a competitor to branch and bound approach.

Computer Business Games BUSINESS COURSE

D.A. ТІМОКНОV (Moscow State University, OR Dept.)

The whole world knows that active forms of training are the most effective. The important place among them a method of computer business games occupies. Its essence is a management of an economic object, which activity is simulated by a computer.

The BUSINESS COURSE project began in 1994 by specialists of the Moscow State University. The project had the idea to create the computer business games on firm's management which meet the requirements of world standards and at the same time take into account the Russian legislation. There is complex mathematical model in the base of this business game. The model creation is based on wide experience of simulating economics processes: from micro to macro levels.

Now, there are three independent software programs being offered:

BUSINESS COURSE: Enterprise. BUSINESS COURSE: Corporation.

BUSINESS COURSE: Corporation Plus.

These programs differ in the scope of management decisions and the size of the accounting and analytical information. They offer a step-by-step development of management skills, give concrete economic knowledge, and cultivate an economic way of thinking. The programs have two editions. The individual edition (personal release) can be used for self-education and self-training in class courses. The collective edition (group release) is intended for inclass seminars guided by an instructor.

Today, there are about 40 educational institutions, including Moscow State University, Academy of National Economics, and Moscow State In-stitute of International Relations, using the BUSINESS COURSE games. The research is supported by grant "Scientific schools" N. 00-15-96141.

Genetic Algorithms for Shedule Construction that Minimizes Number of Processors A G TREKIN

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In this work, the application of genetic algorithms (GA) for a solution of the schedule construction problem with simultaneous minimization of number of processors is considered. In the report, we consider the construction and usage of GA on the example of the following task. For given history of the program behavior and directive time of the program execution, it is required to determine minimal number of processors in computing system (CS), which is necessary for the program execution

in the directive time, and to construct the schedule. The task of CS architecture synthesis in the given settings is concerned with the class of mixed problems of combinatorial and integer optimization. The optimized parameter "number of processors" belongs to a set of integers, but the parameter "schedule of program execution" belongs to a set of combinatorial structures.

combinatorial structures. In the report, the substantiation of GA usage for solving given class of optimization problems is given. The solution of the coordinated problem of parameter modification is considered permitting a guarantee to avoid deriving inadmissible variants of the schedule at all iterations of the algorithm. This is achieved using parametric modes of the schedule representation. Namely, the rations of binding and order are set by values of some parameters, on which, with usage of restoring algorithm, these rations can be univalently restored. The parametric representation admits an independent modification of all variables in the given intervals. This allows to guarantee deriving admissible schedule variants at execution of genetic algorithm operations. In the report, the outcomes of various methods of the parametric schedule representation will be shown. Also various modes of the goal function representation (fitness function for GA) are considered, e.g., linear, "smoothed", with two-sided penalty, and degree. Experiments confirm effectiveness of GA application for the problem of

Experiments confirm effectiveness of GA application for the problem of schedule construction. The values of GA parameters providing acceptable quality of the algorithm are selected. It is shown, what parameters of the initial task and what GA settings render the most essential influence on operation and convergence of the algorithm.

Binomial Model for Option Value for a Portfolio of Capital Projects under Uncertainty

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Consider a firm which compares two investment opportunities. Every capital project j requires a fixed sunk cost I_j at the start of its realization and then brings a profit $P_j(t)\Delta t$ at time $t = 0, \Delta t, 2\Delta t, ...$ Let $P_j(t)$ satisfy equation $P_j(t + \Delta t) = P_j(t) \exp(Y_j(t + \Delta t))$ where $Y_j(t + \Delta t)$ for t = $0, \Delta t, 2\Delta t, ...$ are independent random variables taking values $\pm \sigma_j \sqrt{\Delta t}$ with probabilities $0.5(1 \pm \tilde{\alpha}_j \sigma_j^{-1} \sqrt{\Delta t})$ where $\tilde{\alpha}_j = \alpha_j - 0.5\sigma^2$. Let $P_j =$ $P_j(0)$. The firm can invest to the project j immediately and get the NPV $V_j(P_j) \approx P_j/(\rho - \alpha_j)$. But it can delay an investment and wait for the greater value of the profit. The problem is to evaluate the portfolio option and to find an optimal investment decision rule. We show that it can be formulated as a free boundary problem for Hamilton-Jacobi-Bellman (HJB) equation. Dixit and Pindyck [1] solve the problem in continuous

model for a single investment project j. They have found an optimal threshold decision rule and a corresponding option value $F_j(P_j)$: to wait if $P_j < P_j^*$ and to invest if $P_j \ge P_j^*$. Thus, $F_j(P_j) = A_j P_j^{s_j}$ in the first case and $F_j(P_j) = V_j(P_j) - I_j$ in the second one. Here s_j is a positive root of characteristic equation $\frac{1}{2}\sigma_j^2 s_j^2 + \tilde{\alpha}_j s_j - \rho = 0$. We present a numerical solution of the problem for two incompatible

projects and suggest an investment strategy based on threshold functions. These functions cut a domain D in the plane of points $P = (P_1, P_2)$. If current point P(t) belongs to D then the firm delays an investment and F(P(t)) meets HJB equation. If P(t) reaches the border of D then the firm invests to one of the two projects. The optimal value F(P) and decision rule are found for certain set of threshold functions.

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Reference 1. Dixit, A.K., Pindyck, R.S. Investment Under Uncertainty. Princeton University Press: Princeton, 1994.

Tax Optimization under Tax Evasion: the Role of **Informational Asymmetries**

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Dealing with tax optimization with respect to firms, the government meets uncertainty of two kinds. First, at the time when a firm decides whether to start its activity and become a taxpayer or not, both the government and the firm share uncertainty about her future income which depends on random factors. Before reporting her income the firm finds out its value, and the first informational asymmetry occurs.

Second, the government typically does not precisely know the type of each firm, in particular, its distribution of income if it starts production, and its alternative income if it does not. So the government in general cannot propose to each firm a type-specific contract.

The known Welfare Theorem (see Atkinson, Stiglitz, 1980) relates to the case where all taxpayers are risk-neutral, their participation constraints concern the expected after tax incomes, and the government knows the type of every agent, in particular, her probabilistic distribution of income. Then the first informational asymmetry is not essential: according to the theorem, the government can reach the first best result by means of the type-specific lump-sum tax, and audit is unnecessary.

However, in practice taxpayers cannot be completely risk-neutral under arbitrary tax policy. For instance, for a firm there typically exists such threshold (dependent on her type) that if by chance the income after tax falls below this value then the firm cannot get a credit and becomes a bankrupt. The known models of optimal taxation under tax evasion take this condition into account in the form of a participation constraint. So, the income of an agent after paying the tax and the fine is supposed to be non-negative under any behavior of the taxpayer and any random outcome. In this case the optimal tax in general depends on a taxpayer's income and determination of the optimal tax policy becomes a non-trivial task.

Our first model includes the government and a group of taxpayers with equal but independent random distributions of incomes. The government knows the distribution, but does not know who has what income, while each taxpayer knows her own income. The principal may audit an agent in order to verify her income. An audit always determines a true income and has a fixed cost. The government sets a tax schedule (or pre-audit payment) dependent on the reported income, a penalty schedule (or postaudit payment) dependent on the actual and reported incomes and the probability of inspection dependent on the probability of the reported income.

We assume that every taxpayer is risk-neutral and aims to maximize her expected after tax income. Since we focus on taxation of firms and the participation constraints below prevents tax bankruptcy, the first condition seems to be not very restrictive. We consider 2 types of participation constraints.

(1) An expected income of an agent under her optimal behavior should exceed a fixed level (desirable value) which is called an alternative income.

(2) We require that under optimal behavior and the worst random outcome the income of an agent should exceed a minimal value necessary for survival of a taxpayer.

The present paper studies the tax optimization problem under different penalty constraints: a) penalty is proportional to detected unpaid tax, b) pure penalty is proportional to hidden income; c) penalty is bounded because of the given minimal income of an agent; d) the payment after audit is proportional to detected hidden income.

Our main results are as follows. While the optimal contract is always evasion proof under the penalty constraints c and d, tax evasion may be optimal for the government revenue if the pure penalty is proportional to the unpaid tax or hidden income (the variants a, b) and the proportionality coefficient is sufficiently low. For any penalty constraint a, b or d, the optimal evasion proof contract has the same structure: either the optimal tax is equal to the whole income above the minimal level for small incomes and is flat for higher incomes, or it is flat for all incomes. In the former

case the optimal audit strategy is a probabilistic cut-off rule (cf. Sanchez and Sobel, 1993): every reported income below the certain threshold is audited with the probability, which makes underreporting unprofitable, and every higher report is not audited. With respect to taxation of firms this means that the optimal tax is favorable for fortunate producers.

The second model (Marhuenda, Vasin, Vasina, 2000) considers taxation of firms with lump-sum and sales taxes under the other type of informational asymmetry between the government and agents. The government knows the production capacities of every firm and the cost distribution for every type of production capacities. Each firm chooses the total production volume and the registered amount of production. The rest is sold at the informal market for cash. In order to prevent tax evasion the government organizes tax inspection and penalizes detected tax evasion. The purpose is to characterize the optimal tax structure, tax rates, and auditing strategy that maximize net tax revenue under the given participation constraints and penalty for tax evasion.

In the general model, a firm controls several units with different production costs. First, we study a model where every firm has a unit of production capacity and a fixed cost. The latter value is unknown to the government. The difference in costs can be connected with local conditions and/or qualification of the managers or owners of the firm. In contrast to the previous model and usual setting of the tax enforcement problem, we do not consider firm specific risks and permit to exclude from the economy the firms that cannot pay taxes. We determine the optimal auditing strategy under some exogenously given tax rates. This strategy turns out to be a "cut-off" rule with respect to the registered production volume. Actually, this auditing strategy determines the optimal level of exclusion for firms with high production costs.

We show that, whenever the agents can adjust the property structure to the tax system, sales tax is unnecessary, and the optimal lump-sum tax is proportional to the production capacity of the firm. Another advantage of the lump-sum taxation is that it facilitates the redistribution of the property to the more efficient owners.

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On Endogenous Determination of Utility Functions A.A. VASIN (Neuron State University OR Det)

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Models of evolutionary game theory (EGT) were initially developed for biological populations (Maynard Smith, 1982), but they also may give use-

ful insights for social systems. One of the main differences from traditional approach is that these models determine utility functions endogenously.

The basic model of EGT, the replicator dynamics considers a population of interacting individuals with a given set S of possible strategies. For any strategy s fertility $fer_s(\pi)$ and viability $v_s(\pi)$ of individuals playing this strategy depends on the distribution $\pi = (\pi_s, s \in S)$ of a strategies in the population. The model assumes that every individual does not change his strategy during his life and new individuals inherit strategies from their parents. The evolutionary dynamics of such population is described by equations

$$\pi_s(t+1) = \pi_s(t) f_s(\pi(t)) / \sum_r \pi_r(t) f_r(\pi(t))$$
(1)

where $f_s(\pi) = fer_s(\pi) + v_s(\pi)$ is a fitness of a strategy s.

Those who survive act as if they aim to maximize their fitness which serves as an endogenous utility function under such interaction. The formal results are as follows. Theorem 1 (Bomze, 1986, Passekov, 1988). Every Lyapunov stable

Theorem 1 (Bomze, 1986, Passekov, 1988). Every Lyapunov stable distribution of the model (1) is Nash equilibrium of the population game $G = \langle S; f_s(\pi), s \in S \rangle$.

Theorem 2 (Taylor, Jonker, 1978). Every evolutionary stable strategy (in particular, strict Nash Equilibrium) of the game G is asymptotically stable for the model 1.

Theorem 3 (Vasin, 1989, Nachbar, 1990). If a strategy s is iteratively strictly dominated in the game G then the share $\pi_s(t)$ tends to 0 as t tends to infinity for any $\pi(0) > 0$.

Of course, these results strongly depend on the evolutionary mechanism. If we consider a model where a new individual follows a strategy of a randomly chosen adult then viability turns out to be an endogenous payoff function of individuals in the corresponding dynamical process.

The following model of evolutionary mechanism selection restores priority of the fitness function. Consider a society including several populations with the same set of strategies, fertility and viability functions dependent on the total distribution over strategies, but with different evolutionary mechanisms. For each population such mechanism determines dynamics of the distribution over strategies in this population. For instance, one population develops according to the replicator dynamics. Another one develops according to the random imitation and so on.

Consider the total distribution $\pi(t)$ over strategies in the society. The collection of evolutionary mechanisms determines dynamics of this distribution.

Theorem 4. Let there exists at list one population with the mechanism of the replicator type. Then if $\pi(t)$ tends to π as t tends to infinity

or π is Lyapunov stable then π is a Nash equilibrium for the game where fitness is a payoff function. Under the same conditions, every strict equilibrium of the game is an asymptotically stable distribution of the dynamical model. Moreover, if the set of evolutionary mechanisms is sufficiently rich in some sense then for every iteratively strictly dominant strategy of the game its share tends to 0 as t tends to infinity.

Conventional opinion of biologists is that behavior of many species does not contradict with fitness maximization (see materials of XII and XIII Congresses of the ESEB). Several well known exceptions, in particular, cooperation and altruism in families of ants and bees may be explained (Maynard Smith, 1982, Vasin, 1991) as fitness maximization for the whole family. However, in such case the free rider problem arises.

As to human behavior in the modern societies, it typically does not correspond to fitness maximization neither for an individual, nor for his family or population. Though fitness maximization does not contradict with the general concept of the utility function optimization adopted in economics. Reproduction requires some goods and a free time to grow up the offspring. The contradiction occurs when utility is treated as satisfaction of consumption.

Besides the concept of individual natural selection, an alternative approach to behavior evolution in biological systems considers animal interactions as a game of macroplayers: biological populations or societies (see Odum, 1986). In the previous example the ant family may be considered as a macroplayer which suppresses reproduction of some individuals but supports the whole population. In other cases one macroplayer (a population of predators) uses another one (a population of prays) as a resource for reproduction. The cheapest way to do it is to modify the prays utility function. There exist several interesting examples of such modification.

I think that models based on the concept of macroplayers impact on individual utility functions may be useful for understanding behavior dynamics in human populations. Such approach may be especially important for transitional economies where the radical change of environment creates favorable conditions for manipulation with individual behavior.

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A Model of Tax Enforcement Optimization with Two Possible Levels of Income POLINA VASINA

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In the known models of tax optimization (see Chander, Wilde, 1998) the optimal tax enforcement strategy is evasion proof. However, these models consider penalty constraints that are not used in practice. Our purpose is to find out whether tax evasion may be optimal for the government under constraints corresponding to the actual legislation.

The model. Taxpayers get a high income I_H with probability q and low income I_L with probability 1 - q. The government strategy includes taxes T_H and T_L on these incomes, a probability p of auditing low income reports and a penalty F on tax evasion. An audit reveals true income and costs c. Denote $\Delta T \stackrel{def}{=} T_H - T_L > 0$, $\Delta I \stackrel{def}{=} I_H - I_L$. Under the high income taxpayer's strategy is her report $I_r \in \{I_H, I_L\}$. Since a taxpayer maximizes her expected income under a given government strategy,

$$I_r = \begin{cases} I_L, \ pF < \Delta T, \\ I_H, \ pF \ge \Delta T. \end{cases}$$

The government aims to maximize expected net tax revenue, so the formal problem is to find

$$(T_L, \Delta T, F, p) \leftrightarrow \max R$$

where $R = T_L + q\Delta T - p(1-q)c$ if $pF > \Delta T$, otherwise $R = T_L + qpF - pc$, subject to the following participation constraint:

$$I_L + q\Delta I - T_L - q\min\{pF, \Delta T\} \ge I_{alt},$$

that is, the expected income of a taxpayer should exceed his alternative income (some desirable level of taxpayer's expected income);

$$I_L - T_L \ge I_m$$
, $I_H - T_H \ge I_m$ if $I_r = I_H$, otherwise $I_H - T_L - F \ge I_m$, (1)

that is, under the optimal behavior and the worst random outcome the income of a taxpayer should exceed the value $I_m(\langle I_{alt})$ necessary for "survival" of a taxpayer.

The four variants of penalty constraints we consider in the model are a) $F = (1 + \delta_a)\Delta T$ (penalty proportional to detected unpaid tax);

b) $F = \Delta T + \delta_b \Delta I$ (pure penalty is proportional to hidden income);

c) $I_L + \Delta I - T_L - \max(\Delta T, F) \ge I$ (penalty is bounded because of the given minimal income of an agent);

d) $F = \delta_d \Delta I$ (the payment is proportional to detected hidden income).

For the cases c) and d) Chander and Wilde show that the optimal strategy of the tax authority always implies the honest behavior of tax-payers.

Our results are as follows. In case of penalty constraints a) or b) which are used in practice, tax evasion may be optimal for the government. This

may happen if the penalty for evasion is sufficiently soft and the difference between the alernative and minimal incomes is a little less than $q\Delta I$.

Proposition. First, consider the case where $c < q\Delta I$. Then for any fixed $c, q, \Delta I$, and δ_a , there exist three variants of the optimal government strategy dependent on the difference $I_{alt} - I_m$. If

$$q\Delta I \leq I_{alt} - I_m$$

then the optimal strategy is to set lump sum tax $T_L = \Delta EI$ (= $I_L + q\Delta I - I_{alt}$), $\Delta T = 0$, p = 0.

In the area

$$q\Delta I(1 - \frac{1-q}{1+\delta}) < I_{alt} - I_m < q\Delta I$$

the optimal strategy includes: $T_L = T_{LM} (= I_L - I_m)$, the additional tax $\Delta T = \Delta I p_a^*$ (hence, $F = \Delta I$), the audit probability $p = \frac{\Delta E I - T_{LM}}{q\Delta I} (= 1 - \frac{I_{alt} - I_m}{q\Delta I}) < p_a^* = 1/(1+\delta)$, so tax evasion is optimal for the government. The gross revenue is $\Delta E I$ [that is maximal under (1)], but the net revenue is $R = \Delta E I - pc$.

In the rest area

$$0 < I_{alt} - I_m \le q\Delta I \left(1 - \frac{1-q}{1+\delta}\right) \tag{2}$$

the optimal strategy is $(T_L = T_{LM}, p = p_a^*, \Delta T = (\Delta EI - T_{LM})/q)$. Honest behavior of taxpayers is optimal, and the gross revenue is again ΔEI while the net revenue is $R = \Delta EI - (1-q)p_a^*c$.

Now consider the case of $c(1-q)p_a^* < q\Delta I \leq c$.

In the area

$$q\Delta I - c(1-q)p_a^* \le I_{alt} - I_m < q\Delta I$$

the optimal strategy is $(T_L = T_{LM}, p = 0, \Delta T = 0)$. The gross and net revenue under this strategy is $R = T_{LM} < \Delta EI$.

In the area

$$0 \le I_{alt} - I_m \le c(1-q)p_a^*$$

the optimal strategy is $(T_L = T_{LM}, p = p_a^*, \Delta T = (\Delta EI - T_{LM})/q)$ as in the area (2) under $c < q\Delta I$.

Finally, under $q\Delta I \leq c(1-q)p_a^*$ lump sum tax provides the maximal revenue. If $I_{alt} - I_m < q\Delta I$ then the optimal strategy is $(T_L = T_{LM}, p = 0, \Delta T = 0)$.

A similar proposition holds under penalty constraint b.

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