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ABSTRACTS

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SECTIONS

Plenary: Plenary session
MDM: Multiple objective decision making
S: Social and political decision making
E: OR in economics
F: OR in financial engineering
RF: Financial engineering in Russia (computational intelligence)
O: Optimization methods in OR (nonlinear case)
LP: Linear programming
LSO: Large-scale optimization
N: Networks and data processing
G: Game theory
M: OR models
A: Applications

Generalization of VaR-Criteria on Option Market

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The work deals with the problem of optimal behavior of an investor at the one-period option market with his own view on the market properties. We suppose that the market is risk-neutral and the risk-free rate is equal to 0. Moreover, continuous strikes are quoted on the market. We assume the difference between investor's and market probability functions of future prices of underlier. In this case, the investor could gain in the average income. If the investor is guided by the popular value-at-risk criteria (VaR-criteria) in the traditional form then the results for the full market prove to be absurd.

The VaR-criteria implies the maximization of the average income under the condition that the investor's income is greater than a critical level. The point is that the critical level is relatively low. Thus, the investor receives a low income with the probability equal to 1. Besides, investor's income contains a singular component that guarantees the maximum of average income. All that could hardly be his or her initial desire.

A modified continuous version of VaR-criteria is introduced. It reflects market players' preferences more precisely and is free of the indicated shortcoming. In order to use this method in practice it is necessary to use a multistage version of VaR-criteria. We consider an increasing function B of critical incomes on the segment of critical probability levels [0,1]. We check that the VaR-criteria is fulfilled for every point of this segment starting from the point zero as far as possible. To solve this problem we form the likelihood ratio of market-to-investor probability densities and use the Neumann-Pearson statistic criteria. Thus, we find the system of sets X on $\mathbf{R}$ (the set of all real numbers) with certain optimal property. Then we consider instruments $G[X]$ with the payoff profiles equal to the indicator function of X. These are the blocks of optimal portfolio of the investor.

If the initial investment amount A is given, we could determine whether the problem may be solved completely or partly. We calculate a function $a(p)$ on the segment [0,1] that shows how much of investment resources are needed to solve the problem right up to an arbitrary point p at the segment [0,1]. If and only if A exceeds $a(1)$, we can solve the problem completely. If so, we have to direct the residual $A - a(1)$ towards maximizing the average income. We provide it by investing the residual to the single point. This is the point of the likelihood ratio minimum. Again, the income component proceeding from the residual investment is singular. To avoid this the investor has to choose certain parameters of the function B to satisfy the equality $A = a(1)$.

An example of two-sided exponential probability distribution with different parameters for the investor and the market and the power function of critical income levels, depending on two parameters, demonstrates the peculiarities of constructions proposed.

Guarantee of Survivability of Symmetrical Hierarchical Networks

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A multicommodity flow network [1] is called symmetrical hierarchical if its logical graph has the structure of a star (i.e., the source-sink pairs are given in the form (v_0, v_i) , $i \in M \stackrel{def}{=} \{1, \ldots, m\}$, with the common source v_0) and all the demands are equal to d $\forall i \in M$.

It is known that the physical structure of a star possesses poor properties of survivability. Therefore in this work, we suggest to create an additional circular structure connecting all the sinks. Thus we meet the aim of reserving a connection in the case of losing edges between v_0 and one or more v_i , $i \in M$.

In this paper, a formula is given for calculating the reserve value which is necessary and sufficient to guarantee the level $\theta^g_\gamma(d, c) = 1 - \gamma$ $\forall \gamma \in [0, 1]$. Here γ is a parameter which characterizes the power of the network destruction. The function $\theta^g_\gamma(d, c)$ denotes the guaranteed level of demand satisfaction [2] depending on the reserve capacity c and demands d.

In this case $\theta^g_\gamma(d, c)$ is defined as follows:

$$\theta^g_\gamma(d, c) = \min_{y \in Y_\gamma(d)} \max_{z \in Z(y,c)} \min_{i \in M} z_i / d$$

where $Y_\gamma(d) = \{y \geq 0 | \sum_{i=1}^{m} y_i = (1 - \gamma) md \ , \ y_i \leq d \quad \forall i \in M\}$ and $Z(y, c)$ is the set of all feasible multiflows $z = (z_1, \ldots, z_m)$ in the network with circular reserve $(c, \ldots, c)$ and capacity vector y of radial edges. The multiflow $z \geq 0$ is feasible if there exists the flow $f \geq 0$ such that z and f satisfy the condition of the flow conservation in transit nodes and f satisfies the capacity constraints.

The flow conservation conditions for the hierarchical network are

$$\begin{cases} \displaystyle\sum_{i=1}^{m} f_i^j = z_j & \forall j \in M, \\ f^j_{m+i+1} + f^j_{2m+i} - (f^j_i + f^j_{m+i} + f^j_{2m+i+1}) = 0 & \forall j \in M, \ i \notin \{j, m\}, \\ f^j_{m+1} + f^j_{3m} - (f^j_m + f^j_{2m} + f^j_{2m+1}) = 0 & \forall j \in M \setminus \{m\}. \end{cases}$$

Here the variables f_i^j denote the share of flow z_j between v_0 and v_j which passes by edge (v_0, v_i). The variables f_{m+i}^j and f_{2m+i}^j denote the shares of the flow z_j between v_0 and v_j that passe by the edge (v_{i-1}, v_i) clockwise and counter-clockwise respectively.

The capacity constraints for the hierarchical network are as follows:

$$\begin{cases} \sum_{j=1}^{m} f_i^j \leq y_i & \forall i \in M, \\ \sum_{j=1}^{m} (f_{m+i}^j + f_{2m+i}^j) \leq c & \forall i \in M. \end{cases}$$

Theorem: $\min\{c | \ \theta_\gamma^g(d, c) = 1 - \gamma\} =$

$$\begin{cases} (d(1-\gamma)(m - \lceil m(1-\gamma) \rceil))/2 & \text{if} \quad (1-\gamma)m - \lfloor (1-\gamma)m \rfloor > 1 - \gamma, \\ (\lfloor (1-\gamma)m \rfloor d\gamma)/2 & \text{if} \quad (1-\gamma)m - \lfloor (1-\gamma)m \rfloor \leq 1 - \gamma. \end{cases}$$

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Optimization of Transfer Distribution

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We consider a problem of optimal procedure for allocation of money means from certain regions (donors) and their transference to other ones being in need of assistance. Budget security of a region is determined by ratio of returns D_r collecting on its territory to requirements R_r of budget consuming branches. The allocation of means is carried out in order to maximize the budget security of the region, which is provided worse than others, having in mind restrictions for deduction from regions-donors.

The result effectiveness is determined by effectivenesses for the separate regions as

$$f = \min_r f_r, \qquad f_r = \frac{D_r(1-x_r)+Y_r}{R_r}.$$

It is required to find

$$f^0 = \max_{x_r,\, Y_r} f$$

subject to the constraints

$$Y_r \geq 0, \quad 0 \leq x_r \leq \bar{x}_r, \quad \sum_r D_r x_r = \sum_r Y_r,$$

where $D_r > 0$, $R_r > 0$, $0 \leq \bar{x}_r < 1$.

For finding a solution we use the transition to new variables and the principle of equalization. In general the solution is non-unique. To receive unique solution it is naturally to use the condition of absence of opposite money streams. It is possible to introduce the condition $\min\{x_r, D_r\} = 0$, then the solution will be unique. The optimal value f^0 is determined by the equation

$$B = \sum_r \max\{AR_r,\, D_r(1-\bar{x}_r)\}. \tag{1}$$

If $B = \sum_r D_r$ then $A = f^0$. The optimal values x_r^0, Y_r^0 are determined from the formulae

$$\begin{array}{lll}
x_r^0 = \bar{x}_r, & Y_r^0 = 0 & \text{if} \ \ f^0 \leq D_r(1-\bar{x}_r)/R_r, \\
x_r^0 = 1 - f^0 \frac{R_r}{D_r}, & Y_r^0 = 0 & \text{if} \ \ (1-\bar{x}_r)\frac{D_r}{R_r} \leq f^0 \leq \frac{D_r}{R_r}, \\
x_r^0 = 0, & Y_r^0 = f^0 R_r - D_r & \text{if} \ \ D_r/R_r \leq f^0.
\end{array}$$

The function (1) B of A is continuous and increasing. The solution is obtained in analytical form, generally implicit. Numerical search is accomplished in one-dimensional space. The direction of movement to the extremum is known at every step. We find the global extremum. A numerical method results in the exact solution.

If $\sum_r D_r / \sum_r R_r \geq \max_r D_r(1-\bar{x}_r)/R_r$, we have the explicit solution

$$f^0 = \sum_r D_r / \sum_r R_r, \; x_r^0 = \max\{0,\, 1 - f^0 \frac{R_r}{D_r}\}, \; Y_r^0 = \max\{0,\, f^0 R_r - D_r\}.$$

Guarantee Fund of Futures Exchange: the Sufficiency Estimation with the Aid of Kinetic Models

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Under conditions of high volatility (which are inherent for emerging markets), the questions of credit risk, arising in use of derivatives, become very important. One of such questions is indemnifying the risk of the insolvency of trade participants in the case of prompt changes of market conditions (i.e., drastic prices movements). The most widespread way of minimizing similar risks in futures exchange is to create the guarantee fund, for example, formed from the entrance fees of exchange members or other sources. Methods for estimating the sufficiency of the guarantee fund size were supplied by V.E. Kuznetsov in [1]. In the present paper, the approach based on application of a kinetic model for exchange trade is offered. Similar models are mathematical objects used for the description of nonequilibrium processes of evolution of macroscopical systems [2] with pair interactions.

Attempts to apply kinetic models in economics were undertaken earlier. In particular, many researchers used Boltzmann equation whose physical analogue is widely applied in many problems. However, in the computer calculations there arise some artifacts of digitalization, so-called "superfluous" invariants, and also the equilibrium conditions appropriate to them, which are absent in the continuous case, i.e., the obtained solutions have no analogies in reality. This situation or, more precisely, its misunderstanding by researchers did not allow to use the kinetic approach in economics effectively. Recently by V.V. Vedenyapin, the procedure of constructing discrete models without superfluous invariants was offered. It opens the road to the further use of the kinetic approach in economics.

Let us consider an exchange that trades a contract of one type only (i.e., there is one type of underlying asset with a fixed date of contract expiration). Under some assumptions we can obtain the distribution function for the participants in their position and amount of cash. We can use this distribution function to estimate the risk of participant default, which can not be covered by guarantee fund, i.e., the credit risk and actual efficiency of our hedge operations. It is possible to generalize the kinetic models for the instruments of other organized markets, such as deposits, currencies or securities. The approach can be naturally extended for the case of many instruments. Extending for option trading is very easy: we simply treat option positions of sellers as short (for put options) or long (for call options) positions in underlying futures contracts. The next order of accuracy for this case is concerned with usage of pay-off profiles for short option positions. These profiles can be theoretically determined by various option pricing models.

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Tax Policy Influence on Investment under Uncertainty

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The paper proposes a model which allows to study an influence of corporate income taxation on investor's behavior. A behavior of investor is assumed rational, i.e., he chooses an optimal timing for investment (stopping moment) depending on observations for economic environment.

It is considered a project of creation of a new enterprise (in production) as an object of investment. The most important feature of the model is the assumption that at any moment the investor can either *accept* the project and start with the investment or *delay* the decision before obtaining new information on its environment (prices of the products, demand, etc.).

Investments I_τ, necessary for launching the project at the moment τ are considered to be instantaneous and irreversible (sunk costs).

Let the gross income from the project at time t (after investing at time τ) be $(x_t^\tau,\ t \geq \tau)$ and production cost at this moment equal to $y_t^\tau + D_t^\tau$, where y_t^τ is material cost (including both wages and allowable taxes) and D_t^τ is depreciation charge. If γ denotes corporate income tax rate, net after tax cash flow of investor at moment t equals

$$x_t^\tau - y_t^\tau - \gamma(x_t^\tau - y_t^\tau - D_t^\tau) = (1-\gamma)\pi_t^\tau + \gamma D_t^\tau,$$

where $\pi_t^\tau = x_t^\tau - y_t^\tau$.

Economic environment can be influenced by different stochastic factors (uncertainty in market prices, demand, etc.). For this reason we consider that the "profits" $(\pi_t^\tau,\ t \geq \tau,\ \tau \geq 0)$ are described by a family of random processes, defined in some probability space $(\Omega, F, \mathbf{P})$ with the flow of σ-fields $\mathcal{F} = (\mathcal{F}_t,\ t \geq 0)$, and random variables π_t^τ are supposed to be $\mathcal{F}_t$-measurable. $\mathcal{F}_t$ can be considered as available information on the system up to the moment t. Besides, the amounts of necessary investment

$(I_\tau,\ \tau \geq 0)$ are also described by a random process, adapted with the flow of σ-fields $\mathcal{F}$.

According to current Russian laws newly created enterprises are exempted from corporate income tax for some time after its creation (tax holidays). Let ν be a duration of such tax holidays.

Let the project be launched at the moment τ.

The expected present value of investor from the project discounted to the investment moment is written by the following formula

$$V_\tau = \mathbf{E}\left(\int\limits_{\tau}^{\tau+\nu} \pi_t^\tau e^{-\rho(t-\tau)}\, dt + \int\limits_{\tau+\nu}^{\infty} [(1-\gamma)\pi_t^\tau + \gamma D_t^\tau] e^{-\rho(t-\tau)}\, dt \,\middle|\, \mathcal{F}_\tau \right),$$

where ρ is the discount rate, and the notation $\mathbf{E}(\cdot|\mathcal{F}_\tau)$ stands for the conditional expectation provided the information about the system until the moment τ.

The purpose of the investor is to find a moment for investment (investment rule), which depends on previous (but not future) observations of the environment, so that its net present value (NPV) will be maximal within given tax system, i.e.,

$$\mathbf{E}\,(V_\tau - I_\tau)\, e^{-\rho\tau} \to \max_\tau,$$

where maximum is considered over all Markovian moments (regarded to the flow of σ-fields $\mathcal{F}$) τ ("investment rules").

We assume that the amounts of necessary investment are described by the process of geometric Brownian motion

$$I_t = I_0 + \int_0^t I_s(\alpha_1\, ds + \sigma_1\, dw_s^1),$$

and investor's profits flow is described by the following family of stochastic equations (with diffusion and jump components) depending on parameter τ:

$$\pi_t^\tau = \pi_\tau + \int_\tau^t \pi_s^\tau(\alpha_2\, ds + \sigma_2\, dw_s^2) - \sum_{j=1}^{N_t^\tau} \xi_j \pi_{\theta_j - 0}^\tau, \quad t \geq \tau,$$

where π_τ is $\mathcal{F}_\tau$-measurable random variable , α_i, σ_i $(i = 1,2)$ are real numbers $(\sigma_i \geq 0,\ i = 1,2)$, and $(w_s^i,\ s \geq 0,\ i = 1,2)$ are standard Wiener processes, $(\theta_j,\ j \geq 1)$ are the moments of jumps, $N_t^\tau = \#\{j:\ \theta_j \leq t\}$ (the number of jumps up by t) is Poisson process with parameter λ, $(\xi_j,\ j \geq 1)$ are shares of profits' losses (independent random variables with the same mean $q = \mathbf{E}\theta_j$). These jumps model the process of institutional risks. The process $(\pi_\tau,\ t \geq 0)$ is also specified as geometric Brownian motion

$$\pi_t = \pi_0 + \int_0^t \pi_s(\alpha_2\, ds + \sigma_2\, dw_s^2), \quad t \geq 0,$$

with the same parameters as in (π_t^τ) and given initial state π_0. Such assumptions are typical in financial stochastic models. It is suggested that Wiener processes w_t^1 and w_t^2 are mutually correlated, and $\mathbf{E}(dw_t^1 dw_t^2) = r dt$.

Depreciation charges are represented as $D_t^\tau = \psi I_\tau a_{t-\tau}$ where ψ is a given share of initial depreciation base in total amount of investment, and $(a_t,\ t \geq 0)$ is depreciation density.

It is proved that if parameters of profits and investments processes satisfy the relation $\alpha_2 - \frac{1}{2}\sigma_2^2 > \alpha_1 - \frac{1}{2}\sigma_1^1$, then optimal moment for investment τ^* is the following:

$$\tau^* = \min\left\{t \geq 0 :\ \pi_t \geq p^* I_t\right\}, \quad p^* = \frac{\beta}{\beta - 1} \cdot \frac{\rho + \lambda q - \alpha_2}{1 - \hat{\gamma}}(1 - \gamma\psi A)$$

where

$$\hat{\gamma} = \gamma e^{-(\rho + \lambda q - \alpha_2)\nu}, \quad A = \int_\nu^\infty a_t e^{-\rho t}\, dt,$$

and β be a positive root of the quadratic equation

$\frac{1}{2}(\sigma_1^2 + \sigma_2^2 - 2r\sigma_1\sigma_2)\beta(\beta - 1) + (\alpha_2 - \alpha_1)\beta - \rho + \alpha_1 = 0.$

The optimal threshold p^* for the ratio of the processes π_t/I_t can be considered as one of the main indexes of investment activity. It allows us to obtain explicit formulas for some economic indexes associated with the investment project, such as investor's expected net present income, expected tax payments from the project into the budget, expected duration of investment waiting. Using these formulas we study an influence of different factors (most of all tax system parameters) on investment activity.

In particular, it turned out that a general type of dependence of optimal threshold p^* on tax holidays ν is defined by a sign of the value $\Delta = \mathbf{E}(D_{\tau^*+\nu}^{\tau^*} - \dfrac{\beta - 1}{\beta}\pi_{\tau^*+\nu}^{\tau^*} \mid \mathcal{F}_{\tau^*})$, i.e. by a relation between predicted amounts of depreciation charges and profits at the end of tax holidays $\tau^* + \nu$. If profit is large enough, and $\Delta < 0$, then increase in tax holidays leads to decrease in threshold p^*, and, therefore, encourages investment. However, if depreciation is large ($\Delta > 0$), we have enough surprising result: threshold p^* increases in ν and, hence, increasing tax holidays implies increasing time of investor's entry, i.e. discourages investment.

As for a dependence of optimal investment threshold on depreciation rate, we research two main depreciation methods: straight-line and declining-balance. It is established that if depreciation rate does not exceed a certain "critical" level (specified by tax holidays and discount rate) then increasing depreciation rate implies decrease in threshold p^* and, therefore, earlier investor's entry. Whenever depreciation rate is high enough (exceeds critical level) then increasing depreciation rate has an opposite effect and discourage investment.

The analogous results are valid for expected net present incomes from the project under optimal investor's behavior.

Thus, the mutual usage of tax holidays and accelerated depreciation can lead to emergent effects that influence negatively on investment activity.

Investigation of the above problem turns into solving an optimal stopping problem for two-dimensional geometric Brownian motion. We apply Feynman-Kac formula and variational inequalities as basic methods for solving this problem.

The proposed model of investor's behavior is connected with real option theory and develops the known McDonald-Siegel model (see [1] and [2]). We use also the paper [3], there they state the precise conditions for optimal stopping problem with a difference of two geometric Brownian motions.

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The Model of Industry under Working Capital Deficit

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A generalization of Houthakker - Johansen model is considered. The model takes into account an influence of working capital on the production industry work. Some average characteristics of enterprise work are calculated. There are output, input, liquidity actives, and debt among these average characteristics. The models of industry where the financial resources are controlled by one of the following agents are proposed: labor collective, commercial bank or owner. In these models working capital is deficit. It is shown that the deficit of credit may cause in the non-efficient distribution of production factors in the industry.

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A Model for the Vortex Forecasting System Performance

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The ICAO rules give the minimal separation between aircrafts to guarantee their full safety. The Vortex Forecasting System (VFS) gives the possibility of estimating the minimal separation for the given aircraft pair the leader - the follower under the current conditions. The latter separation is often less then the former one. Thence, the VFS could increase airport capacity.

The proposed model estimates the runway capacity in Instrumental Flight Rules. All the aircrafts fly before landing in the airport area with the radius R from an echelon and the common speed v^*. Their deceleration fill the landing speed v_i for aircraft of type i starts along the glidepath. There are several windows along the glidepath. We estimate for each of the windows the time when the window is free from wake vortices taking into account the current wind, temperature, etc. Simultaneously the problem of the danger zone evaluating for the follower j with respect to the leader i is being solved. As a result, we have the safe time separation $\bar{t}_{ij}$, which determines the safe distance d^*_{ij} between the aircraft i and j, as well as the safe time interval t_{ij} between their landing moments.

Let p_i be the landing frequency of an aircraft of type i. We assume that the service period of aircraft is a random quantity which value t_{ij} has the probability $p_i p_j$. All aircraft arrive according to the Poisson law with the parameter λ (the average number of the aircraft arriving in the time unit). Let the probability of the interrupted landing be p^0. Then, on the basis of the quantities d^*_{ij} and R, we can calculate the maximum value λ^* of the parameter λ that guarantees the aircraft landing echelon be less than R with the probability not more than p^0. At last, we compare the averaged (with respect to the airport wind rose) value λ^* with the runway capacity corresponding the ICAO standards.

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Construction of the Optimal Surrender Charge System for Endowment Zero Loading Policy

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The n-years endowment policy with one unit payable at maturity or in the case of death (at the end of the year of death) is considered. It is assumed that in the case of surrender the insurer has a right to charge the amount of reserve in order to cover its initial costs. From another point of view the construction of flexible surrender charge system allows to control policy-holder's behavior and stimulates prolongation (cancellation) of a policy.

Assume that in the case of policy cancellation the charged amount of policy reserve, the surrender value, is payable at the end of the policy cancellation year. Thus from the simulation point of view, the maturity of the policy at n^{th} year is equivalent to surrender at this year; and the charge at n^{th} policy year should be less than or equal to zero. The negative charge means that at certain policy years the insurer increases the amount of surrender value related to usual profit participation system when a certain amount of excess interest (above guaranteed value) is distributed to policy-holder.

Let i be guaranteed interest rate,

i_1 be interest rate of policyholder that he can receive without the usage of insurance (e.g., in a bank),

and i_2 be actual interest rate of the insurer.

Let assume that interest rates i, i_1, and i_2 are fixed; and $i_2 > i_1 > i$. Annual net premium is calculated by the following formula:

$$NP = \frac{A^1_{x:\overline{n|}} + {}_nE_x}{\ddot{a}_{x:\overline{n|}}}.$$

The large amount of this premium is related to investment part of the policy whereas risk component connected to benefits payable in the case of death is comparatively low. Therefore let only take into account the investment part of the policy with premium P derived from the equation

$$NP = \frac{v^n}{\ddot{a}_{\overline{n|}}} + \frac{A^1_{x:\overline{n|}} - {}_nq_x * v^n}{\ddot{a}_{\overline{n|}}} + \frac{\ddot{a}_{\overline{n|}} - \ddot{a}_{x:\overline{n|}}}{\ddot{a}_{\overline{n|}}} * NP.$$

In other words $P = \dfrac{v^n}{\ddot{a}_{\overline{n|}}} = \dfrac{v^n * (1-v)}{(1-v^n)}$.

The amount of reserve for such policy is defined by the formula

$${}_tV = v^{n-t} - P * \ddot{a}_{\overline{n-t|}} = \frac{v^{n-t} - v^n}{1 - v^n}, \quad t = \overline{1, n}.$$

Let n be the term of the policy,

x_i be the charge at i^{th} policy year, $i = \overline{1, n}$,

and y_i be the policy-holder behavior at i^{th} policy year. If $y_i = 1$ then policy-holder cancels the contract at this year, if $y_i = 0$ then there is no cancellation at this year.

Let define goal functions of the insurer and the policy-holder as net present value of their in- and outflows, respectively. Thus

$$F(x, y) = \sum_{t=0}^{n-1} P * v_2^t * \prod_{j=1}^{t} (1 - y_j) - \sum_{t=1}^{n} {}_tV * (1 - x_t) * v_2^t * y_t$$

is the goal function of the insurer and

$$G(x, y) = -\sum_{t=0}^{n-1} P * v_1^t * \prod_{j=1}^{t} (1 - y_j) + \sum_{t=1}^{n} {}_tV * (1 - x_t) * v_1^t * y_t$$

is the goal function of the policy-holder, where

$$x \in X = \Big\{ (x_1, \ldots, x_n) \; \Big| \; x_i \le 1, \;\; i = \overline{1, n-1}, \;\; x_n \le 0 \Big\},$$

$$y \in Y = \left\{ (y_1, \ldots, y_n) \; \middle| \; \sum_{t=1}^{n} y_t = 1, \;\; y_t \in \{0, 1\}, \;\; t = \overline{1, n} \right\}.$$

The aim of the insurer is to construct the optimal charge system, which maximize its guaranteed result,

$$X^* = \operatorname*{Argmax}_{x \in X} \min_{y \in Y(x)} F(x, y), \;\; Y(x) = \operatorname*{Argmax}_{y \in Y} G(x, y).$$

Throughout charge systems we are interested in those systems which charge values are minimal, for example,

$$X^{**} = \operatorname*{Argmin}_{x \in X^*} \sum_{t=1}^{n} x_i.$$

It is proved that the optimal surrender charge system, which satisfies the aforesaid conditions, can be calculated by the following formula:

$$x_t^{**} = \begin{cases} 1 - \dfrac{P * (v_1 - v_1^t)}{{}_tV * v_1^t * (1 - v_1)}, & \Delta B < 0, \\[2ex] 1 - \dfrac{v_1^n - P * \dfrac{v_1^t - v_1^n}{1 - v_1}}{{}_tV * v_1^t}, & \Delta B \ge 0, \end{cases}$$

where

$$\Delta B = -P * \frac{v_1 - v_1^n}{1 - v_1} + v_1^n = v_1^n - P * (\ddot{a}_{\overline{n|}} - 1).$$

To make an example, the system of surrender charges for cases with different signs of ΔB are presented in the following table:

$n = 10, \;\; i = 4\%,$

1. $i_1 = 5\%, \;\; \Delta B = 0,045 > 0,$
2. $i_1 = 9\%, \;\; \Delta B = -0,058 < 0,$

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
0,436	0,215	0,138	0,096	0,070	0,050	0,034	0,021	0,010	0
1	0,486	0,298	0,190	0,115	0,054	0,001	−0,046	−0,092	−0,136

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New Optimization Problems for Large Socio-Economical Models

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There exist many operations research problems in different fields of science and technology. On the contemporary stage, the social-economical processes require the selection of optimal scenarios among the possible ones. Therefore, the large socio-economical systems are among new fields, where such problems are emerging. For these fields we have used the new models with associative memory [1]. Now we consider one of the first results in formulating and solving a problem of optimal control for models of Makarenko. The theorems of existence are proved for optimal processes in discrete dynamical systems similar to Neural Networks.

The direct applications may be found in time-dependent problems of economical competition. One of the concrete realizations is the problem of syntheses of Hierarchical Neural Networks structures [2]. Second type of the problems may be found in geopolitics and geoeconomics.

Now in the global World all areas of human affairs are exposed to rethinking and reconstruction. One of such fields of interest is the field of international relations, solutions of conflicts, and security of countries. This field of activity attracts great attention and efforts. Such problems should be solved in current practice of governments, diplomats, businessman, international organizations, and so on. From the antiquity, such questions have been considered by philosophies. As a scientific discipline international relations have been developed since 20th century by many researches. In proposed report we consider further development of new approaches to geopolitical and regional problems described earlier.

The background for the approach is a new neural network model for geopolitical relations. The states of regional assemblies of countries are considered as steady states of some models with associative memory. Further development allows to consider the geopolitical and geoeconomical problems together. Such objects as civilization, the border between civilizations introduced explicitly by C. Chantington, and the ideas by Z. Bzesinsky can obtain strict definitions and modeling concepts. New game-theory models and optimization problems in foreign relations are described. Also models with variable structure are proposed. Some examples of applications of developed algorithms to geopolitical prognosis are discussed.

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Experimental Software of Nonlinear Feasible Goals Method

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Feasible Goals Method (FGM) is the multiple criteria decision support tool based on the single-shot identification of a preferable feasible goal. The method is based on the transformation of mathematical models into qualitative pictures, which contains the approximation of the Feasible Set in the Criterion Space (FSCS). The FGM has found real-life application in the case of linear models (Lotov et al., 1999). The problem is much more complicated in the case of nonlinear models, for which the FSCS is usually non-convex. The method of approximating the FSCS for nonlinear models was proposed in (Kamenev and Kondrat'ev, 1992). It is based on covering the FSCS by a system of balls, which centers are computed as outputs of filtered random feasible decision points.

Let the decision set be X, the criterion space be R^m, and the mapping from X to R^m be f. Then, the FSCS is $Y = f(X)$. We assume that X and Y are bounded. Let μ be the uniform measure defined on X, $\mu(X) = 1$. Let T be a covering base, i.e. a finite number of points of Y, and $(T)_\epsilon$ be the ϵ-neighborhood of T, $\epsilon > 0$. The set $(T)_\epsilon$ is called the (ϵ, η)-approximation of Y if $\mu(f^{-1}((T)_\epsilon \cap Y)) \geq \eta$, where $0 < \eta < 1$. The method is aimed at constructing the (ϵ, η)-approximation of Y with given precision ϵ, completeness η, and estimation reliability $\chi < 1$. An iteration has the following steps:

1. N random independent points from X are generated and their outputs are computed (N depends on η and χ);

2. the outputs of the random points are used for testing the termination condition for the covering base T that was constructed on previous iterations; the condition

$$\mu(f^{-1}((T)_\epsilon \cap Y)) \geq \eta$$

is tested with a given reliability χ;

3. if the condition is not satisfied, T is augmented by the output vector that is the most distant from T.

A software tool was coded in the form of the add-in tool for MS Excel. It consists of four subsystems. The first subsystem helps to formulate the initial model. The second one helps to specify criteria and approximation parameters. In the third one, the covering base in the form of a database is constructed. The last subsystem visualizes the approximation on-line, helps to select a preferable goal on display and to find a related decision in the database.

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A Game of Spatial Competition on the Line

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In this paper, the approach from [1] is extended to the case of n-person game.

Let n firms be located on a line, the locations of the firms are denoted by $x_1, ..., x_n$. They sell the homogeneous product with zero cost (the cost can be normalized to zero). Customers are assumed to be distributed over the line according to a cumulative distribution function F. Each customer purchases one unit of the product. The transportation costs are assumed to be linear with coefficient k.

Let $p_i(z)$ be the price charged by the firm i for the location of z's customer. Let C_i be the region served by the firm i, then

$C_i = \{z : \quad p_i(z) + k|x_i - z| \le \min_{j \ne i}(p_j(z) + k|x_j - z|) \quad \text{and}$
$|x_i - z| \le \min_{j \ne i} |x_j - z|\}.$

Let $p_i^*(z) = k \max\{\min_{j \ne i} |x_j - z| - |x_i - z|, 0\}, \quad i = 1, ..., n$. The payoff functions are given by

$$H_i(p_1, ..., p_n, x_1, ..., x_n) = \int_{C_i} p_i(z) dF(z), \quad i = 1, ..., n.$$

Let the locations of the firms be fixed then we have n-person price game. Following [1], the Theorem 1 can be proved.

Theorem 1. The price strategy $(p_1^*(z), ..., p_n^*(z))$ is a point of Nash equilibrium.

Let all firms charge $p_i^*(z)$, $i = 1, ..., n$, respectively. We consider n-person location game under price equilibrium. The payoff function of the firm i is given by $H_i(p_1^*, ..., p_n^*, x_1, ..., x_n)$. Each firm can choose the location.

Theorem 2. Let $F(z_0) = 0$ and $F(z_1) = 1$ for some z_0, z_1 and $F(z)$ be continuous function.

(1) If Nash equilibrium in pure strategies exists then the ordered permutation of components of any equilibrium point is a solution of the system of equations

$$F(\frac{x_1 + x_2}{2}) - 2F(x_1) = 0,$$

$$F(\frac{x_{i-1} + x_i}{2}) + F(\frac{x_i + x_{i+1}}{2}) - 2F(x_i) = 0, \quad i = 2, ..., n-1,$$

$$1 + F(\frac{x_{n-1} + x_n}{2}) - 2F(x_n) = 0.$$

(2) If $z_0 = 0$, $z_1 = 1$ and $F(z)$ corresponds to the uniform distribution then any permutation of ordered set $(\frac{1}{2n}, \frac{3}{2n}, ..., \frac{2n-1}{2n})$ is the point of equilibrium.

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Application of Edgeworth-Pareto Hull Approximation Techniques in Water Quality Planning

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Search for Pareto-efficient decisions of multiple-criterion decision problems can be based on the Feasible Goal Method (FGM) (A. Lotov, 1973). The FGM involves approximation and visualization of multi-dimensional sets that are formed by all the feasible combinations of criterion values and by all dominated criterion points, i.e., approximation of Edgeworth-Pareto Hulls (EPH) of Feasible Sets in Criterion Space (FSCS). The FGM supports exploration of dependencies between non-dominated criterion values and helps to identify and study acceptable solutions to problems.

To approximate a convex EPH, the FGM applies iterative approximation by polyhedra. The polyhedra are constructed on the basis of evaluations of the support function for the approximated body in a number of directions. As the use of directions given a-priori did not make it possible to approximate multi-dimensional sets efficiently (G. Sonnevend, 1983), an idea of adaptive iterative approximation was put forward and the Estimate Refinement method (the ER method) was developed (V. Bushenkov, A. Lotov, 1982).

The analysis of its practical applications has showed that the ER method used too many evaluations of support function at each iteration (up to 500 for 7-dimensional bodies). As in real-life problems each such evaluation takes a lot of time, too much time is needed for the ER method to approximate multi-dimensional EPHs.

A new adaptive iterative method for approximation of multi-dimension convex bodies was developed (L. Bourmistrova, 2000). It is the Modified Converging Polytopes method (the MCP method). It has the same rate of convergence as the best approximating polytopes for both smooth and non-smooth convex bodies while it requires a small number of evaluations of the support function for the approximated body at each iteration.

In the framework of the federal programme "Revival of the Volga River" the MCP method was applied in a study of a real-life problem. In the programme a decision support system for water quality planning in large rivers was designed and developed. The system was provided with an integrated linear model, describing the problem of water quality planning in the Oka river, which is one of the largest tributaries of the Volga river. The model had 539 variables, more than 500 limitations and consisted of three components which were a wastewater discharge sub-model, a wastewater treatment sub-model, and a pollution transport sub-model.

The MCP method was put to test in approximation of non-smooth convex EPHs for some multiple-criterion decision problems that had been formulated by specialists in water quality planning and were typical for the studied problem. A comparative study of the ER and the MCP methods shows that the use of the MCP method made it possible to reduce considerably time spent on approximation. For example, in a typical 5-dimensional problem the MCP method provides approximation of the EPH with 1%-accuracy four times faster than the ER method.

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The Technologies of Financial Engineering for the Reception of the Investments to the Russian Enterprises

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The simplest analysis of structure of the Russian enterprises gives the basis to believe, that not less than 75% of enterprises in the general sectors of economy are capable to enter a stock market directly, with the purpose of reception of resources for business development. In some branches this percentage achieves a size of 85-95%. For this purpose it does not necessary to wait for development of the national capital market. European and North American markets concentrated the most of the world resources and have enough developed mechanisms to hand out the credit on any period: from several hours up to "infinity". The Asian markets for a long time and successfully applied it. The notorious schemes of "tigers" are the only one of few synthetic tools created for reception of the capital from foreign sources. Let's consider some possible schemes.

1) The most known and simple schemes are the release and accommodation of ADR. Practically, no one enterprise received any appreciable money inflow from this source.

2) More difficult, expensive, and long way is to enter the foreign markets with the own shares. Investments are large, but the returns are unknown. The basic disadvantage is the long time of reception of a "good will" and high operational costs.

3) The third way is creation of the constantly working channel of reception of the investments through the specially created mechanisms. One of such, conventional, is the scheme based on mutual takeover (merger).

4) The programs of capital reception through extraction of tax advantages are the most attractive. One of the most simple and convenient schemes is financial leasing.

The comparative benefits of the offered schemes are given in the paper.

Homogeneous Problem of Dynamic Stochastic Programming for Investment Strategy of a Proprietor

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Attempts of creation of favorable conditions for return the capitals to Russia are now undertaken. Experts assert, that it is unreal to expect foreign investments without repatriation of the capitals. Taking into account that the so-called middle class in Russia is practically absent (and the poor does not save), the nearest prospect of development of the Russian economy will be determined by the investment strategy of large proprietors.

The formal model of rational behavior of a large proprietor is considered. The model is suitable for the forecasts and quantitative estimations. It may be treated as a dynamic problem of wealth allocation between "profitable" and "useful" assets. Qualitatively the model reflects the basic alternative of a large Russian proprietor between low profitable but reliable investments in foreign securities and risky but profitable investments in the Russian economy. Quantitatively the model can describe the capital flows in and out of the country and the investment flows reallocation among different sectors of the national economy depending on deterministic and stochastic characteristics of socio-economic situation.

The model is based on the description of rational behavior of the proprietor within the framework of homogeneous stationary stochastic Markovian problem of allocation of assets of different profitability, reliability, and usefulness. The model allows the proprietor to operate at the stock market as well as to undertake the direct management of assets at a technological level. The later possibility simulates the behavior of the proprietor, having direct access to industrial or trade technologies, in the spirit of Neumann-Gale and Leontieff models.

The homogeneity of the problem is the result of the assumption of homogeneity of the utility function and linear homogeneity of the traditional descriptions of production processes. The assumed homogeneity allows to reduce the Bellman equation on a compact, that in turn enables the effective computing of the solution of this equation by monotonous iterations.

The model has good qualitative properties. In particular, it catches effects of investment diversification and describes speculative stocks. The optimal strategies in the model occurs to be linear with respect to cumulative capital of the proprietor. Their description can be given in the terms habitual for economists such as propensity to consume and propensity to save. Within the model these propensities are calculated. The linearity of optimal strategy allows to aggregate the description of proprietors with identical characteristics, which is rare for models of economic agents.

The linearity of the optimum strategy takes place in the important case when the wealth of the proprietor can be adequately estimated by a scalar value. This case corresponds to the economy with low transaction costs of exchange of different assets or, that is the same, the economy with free markets of all assets. But the offered method of solving Bellman equation is not limited to this case. It may be used in the case of multidimensional wealth as well. The results obtained concern not only the concrete model but a class of such models that within their framework the analysis of different economic questions is possible.

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Selecting Effective Points by Means of Aspiration Level Metod

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Multiple criteria optimization methods usually assume that a multiple criteria decision-making problem is converted into an auxiliary parametric single-objective problem which solution provides a Pareto-optimal point. Different methods apply different convolutions but typically they use weighting coefficients to introduce preferences of a Decision-Maker. This paper presents the interactive procedure in which Decision-Maker's whishes are expressed in the form of aspiration levels. The aspiration levels are values of objectives, which the Decision-Maker would like to achieve taking into account real situation.

First, the Decision Maker specifies aspiration point t. Second, the points, which are the nearest to t in the sense of Chebyshev norm, are searched in the set of feasible goals. Let all such points refer to as $F(t)$. It is proved in the paper that these points are half-effective. The necessary condition of half-efficiency is also proved. If we maximize the sum of criteria on the set $F(t)$, we come to an effective point (the single one). The Decision-Maker analyzes this point. If he is not satisfied, he proposes another aspiration point and so on.

The statements about parameterization and approximation (in the sense of Hausdorff norm) of Slater's and Pareto's sets are formulated and proved applying to the presented procedure. Thus, the aspiration point is a good parameter for examining half-effective and effective sets.

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Power Energy Wholesale Market Model

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In this talk a model of the concurrent wholsale power energy market is considered. The model makes use of a set of linearized power flow equations that provide a suitable approximation for the initial non-linear OPF problem as well as allowing for a more flexibility in problem formulation, size and solution speed. The problem's dual solution provides an instantly computable nodal prices that could be used for the market clearance. The model accounts for the power losses in the transmission network and proposes a loss allocation scheme among the market participants.

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Ruin in Dynamic Risk Model

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The purpose of this article is to present a dynamic model to describe the amount of an insurer's surplus. For $n \geq 0$ let U_n denote the surplus of the insurer at time n. We assume that the premium are received at a rate $c_n > 0$ and the aggregate claim at nth period is W_n. The basic relation is

$$U_{n+1} = U_n + c_n - W_n.$$

At time n the insurer can divide the premium c_n into two parts c_n^1 and c_n^2 where c_n^2 is the reinsured premium. Thus, there are different problems to minimize risk insured in this model and to find optimal choice of c_n^2.

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Assets-Liabilities Management of Commercial Bank Based on Statistical Analysis of Financial Markets

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A problem of bank assets-liabilities management is the profit maximization under bank technological restrictions, taking into account various economical risks. In practice, known methods of portfolio theory are often not applicable: detailed information on revenues and risks of assets is absent, these revenues are correlated in a complex way, etc. Really, the portfolio revenue is an unknown nonlinear stochastic function of its parameters.

Our approach consists of using a data set of practical results of bank operations for design (identification) of the revenue function [1]. Then the problem of bank assets-liabilities management is represented as a complex global optimization problem of a large dimension. This problem can be reformulated as a stochastic programming problem with linear constraints. For searching a global optimum of the problem, a special version of the method of branches and bounds is proposed.

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Problem-Independent Estimate of Asymptotic Efficiency for Polyhedral Approximation of Convex Feasible Sets in the Criterion Space

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Approximation of the multiple-dimensional convex bodies is the important feature of the Feasible Goals Method (FGM) that is the multiple criteria decision support method based on visualization of the feasible sets in criterion space (see Lotov et al., 1999). We consider the problem of algorithmic approximation of convex bodies with any given accuracy. Let $\mathbf{C}$ be the class of compact convex bodies C in $\mathbf{E}^d$, d ≥ 2, with the support function $g(C,u)$. Let $\mathbf{C}^s$ be the class of compact convex bodies with s times continuously differentiable boundaries and positive Gaussian curvature. Let $\mathbf{P}$ be the class of inscribed polyhedra for C. Let $P \in \mathbf{P}$ and $m^t(P)$ be the number of its vertices. Let $\mathbf{P}(m)$=$\{P \in \mathbf{P}$: $m^t(P) \leq m\}$. Let us consider algorithms based on adaptive augmentation iterative scheme (Kamenev, 1992).

Augmentation scheme. Let $P^n \in \mathbf{P}$.
Step 1. Choose the point $p_n \in \partial C$.
Step 2. Construct P^{n+1}=conv$\{p_n, P^n\}$.

We define the sequence F=$\{P^n\}$ as H-sequence for C with constant γ (H(γ, C)-sequence) if there exists a constant $\gamma > 0$ such that $\delta(P^n, P^{n+1}) \geq \gamma\delta(P^n, C)$ for Hausdorff metric δ. We define $F_1 = \{P^n\}$ as $H_1(\gamma, C)$-sequence if there exists a constant $\gamma > 0$ such that for each n there exists an external unit normal u in p_n, such that $g(u,C) - g(u,P^n) \geq \gamma\delta(P^n, C)$.

G. Kamenev (1999) proved that there are some examples of iterative algorithms generating H and H_1-sequences for convex bodies.

Let $\delta(\mathbf{P}(m), C)$=inf$\{\delta(P, C)$: $P \in \mathbf{P}(m)\}$. Then we define the lower asymptotic efficiency of the sequence F as

$$\eta(\mathrm{F}) = \lim\inf \delta(\mathbf{P}(m^t(P^n)),\mathrm{C})/\delta(\mathrm{P}^n, C) \text{ as n} \to \infty.$$

Let $C \in \mathbf{C}^3$. Kamenev (1992) proved that for $C \in \mathbf{C}^3$ it holds $\eta(F) \geq \eta_d \gamma^{2d/(d-1)} \rho/\rho_1$, where ρ and ρ_1 are the minimal and maximal curvature radii of ∂C, respectively, and limη_d=1/4 as $d \to \infty$. Now this result is improved (for d=2 it was proved by Efremov in 2000).

Theorem. Let $C \in \mathbf{C}^2$, F be H(γ, C)-sequence, and F_1 be $H_1(\gamma, C)$-sequence. Then

$$\eta(\mathrm{F}) \geq (1-(1\text{ -}\gamma)^{1/2})^2/4 \quad \text{and} \quad \eta(\mathrm{F}_1) \geq \gamma^2/4.$$

Note that in the case of $\gamma \approx 1$ the both formulas coincide.

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A Control System for Commercial Bank (BCS)

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The control system for a commercial bank has to support the choice of the rational plan and the decision making process during monitoring of the plan execution. The calculation of future bank positions for the given time period is based on the special simulation model that allows to get the incomes, risks, and other financial parameters for any given plan of resource management. This model contains three blocks of the data:

1. initial conditions of the bank;
2. the scenario of the market development;
3. strategies (plans) of the bank resource management for the forthcoming period.

The bank evolution is the set of the bank positions as the functions of the time. For representation of the bank positions in the frameworks of BCS one usually uses the following technology:

1. the assets, the capital, and the liabilities of the bank are divided into some real aggregated positions;
2. the market positions are formalized as a set of indicators (exchange, exchange quotations, share indexes, interest rates, etc.), which dynamically changes with the scenario of the external situation;
3. for each aggregated position the appropriate model is elaborated.

The simulation experiments have the iterative character. The operation researcher changes the blocks of the data to study the influence of these parameters on the bank evolution.

In the report, the applications of the offered approach are discussed.

Formation of Financial Engineering in Russia and Computational Intelligence

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The most appropriate definition of financial engineering is the one done by J. Finnerty: "Financial engineering includes designing, development, and realization of innovative financial tools and processes as well as creative search of new approaches to the decision of problems in the sphere of finance".

The team of financial engineers consists of lawyers, economists, accountants, tax experts, mathematicians, programmers, and the leader. Of course, that is the basic bearer of ideas and conceptions of the project. At once, we have highly valued large perspectives of financial engineering in the sphere closely connected with our interests, i.e., with modeling and creating program systems. All our previous experience connected with designing space and defense systems and reforming socio-economic complexes shows that in the new sphere of designing and creating innovative financial tools, the participation of system analysts, mathematicians, and programmers will be productive as well.

Just as our colleagues in America, we intend to follow the same path in Russia. We are going to propagandize ideas and methods of financial engineering and to take part in commercial and public projects. We also intend to organize the publication of the "Journal of Financial Engineering" and to create the Russian branch of the "International association of financial engineers". Those were our projects in 1996. And now we can say that in essence, the program is carrying out, although not so rapidly as we want.

As we can judge by the experience of USA, the researchers deal with very advanced markets of financial products.

On the contrary, our problem is that at present, security markets don't actually exist in Russia. Nevertheless, the barest necessity of solving the problems of financial design exists. This can be seen from the Internet-information (see "gray schemes", www.rusgal.ru) and [1], where real problems of economy and real behavior of the economic agents are considered. This can be also seen from discussions about overcoming debt crisis, problems of the external debt, the return of the fluent capitals, acquaintance with real organizational problems of real operators in markets, etc. Economists consider the problems of constructing the processes and schemes to be ambiguous. They consider financial schemes and projects as folklore creatures, even with "gray", i.e., "semi-legal", elements.

We believe that development of financial projects needs an approach that follows traditions of the domestic school of operations research and designing automation. The most essential effect from participation of system experts is that the developed software allows carrying out numerous calculations for various scenarios of economic development and various financial schemes.

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Computational Experiments with Models of Passive Evolution for Commercial Bank Process

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One of the most "deep" tools of the financial analysis and management in a bank is modeling the bank firm as a whole. The model of passive evolution (MPE) is a model of attenuation of a bank payment flows under condition of termination of its active actions. MPE consists of an estimation of the bank ability for the further functioning under condition of the situation, when bank freezes all active operations on accommodation and attraction of means. Bank proceeds payments only with available debtors and creditors. Note that MPE is intended not for the forecast, but for an estimation of the current bank condition. In addition, this estimation is comparative.

The scheme of passive evolution consists of the following: for each day the total withdrawal payment is calculated, which is covered with the expense of cash money resources and sale of a part of assets. In the base model, it is assumed that the bank has assets of the given amount and "life" intervals and obligations with intervals of their existence.

Model 1. The deterministic case.

Model 2. A stochastic problem: limited case.

Model 3. A stochastic variant of the base model.

Model 4. The base model in the case of "panic" expectations and creditors' decisions.

The description of the complex, which consists of 4 programs, is resulted. The use of each program represents (in an aggregate form) the computer technology, developed by the author, for the research of the initial base problem.

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Local-Optimal Strategies in a Problem of Security Portfolio Management

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A problem of stochastic optimal control with discrete time is considered for the fixed period of control. The state set consists of the deterministic variable (capital) and random factors (prices) at each step. The probability distribution of random prices at any moment is a function of prices at several previous moments. This means the random variables of Markovian process with the fixed depth of the memory. The control strategy is a function of the process states. The transition function is introduced in the standard way for stochastic control systems. The pay-off function for each moment is given as a function of the state and control. A probability distribution for the initial states is given as well. The given distribution, transition functions, and control strategies determine the distribution on the set of trajectories, which allows to define the mathematical expectation of the system criterion.

It is shown that the optimal control coincides with a local-optimal control in the case of the payoff function for the whole period equal to the sum of payments at any instant and the transition function depends on control and random parameters only. For wide classes of problems, the conditions are given, under which the local-optimal strategy gives a good approximation for the optimal decision. As well, the model is considered with payoff functions that depend on the process states insignificantly. It is shown that the local-optimal strategy gives approximately the same optimal value of criterion.

Mathematical Model of Conversion

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Production conversion model is transformed into partially integer programming problem with Boolean variables. The problem is solved by the Benders method. Due to a special structure of constraints for continuous variables, there is no need in iterative processes. Constraints of the Benders method form a final problem.

Monotone Numerical Methods for the Option Pricing and Hedging Models

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To consider the option pricing problems, there are models that admit partial differential equation (PDE) formulation. For instance, the fair value of the European put and call options can be found as the solution of the Black-Scholes differential equation.

To derive the solutions for the wide variety of PDEs, the sophisticated numerical methods should be engaged. The finite difference method proved to be a flexible and robust tool for PDE solution. As soon as the differential scheme is developed, it is necessary to test it and to find out its properties. To satisfy the contemporary requirements, it is to be stable and monotone. The latter means that the scheme does not produce spurious oscillation of the numerical solution.

It was shown that the popular Cranck-Nickolson scheme (symmetrically weighted scheme), which is traditionally used to solve the parabolic PDE, proved to be non-monotone, as long as it produced remarkable parasitical oscillations of the solution. The authors managed to develop the alternative method based upon the Samarskiy-Fryazinov scheme using the directed differences technique.

It should be emphasized that the differential scheme can be non-monotone, though stable at the same time. It goes that the stable differential method can bear spurious oscillations even if the scheme is prove to be stable.

Various hedging problems use the "Greeks" technique, involving the calculation of the partial derivatives of the solution. It goes that the partial derivatives computational issues should be treated with care, leading to financial strategy faults and money losses otherwise. That is why one should pay much attention to the choice of differential scheme.

It was shown that the popular Cranck-Nickolson approach produces spurious oscillations both in the solution function profile and in the partial derivatives. It should be mentioned that the oscillations' amplitude even increased. The alternative method based upon the directed differences technique has shown the proper results, i.e., the monotone partial derivatives profiles.

All of these results proved to be true for more complicated cases. It is well known that the solution of the Black-Scholes equation can be derived in the closed form, therefore it was used in our research for the testing purposes. The pricing and hedging problem for the passport options, a relatively new and popular derivative security, can be presented in the form of non-linear differential equation. In that case the closed-form solution cannot be achieved, hence it is inevitable to engage sophisticated numerical methods. Similarly, the Cranck-Nickolson approach generates significant oscillations both in the solution and partial derivatives function profiles.

It might be noted in passing that the Samarskiy-Fryazinov scheme has the first order accuracy in time, at the same time as the Cranck-Nickolson method is the second order scheme. But the calculation has shown that the directed differences scheme's accuracy satisfies our calculation time requirements.

To summarize, when the option pricing or hedging problem is concerned, in many cases it admits partial differential equation model formulation and can be easily solved using finite difference technique. However, it is vital to pay significant attention to the aspect of the differential scheme monotony, for the reason that the strategy based on the data containing spurious oscillations would lead to notable financial losses otherwise.

Network Routing Model with Vector Cost Criteria

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1. Introduction. Conspicuous annual development of large area networks and high increase of transmitted data volumes are incentives for working out new methods of routing control. All network users are interested in improvement (minimization) of some quantities such as time or cost of their own message transmission. It is quite natural that routing control would pursue the objects. Mathematical statement of corresponding optimization control problems could be based on the well known notions of Nash equilibrium and Pareto optimality. Algorithms of routing control must be also distributed all over the network so as it ensures high productivity and reliability of data transmission process. This is taken into consideration in the paper.

2. Network Model. In the report a routing model with multivalue criterion $G = (G_1, G_2, \dots G_m)$ is to be discussed. Cost of message passing between network users, that form any kth sink-source pair, is denoted by G_k. Routing choice in the network is based on Pareto optimization approach. Distribution of routing control is achieved by means of some scheme of the network decomposition.

In the model, control consists in choice of L_j^k, λ_j^k for all sink-source pairs $k = 1, 2, \dots, m$. Here, all L_j^k are routes and λ_j^k are intensities of packets transmission along L_j^k. Any routing fully defines a vector $z = (z_1, z_2, \dots, z_n)$, where z_l is intensity of flow through line $l = 1, 2, \dots n$. Network structure is given by a graph. Users are placed in vertices; lines are edges of the graph. Messages are passed in the form of packets along the graphs routes.

The time of packets' transmission along any line $l = 1, 2, \dots n$ is given by a function $t_l = t_l(z_l)$. It is assumed that transmission cost is directly proportional to the volume of a message (measured in packets) and so can be expressed as $g_l = c_l \lambda_j^k t_l(z_l)$. All this allows to define G_k as a total of the pay-offs $g_l, l \in L_j^k$ for all routes L_j^k in use.

3. Obtained results. Under some assumptions the vector criterion $G = G(\lambda, z)$ has been studied in order to search one of Pareto optimal routings. The following decomposition method has been applied. The initial network is replaced by a set of ring subnetworks. In any of the rings, a routing problem of the same but much more simple structure is to be solved. It is worth mentioning that European part of INTERNET has the needed structure containing only 7 rings.

Decentralized iterative algorithm has been proposed and well-founded. On its iterations, network routing is recalculated using current values of the vector z. (By introducing a metric the algorithm can be included in habitual for a practical scheme.) The method is proved to be convergent to an optimal solution of the model.

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Operations Research for Queuing Systems

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To study a feasible development of a Queuing System (QS) it is necessary to know a model of QS development and parameters of the arriving flow. In the report, it is assumed that ν is the design rate of the service of the units, N is the design number of the units in the arriving flow. Moreover, it is assumed that the customer and the designer are in the marketing relations [1] on markets of resources and services. Let c be a price of one resource, p be a price of a service of a single unit, and $M_\Lambda(\nu)$ be the expectation of the number of the units serviced for the arriving flow with the parameter Λ. Then to serve the incoming flow of the units by the QS a designer can serve the sum $pM_\Lambda(\nu)$, that will make a profit available to him,

$$u(\nu, p) = pM_\Lambda(\nu) - c\nu, \qquad 0 \le \nu \le \nu_o \tag{1}$$

where ν_o is a permissible size of founds [2,3].

Let C_o be the stock of the money of the customer to the consumption and p_o be the value of his economical loss due to a potential skip of a unit. Then after an allocation of ν units of a resource to design QS the customer budget is equal

$$v(\nu, p) = C_o + c\nu - pM_\Lambda(\nu) - p_o(N - M_\Lambda(\nu)), \tag{2}$$

where $c\nu$ is the gain of the customer due to the sale of ν units of the industrial resource at the price c to the designer and $pM_\Lambda(\nu)$ is an expectation cost for a customer to serve $M_\Lambda(\nu)$ units. The latter term is an economical loss of a customer due to a potential skip $N - M_\Lambda(\nu)$ units. The strategy of the price formation

$$(\hat{\nu}, \hat{p}) = \arg\max u(\nu, p) \cap \arg\max v(\nu, p), \qquad (\nu, p) \in S, \tag{3}$$

provides the maximal gain to a designer and the maximal saving of his budget to a customer. In particular for one channel QS with a rejections [4], a form of a permissible set S is defined, in which a strategy with this property exists [5]. The analysis of $(\hat{\nu}, \hat{p})$ performed in this report has exposed an interesting dependence on input data.

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The Solution of a Problem of Resource Distribution under Uncertainty

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I. The nonpreemptive scheduling of a partially ordered set of jobs W is studied; K types of resumable resources are used at execution of these jobs. The amount of resources of each type is known. The following characteristics of each job are given: the execution time, the amount of resources of each type, which is necessary for the execution, and the due date T. At uncertain moments of time the demands to execute jobs of V may be received. These jobs have a higher priority. If so, the execution of the jobs of W is stopped and postponed on later time. Thus the previous schedule for W is broken. The process of execution of jobs of W and reception of demands to execute jobs of V repeats many times. The problem is to elaborate a strategy of constructing schedules of execution of jobs of W, which satisfies the following requirements:

1) restrictions of the partial order of job execution are not broken;

2) restrictions on consumption of resources are carried out;

3) if the demand to execute jobs of V is not received, the execution time of job set W does not surpass T;

4) the probability for the schedule of execution of jobs of W be broken (because of reception of demands to execute the jobs of V) is minimal.

The algorithm of solving this problem consists of two stages. At the first stage sets X and Y are constructed. The set X describes all possible moments to start the execution of jobs of W. Each element of X corresponds to the schedule of execution of jobs of W with a duration less than or equal to T. The set Y describes all possible moments of receiving demands to execute jobs of V. At the second stage the antagonistic game with discontinuous payment function on sets X and Y is defined. The optimal mixed strategy of the first player in this game determines the strategy of constructing the schedule of execution of jobs of W subject to the requirements 1–4. An approximative method to solve this game, based on approximating the infinite game by a matrix games, was elaborated.

II. The problem of optimal stopping time of nonpreemptive scheduling for independent jobs on a multiprocessor system is studied. The processor speeds can be different. Execution time of every job with every processor is given. Approximative algorithms, based on calculation of dynamic priorities of jobs, are elaborated. The comparative analysis of these algorithms was carried out.

III. The problem of preemptive feasible scheduling for jobs on multiprocessor system is studied. Each job is characterized by specific processing requirements for release time and due date. The processor speeds, which can be different, are given. Interruptions and preemptions are allowed. Corresponding expenditures are taken into consideration. An approximative algorithm, based on calculation of dynamic priorities of jobs, corresponding to its relative urgency, was elaborated.

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Human Choice with Individually Difficult Tasks

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Human behavior in multicriteria choice is studied in a specially constructed procedure based on the decomposition of a multicriteria problem into multiple steps of pair-wise comparisons. The experiments with the Russian and Finnish students were conducted on the base of creating an individually difficult task for each person in accordance with his/her preferences. The goal of the experiment was to study human behavior in order to be able to develop easy-to-use decision support systems. The aforesaid studies indicate that multicriteria object comparison is difficult to the human system for processing information. While solving such problems, human beings make errors as well as use simplifying strategies to adapt the problem to their capabilities. It seems reasonable to support the process of decision making by means of (i) a decomposition of multicriteria objects to parts (or units) and (ii) step-by-step comparisons of such units regarding their relative strengths and weaknesses.

We suggest the procedure that has the following features.

1. The Decision Maker (DM) has to choose the best object from some set taking into account the values for a number of criteria.

2. All the comparisons have a qualitative nature ('better', 'worse', 'equal').

3. The DM makes a choice during several stages with an increasing number of criteria to be taken into account (two, three, etc.). At each stage, the DM has to complete the pairwise comparison of the corresponding units of the objects. At each stage, the process of comparison is supported with information obtained from the DM in the previous stages.

4. Several stages, including the comparison of all the pairs of criteria, create a round.

The process consists of two rounds that differ only in the order of the criteria presenting to the DM.

The first finding of the experiments is the importance of individually adjusted tasks for the persons. While solving such tasks, they could make meaningful and multicriteria choices and demonstrate stable strategies in most of the cases. That is why we can recommend this approach for future research of human ability to solve multicriteria tasks.

The second finding is the usefulness of two-round experiment arrangements. The first round gives the opportunity to find a compromise between the criteria and to study the problem. The second round allows the persons to improve the selected strategy or to confirm the previous choice (in a majority of the cases). Most probably, the existence of several rounds coincides with real life decision-making when people collect information, study a problem, and after that make the final choice.

Thirdly, the experiments also demonstrate that the order of the criteria presenting influences the results.

The Model of the Buffer Company That Actualizes the Trading by Small and Super Small Lots on the Russian Shares Market

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In the report, the model of the financial company playing the role of the buffer between small investors and the exchange, trading the large stock lots, is described. The company manages stock investments of small investors depending on the accumulated funds of the small investors. Analytical researches and computer experiments are realized using the statistical data. The results are used as the basis for estimating the economic effectiveness and financial reliability of the project.

The game of quality in Z^n

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Let's consider the game in Z^n. Let the player P (persurer) be in the point x_0^P and the player E (evader) be in the point x_0^E. The dynamics of the players is as follows. The player P being in the point x can attain one of the points of the set $D_P(x)$ which is called the set of attainability of the player P; $D_P : Z^n \to 2^{Z^n}$. The choice of a point from the set $D_P(x)$ by the player P is the move of the player P. The same is for the player E.

Let's make the following suggestions:

1) the sets of attainability of the players are finite;

2) each of the players being in any point can reach any other point using finite number of moves;

3) $\forall x \in Z^n$: $x \in D_P(x)$, $x \in D_E(x)$.

The players make their moves one by one and inform each other about them. Let it be the player E who makes the first move in the game.

The sequence $i_k^P = (x_0^P, x_1^P, x_2^P, \ldots, x_k^P; x_0^E, x_1^E, x_2^E, \ldots, x_k^E)$ of all the moves which the player P knows after kth step is called kth information state of the player P. Let's mark a set of all information states for the given k as I_k^P. Then $I^P = \bigcup\limits_{k=0}^{\infty} I_k^P$ is the set of all information states of the player P. The definitions of i_k^E, I_k^E, and I^E are similarly introduced for the player E.

The pure strategies and trajectories of the players are analogous to the strategies considered in the work [2]. The metric, similar to the metric of Bare, is defined on the set of trajectories and the set of pure strategies. These sets proved to be compact in the metric mentioned above.

We will mark the game trajectory, which corresponds to the situation (σ^P, σ^E) as $< \sigma^P, \sigma^E >$.

Theorem 1. The map $(\sigma^P, \sigma^E) \to < \sigma^P, \sigma^E >$ is continuous.

Let's introduce the payoff function $\Phi(s)$ on the set of game trajectories in the following way:

$$\Phi(s) = \left| \begin{array}{l} 0 \Leftarrow \forall n \in N : \bar{\rho}(x_n^P, x_n^E) > r; \\ 1 \Leftarrow \exists n \in N : \bar{\rho}(x_n^P, x_n^E) \leq r; \end{array} \right.$$

where $s = (s^P, s^E) = ((x_0^P, x_1^P, x_2^P, \ldots, x_k^P, \ldots), (x_0^E, x_1^E, x_2^E, \ldots, x_k^E, \ldots))$ is a game trajectory, $r \geq 0$ is the radius of the player P and $\bar{\rho}$ is a metric (for example, Euclidean metric) defined on the pairs of points in Z^n. Thus we have determined the game

$$\Gamma = (x_0^P, x_0^E, D_P, D_E, \Phi).$$

Lemma. The payoff function Φ is low semicontinuous on the set of trajectories of the game Γ.

Theorem 2. There is a Nash equilibrium in the antagonistic game Γ with the payoff function Φ.

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Solving Certain Problems of Canceling Mutual Debts

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We consider several operations research models, which are connected with cancellation of mutual debts [1, 2, 3].

We denote the set of clients by $N = \{1, ..., n\}$. Let (d_{ij}), (c_{ij}), and (a_{ij}) be given nonnegative matrices for $(i, j) \in E = \{(i, j) | i \neq j, 1 \leq i, j \leq n\}$, where d_{ij} is the debt of the client i to the client j; c_{ij} is the unit cost of the debt; a_{ij} is an upper bound of feasible debt's repayment from the client i to the client j.

For any function f_{ij}, we define the numbers $div_f(i) = \sum_{j\in N} f_{ij} - \sum_{j\in N} f_{ji}$, $i \in N$. The client i is the potential debtor if $div_d(i) > 0$ and the potential creditor if $div_d(i) < 0$.

The problem of the debts' distribution may be formulated as follows:

$$\max \sum_{(i,j)\in E} c_{ij}\, f_{ij}$$

subject to $div_f(i) = div_d(i)$, $i \in N$; $f_{ij} \leq a_{ij}$, $(i, j) \in E$.

The case of $a_{ij} = d_{ij}$ is of particular interest for applications.

The problem of the expedient crediting for cancellation of mutual debts can be formulated as follows: for a given common resource Q, resources Q_i for a specified purpose, and matrices $(d_{ij}), (c_{ij})$, $(i, j) \in A$, $A \subset E$, to find a new debts' distribution attaining

$$\min \Big(\sum_{i\in N} (q_i^0 + q_i) + \sum_{(i,j)\in A} c_{ij}\, (d_{ij} - f_{ij}) \Big)$$

subject to

$$q_i^0 + q_i + div_f(i) = p_i + div_d(i), \quad i \in N;$$

$$0 \le f_{ij} \le d_{ij}, \quad (i,j) \in A;$$
$$0 \le q_i \le Q_i, \quad \sum_{(i,j)\in A} q_i^0 \le Q^0, \; i \in N,$$

where q_i^0 and q_i are common and specified credits given to the client i for cancellation of debts, p_i is a resource of the client i, $0 \le p_i \le Q^0$.

The similar models was constructed for problems of the expedient barter [3].

The effective solution of the aforesaid problems is suggested via transforming into well known minimum-cost circulation problem which is one of the fundamental problems in the theory of algorithms.

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Universal Mathematical Model for Optimal Configuration of the Blood Microvascular Junction

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1. The influence of the hemodynamic factor on the structure of networks is considered with the use of geometrical approach to the structural analysis of vessel junctions. The configuration of a microvascular junction is defined by the law of impulse preservation and is connected with the model of the behavior of blood as a non-Newton liquid. The connection between the optimal configuration of a microvascular junction and hemodynamic factor is described by the universal model, presented as the system of the following equations:

$$A_{3(j)}^2 d_3^4 - A_{1(j)}^2 d_1^4 - A_{2(j)}^2 d_2^4 - 2A_{1(j)}A_{2(j)}d_1^2 d_2^2 \cos\varphi_1 = 0;$$

$$A_{1(j)}^2 d_1^4 - A_{2(j)}^2 d_2^4 - A_{3(j)}^2 d_3^4 - 2A_{2(j)} A_{3(j)} d_2^2 d_3^2 \cos\varphi_2 = 0;$$

$$A_{2(j)}^2 d_2^4 - A_{1(j)}^2 d_1^4 - A_{3(j)}^2 d_3^4 - 2A_{1(j)} A_{3(j)} d_1^2 d_3^2 \cos\varphi_3 = 0;$$

where $j = 1, 2$; if $j = 1$, $A_{i(1)} = |H_i|$; if $j = 2$, $A_{i(2)} = \eta_i$. Here H_i is complete blood pressure, η_i is the coefficient of dynamic of blood viscosity in i-microvessel, d_i is the diameter of i-microvessel ($d_i < 100$ microns), and φ_i is the corner between the axes of symmetry of a lumen of microvessels, thus $0 < \varphi_i < 180°$, $\sum_{i=1}^{3} \varphi_i = 360°$; $i = 1, 2, 3$. Losses of mechanical energy during movement of the blood flow through a vascular junction of the optimal configuration are not significant biologically.

2. A vascular junction is a structural functional element of the blood vascular networks which are carrying out the function of local distribution of blood flow and its control. The quantitative interrelations between the structural parameters of the junction configuration and parameters of the hemodynamic factor follow from the universal model of the optimal configuration of microvascular junction. The mechanism of local distribution and control of blood microflows works on the basis of the aforesaid quantitative interrelations. By setting a direction of a flow of blood through a microvascular junction and a parameter of the complete pressure of blood or a coefficient of dynamic viscosity of blood $A_{i(j)} = t$, the model allows to define the values of these parameters in two other microvessels according to the formulas:

$$A_{1(j)} = t; \quad A_{2(j)} = \frac{d_1^2 \sin\varphi_3}{d_2^2 \sin\varphi_2} t;$$

$$A_{3(j)} = \frac{d_1^2 \sin\varphi_1}{d_3^2 \sin\varphi_2} t.$$

Under the conditions $A_{1(j)} = A_{2(j)} = A_{3(j)}$ and $d_i > 100$ microns, which correspond to the models of the behavior of blood as the Newton liquid, the universal model of the optimal configuration of a microvascular junction is transformed into the model of Murray for the optimal configuration of a microvascular junction, from which the rules of Roux follow. The model of Murray represents a special case of the universal model of the optimal configuration of a microvascular junction.

3. A lumen of a microvascular junction consists of a fragment of an ellipsoid, one plane of the symmetry of which coincides with the symmetry plane of the junction, and three fragments of round cylinders joined to it. All three axes of symmetry of fragments of round cylinders lay in the plane of the symmetry of the junction and are crossed in a single point.

Financial Optimization

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The report is an overview devoted to the modern area of operations research named financial optimization. We discussed, such mathematical problems of financial optimization as portfolio management, risk management and diversification, immunization, performance measurement, and others. The operations research methods are applied to the problems of financial optimization. The material is based on the issues [1]–[3].

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Equilibrium in a Model of Endogenous Political Party Formation

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We develop a model of endogenous party formation in a multidimensional policy space. Party platforms depend on the composition of the parties' primary electorate. The overall social outcome is taken to be a weighted average of party platforms and individuals vote strategically. Equilibrium is defined to obtain when no group of voters can shift the social outcome in its favor by deviating and the party platforms are consistent with their electorate. We provide sufficient conditions for existence and study the properties of sorting equilibria in this model.

Correcting the System of Constraints of a Linear Program with the Aid of Minimax Criterion

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Let the constraints of a linear program in the canonical form $Ax = b$, $x \geq 0$ with $m \times n$ matrix A and $b \neq 0$ be incompatible. The requirement is to construct an optimal correcting matrix H according to the minimax criterion

$$\min_{x,H} \max_{i,j} |h_{ij}| \tag{1}$$

such that the following system is compatible

$$(A + H)x = b, \quad x \geq 0. \tag{2}$$

Lemma 1. For an arbitrary fixed vector x the solution of the problem $\min\limits_{x,H} \max\limits_{i,j} |h_{ij}|$ subject to condition (2) is $h_{ij} = (b_i - a_i x)/\sum\limits_{j=1}^{n} x_j$, $i = \overline{1, m}$, $j = \overline{1, n}$, where a_i is the row of matrix A (it means that all the matrix H rows consist of the equal elements).

The task runs

$$\varphi(x) = \max_{i\in[1:m]} |f(x, i)| \to \min_{x\in\Omega}, \quad \Omega = \{x \in E^n | x_j \geq 0, \forall j \in [1 : n]\},$$

where $f(x, i) = \dfrac{b_i - a_i x}{\|x\|}$, $\|x\| = \sum\limits_{j=1}^{n} x_j$.

Let's introduce the following sets:

$R(x) = \{i \in [1 : n] : \ |f(x, i)| = \varphi(x)\}$,

$$L(x) = \{z = \sum_{i\in R(x)} \alpha_i \nabla f(x, i) \mathrm{sign} f(x, i) | \alpha_i \geq 0, \sum_{i\in R(x)} \alpha_i = 1\},$$

$\overline{\Gamma}^{+} = \{z \in E^n | (z, V) \geq 0 \ \forall V \in \overline{\Gamma}(x)\}$ where $\overline{\Gamma}$ is the cone of the possible directions for the set Ω at the point x. Let's designate

$$\psi(x) = \min_{g\in\overline{\Gamma}^{+}, \ \|x\|=1} \frac{\partial \varphi(x)}{\partial g}.$$

Lemma 2. For the point $x^* \in \Omega$ to be the minimum of the function $\varphi(x)$ on the set Ω it is necessary that $\psi(x^*) \geq 0$ or in the equivalent form $\overline{\Gamma}^{+}(x^*) \bigcap L(x^*) \neq \emptyset$.

Let's consider

$$\rho(x) = \inf_{z \in L(x),\ y \in \overline{\Gamma}^+(x)} \|z - y\|. \tag{3}$$

Since $L(x)$ and $\overline{\Gamma}^+(x)$ are closed sets and $L(x)$ is bounded, the infimum in (3) is attained.

If $\rho(x) > 0$ then vector $g = \frac{y(x)-z(x)}{\|y(x)-z(x)\|}$ is the vector of the steepest descent for the function $\varphi(x)$ on the set Ω at the point x. The initial point is determined as a solution for $\sum_{i=1}^{m}(b_i - a_i x)^2 \to \min_{x \geq 0}$.

The proposed algorithm was tested.

Problem of Approximation with Variation of All Data

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The problem of approximating a function $y = \varphi(x)$, $x \in \mathbf{R}^m$, $y \in \mathbf{R}$ is considered. The function is given by the table of the data (x^i, y^i), $i = 1, \ldots, n$. The problem is to find a function $y = f(x)$ such that the sum of squares of deflections of points (x^i, y^i) from the schedule of the function in $\mathbf{R}^{m+1}$ is minimal.

The probabilistic substantiation of a method which is distinct from those for LS is resulted. For linear approximation function the simple mathematical method of determination of parameters is constructed.

If $x \in \mathbf{R}$, $f(x) = ax + b$, $a \in \mathbf{R}$, $b \in \mathbf{R}$, the criterion is

$$\Phi = \sum_{i=1}^{n} \frac{(ax^i - y^i + b)^2}{a^2 + 1} \to \min_{(a,b)}. \tag{1}$$

Denote $\mu_x = \frac{1}{n}\sum_{i=1}^{n} x^i$, $D_x = \frac{1}{n}\sum_{i=1}^{n}(x^i - \mu_x)^2$, $\mu_y = \frac{1}{n}\sum_{i=1}^{n} y^i$, $D_y = \frac{1}{n}\sum_{i=1}^{n}(y^i - \mu_y)^2$, $\mu_{xy} = \frac{1}{n}\sum_{i=1}^{n} x^i y^i$, $M = \mu_{xy} - \mu_x \mu_y$.

The problem (1) has the single solution $a = \frac{D_y - D_x}{2M} + \frac{\sqrt{(D_y - D_x)^2 + 4M^2}}{2M}$, $b = \mu_y - a\mu_x$ at $M \neq 0$; $a = 0$, $b = \mu_y$ at $M = 0$, $D_x > D_y$; has no solution at $M = 0$, $D_x < D_y$ and any pair (a, b) with $a \in \mathbf{R}$ and $b = \mu_y - a\mu_x$ is a solution at $M = 0$, $D_x = D_y$.

For treating a general case, the problem of approximation is formalized as a problem of minimum variation for incompatible linear system. The approximation function is $f(x) = \langle a, x\rangle + b$, $a \in \mathbf{R}^m$, $b \in \mathbf{R}$. Denote $\bar{y} = (y^1, y^2, \dots y^n) \in \mathbf{R}^n$, $\tilde{x}^i = (x_1^i, x_2^i, \dots, x_m^i, 1) \in \mathbf{R}^{m+1}$, $\tilde{a} = (a, b) \in \mathbf{R}^{m+1}$, $\tilde{X}$ is a matrix, which rows are the vectors $\tilde{x}^i$, H is a matrix of the dimension $n \times (m+1)$ with zero last column, $h \in \mathbf{R}^n$; $M(H, h) = \{\tilde{a}\colon \left(\tilde{X} + H\right)\tilde{a} = \bar{y} + h\}$. Then the problem of variation (approximation) is as follows:

$$\Phi(H, h) = \|H\|^2 + \|h\|^2 \to \min_{(H,h):M(H,h)\neq\emptyset}, \tag{2}$$

where $\|H\|^2 = \sum\limits_{i=1}^{n}\sum\limits_{j=1}^{m+1} h_{ij}^2$.

The solution of the problem (2) is $\min\limits_{(H,h):M(H,h)\neq\emptyset} \Phi(H, h) = \lambda_{min}(D)$, $a = \frac{z}{z_0}$ (if $z = 0$, the problem has no solution), $b = \mu_y - \langle a, \mu_x\rangle$, where $D = B_1^T\left(E - \frac{b_2 b_2^T}{\|b\|^2}\right)B_1$, $B_1 = (-\bar{y}, X)$, X is a matrix, which rows are the vectors x^i, $b_2 = (1, 1, \dots, 1)^T \in \mathbf{R}^n$, $\bar{z} = (z_0, z) \in \mathbf{R}^{m+1}$ is an eigenvector of D, corresponding to the minimal eigenvalue $\lambda_{min}(D)$.

Some Approaches to Simultaneous Use of Forecasts Based on a Mathematical Modeling and Expert Forecasts

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The report considers the techniques of dot and interval forecasting for a process with a discrete time. Report [1] has already dealt with this problems. The techniques offered in the report for dot and interval forecasting specify the simultaneous use of two types of forecasting, namely forecasting with the use of mathematical modeling and expert forecasting.

The report contains different approaches to the simultaneous use of the above mentioned techniques of forecasting. These approaches depend on the nature of the process forecasted and the terminology used by specialists of the appropriate field. For example, expert forecasts based on the use of some expertise can be applied for a correction of a forecast based on the mathematical modeling. In particular expected forecasts can be obtained as a summarized result of the two above mentioned forecasts multiplied by coefficients α and $1-\alpha$. The parameter α satisfies the formula $0 \leq \alpha \leq 1$. For summarized processes the parameter α can be replaced by a function $\alpha(\tau)$. In the formula $\alpha(\tau)$ the variable τ means a current time, τ belongs to the interval $t \leq \tau \leq t + T$. In this formula a parameter T is a range forecast and a parameter t is an initial moment of a forecasting.

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The Choice of the Net Structure and User Strategies in the Internet

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The problems of the Internet services taking into account the various characteristics of the communicative information are described. The significant attention is given to the problems of the information Internet-technologies used on the financial markets. The problems of search for the rational net structure and coordination of user strategies under uncertainty are investigated. The results of the analytical researches and the computational experiments are presented. In particular, a simulation of the new schemes of Differentiated Services for the next-generation Internet is considered. DIFFERENTIATED SERVICE (Diff Service or DC) is a set of technologies, by which network service providers can offer different levels of network quality-of-service (QoS) to different customers and their traffic streams. Traditionally, the providers provide all customers with the same level of performance ("best effort service"). The so-called INTEGRATED SERVICE (Int Service or IS) supports individual flows with specific QoS requirement. On the other hand, in Diff Service, packets of the same QoS specification are grouped together and forwarded in the same manner, e.g., to a given subnet with some level of providing the service. Diff Service provides specific treatment (classification and forwarding) to packets from aggregate streams of the same QoS specification, irrespective of their individual flow. A major disadvantage of Diff Service in comparison with Int Service is that Diff Service does not provide a full guarantee to every application flow, especially for multimedia applications. We have proposed two new schemes of Diff Service to overcome above-mentioned difficulties.

1. ALPHA-FORWARDING DIFFSERVICE SCHEME makes the use of a flexible queue management mechanism for the efficient handling of

DS traffic. Each DS domain node has two different queues associated with each outgoing link: one for the "best effort" packets and the other for DS packets, where the length of the best-effort queue must be greater or equal to ALPHA.

2. DYNAMIC-DISTRIBUTED DIFFSERVICE SCHEME allows every DS to aggregate a fair chance of passing through the router in the same period of time, while guaranteeing an upper bound on the delay time and delay variations for multimedia data packets inside DS domain.

Both the schemes are attractive: they are simple and do not involve maintenance of complicated per-flow information. To simulate the proposed DS-schemes, we use the Network Simulator NS2 from UC Berkeley and develop some new features that support the Diff Service concept. We demonstrate its fair and efficient performance in terms of packets drop rate, mean packet delay, and inter packet delay via simulation of CBR and VBR flows.

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A Bayesian Approach to Uncertainty Aversion

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The Ellsberg paradox demonstrates that people's belief over uncertain events might not be representable by subjective probability. We argue that Uncertainty Aversion may be viewed as a case of Rule Rationality. This paradigm claims that people's decision making has evolved to simple rules that perform well in most regular environments. Such an environment consists of replicas of some basic singular circumstance. When the rule is applied to a singular environment, the behavior may seem paradoxical. We claim that the regular environment in which decisions under uncertainty take place, is described by one decision that spans multiple ambiguous risks, which are positively correlated. We show that when a risk averse individual has a Bayesian prior and uses a rule, which is optimal for the regular ambiguous environment, to evaluate a singular vague circumstance, then his behavior will exhibit uncertainty aversion. Thus, the behavior predicted by Ellsberg may be explained within the Bayesian expected utility paradigm.

JEL classification: D81
Keywords: Ellsberg paradox, rule rationality, ambiguity aversion, risk aversion, subjective probability.

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Financial Design Subject to Regulative Instructions of the Bank of Russia

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The behavior of commercial banks, as basic operators on the financial markets, essentially depends on the instructions and normative regulations of the Central bank of the Russian Federation [1-4]. The model of creditors and depository operations is studied. The ways of the further development of researches directed to the study of this influence on other segments of the financial market are discussed.

In the model, the following actions of the operator of a commercial bank are taken into account: per each trade day he can attract deposits under the announced rates, which depend on terms of attraction, and offer the credits under the announced rates, which depend on terms of granting. The credit resources of trade day develops from deposits attracted in this day, returns of credits given before, repayment of before attracted deposits, the percentage income, the percentage charge, and a credit resource of the last day, if it was not spent during this last day.

If the instructions were absent then if the rates of granting exceed the rates of attraction, the operator would owe, not limiting himself, to attract and to place greatest possible volume of money resources. The rules of the Central bank first of all interfere with such unlimited decision. In the instruction N.1 [1], there are 17 specifications, which should be maintained by bank at its work on the financial market. Four of the specifications are left for formalization. In the work, the task of optimization of actions of the financial operator is formulated, and the profit of the bank is considered as criterion of optimality. The results of the numerical analysis of the model are demonstrated.

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Dual Algorithms for Polyhedral Approximation of Convex Feasible Sets in Criterial Space

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Approximation of the multiple-dimensional convex bodies is the important feature of the Feasible Goals Method (FGM). FGM is the multiple criteria decision support method based on visualization of the feasible sets in criterion space (see Lotov et al., 1999). Let $\mathbf{C}_0$ be the class of compact convex bodies C in $\mathbf{E}^d$ ($d \geq 2$) such as $0 \in \mathrm{int}C$. Let ρ be the distance in $\mathbf{E}^d$, $g(C,u)$ be the support function, and $g^*(x,C) = Ox/Ox_0$, where $x_0 = [Ox) \cap \partial C$. Let $H(u,C) = \{x \in \mathbf{E}^d\colon < u, x > \leq g(u,C)\}$ for any $u \in \mathbf{E}^d \backslash \{0\}$. Then $x/g^*(x,C) \in \partial C$ for any $x \in \mathbf{E}^d \backslash \{0\}$.

Let $C^* = \{x \in \mathbf{E}^d\colon < x, y > \leq 1,\ y \in C\}$ be the polar set for C. Then $g(u,C^*) = g^*(u,C)$ and $g^*(x,C^*) = g(x,C)$. Let $\mathbf{P}^i$ and $\mathbf{P}^e$ be the classes of inscribed and superscribed convex polyhedra for C. For $P \in \mathbf{P}^i$ ($P \in \mathbf{P}^e$) let $M^t(P)$ ($M^f(P)$) be the set of its vertices (facets). Let $S^{d-1} = \{x \in \mathbf{E}^d\colon < x, x > = 1\}$, $S(x,C) = \{u \in S^{d-1}\colon < u, x > = g(u,C)\}$, $T(u,C) = \{x \in \partial C\colon < u, x > = g(u,C)\}$. Approximation algorithms can be based on iterative cutting and augmentation schemes (Kamenev, 1992).

Augmentation scheme. Let $P^n \in \mathbf{P}^i$.
Step 1. Choose the point $p_n \in \partial C$.
Step 2. Construct $P^{n+1} = \mathrm{conv}\{p_n, P^n\}$.

Cutting scheme. Let $P^n \in \mathbf{P}^e$.
Step 1. Choose the direction $u_n \in S^{d-1}$.
Step 2. Construct $P^{n+1} = P^n \cap H(u_n, C)$.

We define the sequence $\{P^n\}$ generated by augmentation (cutting) scheme as H(γ,C)-augmentation (cutting) sequence if there exists a constant $\gamma > 0$ such that $\delta(\mathrm{P}^n, P^{n+1}) \geq \gamma\delta(\mathrm{P}^n, C)$ for Hausdorff metrics δ. We define $\{P^n\}$ as $H_1(\gamma$,C)-augmentation sequence if it is generated by augmentation scheme and there exists a constant $\gamma > 0$ such that for each n there exists $u \in S(p_n, C)$, such that $g(u,C) - g(u,P^n) \geq \gamma\delta^H(P^n, C)$. We define $\{P^n\}$ as $H_1(\gamma$,C)-cutting sequence if it is generated by cutting scheme and there exists a constant $\gamma > 0$ such that for each n there exists $p \in T(u_n, C)$, such that $g(u_n, p/g^*(p, P^n)) - g(u_n, C) \geq \gamma\delta^H(P^n, C)$. Then there holds the following dual property of augmentation and cutting sequences.

Theorem. Let $C \in \mathbf{C}_0$ and $\{P^n\}$ be H(γ,C) ($H_1(\gamma$,C))-cutting sequence. Then there exists the constant $\gamma^* > 0$, depending on γ, C, P^0, such that $\{(P^n)^*\}$ be H(γ^*, C^*) ($H_1(\gamma^*, C^*)$)-augmentation sequence.

This result gives the opportunity to construct cutting algorithms of external polyhedral approximation of convex bodies using optimal iterative augmentation algorithms (see Kamenev, 1992, 1999).

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Architecture of Self-Learning Controller

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The method of designing the archiecture of self-learning controller is considered. Fuzzy information processing is used to describe controller operation. The controller uses a fuzzy neural network controlled by expert information unit. An example of application to the artificial ventilation of lungs is considered.

Hedging of Prices on Commodity Markets with the Use of Put Options

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In the international financial practice, the financial derivative instruments including futures, forwards, swaps, and options are widely used for insurance of prices for different types of assets. Over-the-counter derivatives trading as well as exchange traded instruments markets have developed. The objective of managing price risks is also important for Russian producers of commodity products. For example, price situation in the energy market at the end of 1998, when the prices for Brent fell below 10 dollars per barrel, have put the domestic companies in very severe conditions. Many of them were forced to apply for western credits to finance the production. During the process of making practical decisions related with managing price risks, the industrial companies frequently face certain difficulties in finding out correct analytical approaches to formulation of a hedging problem: in what cases and how to insure? We concentrate here on consideration of put options for the purposes of hedging. In the work, the analytical decision of the problem and results of computational experiments are presented.

Let S be a unit of the asset, which price changes in time under the known law. If the owner of the asset wants to be insured against the losses connected with the fall of the market price for the base asset, he can get options on the sale (put options) with a level of execution X and time of execution T. The cost of such option is calculated under the Black-Scholes formula. If the sum C is enclosed in the insurance, a quantity of the options may be bought. Cost of the portfolio is calculated on the base of the cost of the base asset and bought options at the moment T. The concept of function of a prize, which reflects financial flows given at the moment T, is entered. So the owner task is to minimize expenses for the insurance provided the meaning of the prize function not below than some level of payments with given probability. It is assumed that the owner of the asset wishing to be insured does not pursue the speculative purposes. The decision of the task is reduced to the decision of the algebraic system of the simple form.

Models of Bioelectric Processes in the Myocardium on the Basis of Autoregression at Availability of Martingale-Difference-Type Hindrances

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One of the main directions of the theory of operations research is mathematical modeling. One of the relevant tasks is to design a model of the evolution of bioelectric processes in the myocardium on the basis of the model of time-space evolution of bioelectric potentials [1]. The space model is parametrical with spherical basis functions. The dynamics is described by multidimensional autoregression. We assume that hindrances of observations are available with zero mathematical expectation (spatial estimators of parameters are unbiased).

At availability of hindrances of observations the least-squares method is not applicable [2]. In the paper, the following recurrent procedure of estimating parameters is offered.

The criterion is the ratio of two quadratic forms: the sum of quadrates of aberrations is in the numerator and certain quadratic form, depending upon obscure parameters, is in the denominator. The recurrent algorithm uses the information about gradients of the criteria with respect to the obscure arguments.

Basing on the theory of asymptotic continious models under standard limitations of martingale-difference-type interference, the almost surely convergence of the algorithm is proved.

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Picture State Parameters in the Monitored Space Estimation

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In this paper, we consider mathematical methods, computational algorithms, and programs for solving the problems of space state monitoring using the information system data. The inputs are produced as a statistical field. They are formed on the basis of the information received from various measuring devices of the information system.

The main problems of space state monitoring are

- detection of the enviroment changes;
- suppression of the picture noise;
- determination of the geometric characteristics of objects on a picture;
- estimation of the trajectory parameters and prediction of the movement of geometric centers of objects;
- representation of current information on PC display and the picture classification.

Detection of enviroment state changes is implemented on the basis of the likelihood ratio, linear filter regarding nonstationarity of useful input signal, and nonlinear filter described by 2D Hammerstein's operator. For picture noise suppression Zadeh-Ragaziny's and V.M.Semenov's two-dimensional linear filters are used. The system of equations for calculating weight functions and characteristics of probabilistic errors are developed for these filters. The method for solving linear filtration equation is proposed for the case of quasi-stationary random process. It's proposed to solve this equation with the use of discrete variants of direct methods of variational calculus.

Equations are obtained for the weight function of two-dimensional minimaximal filter by V.M.Semenov. The noise suppression is implemented by two-dimensional nonlinear median filter. The formulas are obtained for calculating probabilistic characteristics of the median filter. They are calculated by using Lagrange's splines. The method is proposed for calculating statistical modes for different distribution functions. The calculations are based on solving a first-order differential equation. An estimation of the picture parameters is made after the noise suppression. In this case, the approximated picture is described by two-dimensional spline function. Two methods are proposed for solving the linear algebraic system with the least-squares method. They are the splitting-up method and the method using conjugate splines. If one of the components of measuring error is bounded then the problem of estimating the picture parameters is represented by a nonlinear mathematical programming problem.

As a result of determination of contours from discontinuous sample, their appoximation is made. The parametric splines of different types and degrees are used for the purpose. With the aid of these splines the following characteristics of objects are calculated: the perimeter length, the curvature contour and radius, the square, and the coordinates of geometric center. The prolongation of trajectories of geometric centres is implemented by polynomials of 1-4th degrees using filters with limited memory.

The high efficiency of proposed methods for natural noises has been confirmed by simulation.

Modeling and Optimization of Interaction between the Investor and Several Regions Taking Into Consideration Their Investment and Financial Policy

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The process of the interaction between the investor and several regions is considered in the paper. The approach proposed earlier by Beklaryan L.A. and Sotsky S.V. for the case of a single region is developed.

The corresponding mathematical models are worked out and the algorithm for searching optimal solutions of the subsequent non-linear optimization problems are suggested.