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From chaos to order. Difference equations in one ecological problem

Abstract: In this paper we consider properties of the difference equations (discrete mappings) obtained in the study of the population dynamics of lemmings. A bifurcation scenario is proposed for obtained equations. Certain stability zones appear under this scenario with periods varying in order of natural series and also zones with more complicated modes. The study of transitional zones ('ordering of the chaos') is performed with the use of analytic calculations and computational experiments. Numerical analysis of mappings uses the methods of approximation of implicitly specified sets allowing us to construct and visualize sets of 'resonance' parameters including the front of the so-called singularity of 'blue sky'.

Keywords: Difference equations, discrete mappings, computational experiment, methods of approximations of implicitly specified sets, dialog decision maps.

MSC 2010: 37M20, 65P30

Since the end of last century the interest in difference equations (DE) was largely driven by environmental problems. The initial impetus was given by R. May in [8] where the term 'chaos' was used in model description of biological populations by a logistic equation, this term was introduced by T. Li and J. York [5], 'the cycle of period three creates a chaos'. More precisely, as was shown earlier by A. N. Sharkovsky [12], the existence of a cycle of period three implies the existence of a cycle of any period (in the continuous mapping of a unit segment onto itself). In fact, with the filing of R. May [8] the logistic equation was the focus of studying the difference equations [2].

Describing the population dynamics of animals within the framework of mathematical models of tundra populations and communities [1, 11, 13], one succeeded in justification of the type of difference equation (DE) differing from conventional logistics. It also represents a unimodal mapping of the unit segment onto itself and consists of three segments, two of which have the absolute value of the derivative exceeding one, and the third is a constant (a segment of horizontal line), the numerical value of this constant is taken as the bifurcation parameter.

For this type of equations and the chosen bifurcation parameters, a scenario of its change was proposed so that stability zones with stable cycles appear sequentially [9]. Inside the zone of stability the period of cycles is constant, in the transition from one zone to another the period changes according to the sequence of natural numbers. Stability zones are separated from each other by transitional domains with more complex (quasistationary) modes. In this case the domains of stability (for parameters of the equation allowing one to reproduce the population dynamics close to the real one) are much wider than the transitional zones. In contrast with traditional studies [2], the emphasis shifted here, the purpose of this research is not just to prove the existence of cycles of different periods, but the search for a region of their stability. The representation of DE in the form of straight line segments makes the problem of determination of periodic orbits and stability domains solvable by analytic methods. This allows one to analyze the so-called region of chaos (where trajectories are very sensitive to small changes of parameters) by using the procedure of consecutive consideration

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