```
> restart;
```

- Load from http://www.ccas.ru/ca/_media/truncatedseries2020.zip the archive with two files: maple.ind and maple.lib.
- Put these files to some directory, for example to ”/usr/userlib”.
- Assign libname := ”/usr/userlib”, libname in the Maple session.

```
> libname := "maple.lib", libname :
> with(TruncatedSeries);
```

$$[FormalSolution, LaurentSolution, RegularSolution] \tag{1}$$

Samples from
S. A. Abramov, A. A. Ryabenko, and D. E. Khmelnov, Truncated Series and Formal Exponential-Logarithmic Solutions of Linear Ordinary Differential Equations // Computational Mathematics and Mathematical Physics, 2020, in print.

```
>
```

Section 3

```
> Example1 := (-x^2 + O(x^4)) θ(y(x), x, 5) + (x^3 + O(x^4)) θ(y(x), x, 4) + (x + O(x^3)) θ(y(x), x, 3)
      + (x^3 + O(x^4)) θ(y(x), x, 2) + (x + O(x^3)) θ(y(x), x, 1) + (-1 + O(x)) y(x) :
> FormalSolution(Example1, y(x));
```

$$\left[e^{-\frac{3}{x^{1/3}}} x^{2/3} \left(\_c_1 - \frac{16\,\_c_1\, x^{1/3}}{9} + O\left(x^{2/3}\right) \right) + e^{-\frac{3\,RootOf\left(\_Z^2 + \_Z + 1, index = 1\right)}{x^{1/3}}} x^{2/3} \left(\_c_2 + \left(\frac{16}{9} + \frac{16\,RootOf\left(\_Z^2 + \_Z + 1, index = 1\right)}{9} \right) \_c_2\, x^{1/3} + O\left(x^{2/3}\right) \right) + e^{-\frac{3\,RootOf\left(\_Z^2 + \_Z + 1, index = 2\right)}{x^{1/3}}} x^{2/3} \left(\_c_3 + \left(\frac{16}{9} + \frac{16\,RootOf\left(\_Z^2 + \_Z + 1, index = 2\right)}{9} \right) \_c_3\, x^{1/3} + O\left(x^{2/3}\right) \right) + e^{\frac{2}{\sqrt{x}}} x^{5/4} \left(\_c_4 - \frac{15\,\_c_4\sqrt{x}}{16} + O(x) \right) + e^{-\frac{2}{\sqrt{x}}} x^{5/4} \left(\_c_5 + \frac{15\,\_c_5\sqrt{x}}{16} + O(x) \right) \right] \tag{1.1}$$

```
>
> Example2 := (-x^2 + O(x^4)) θ(y(x), x, 5) + (x^3 + O(x^4)) θ(y(x), x, 4) + (x + O(x^3)) θ(y(x), x, 3)
      + (O(x^3)) θ(y(x), x, 2) + (O(x)) θ(y(x), x, 1) + (-1 + O(x)) y(x) :
> FormalSolution(Example2, y(x));
```

$$\left[e^{-\frac{3}{x^{1/3}}} x^{2/3} \left(\_c_1 + O\left(x^{1/3}\right) \right) + e^{-\frac{3\,RootOf\left(\_Z^2 + \_Z + 1, index = 1\right)}{x^{1/3}}} x^{2/3} \left(\_c_2 + O\left(x^{1/3}\right) \right) + e^{-\frac{3\,RootOf\left(\_Z^2 + \_Z + 1, index = 2\right)}{x^{1/3}}} x^{2/3} \left(\_c_3 + O\left(x^{1/3}\right) \right) + e^{\frac{2}{\sqrt{x}}} x^{5/4} \left(\_c_4 + O\left(\sqrt{x}\right) \right) + e^{-\frac{2}{\sqrt{x}}} x^{5/4} \left(\_c_5 + O\left(\sqrt{x}\right) \right) \right] \tag{1.2}$$

```
>
> Example3 := (O(x^2)) θ(y(x), x, 5) + (x^3 + O(x^4)) θ(y(x), x, 4) + (x + O(x^3)) θ(y(x), x, 3)
      + (O(x^3)) θ(y(x), x, 2) + (O(x)) θ(y(x), x, 1) + (-1 + O(x)) y(x) :
> FormalSolution(Example3, y(x));
```

$$\left[e^{-\frac{3\,\mathrm{RootOf}(\_Z^2+\_Z+1,\,index=1)}{x^{1/3}}} y_{reg,1}\left(x^{1/3}\right) + e^{-\frac{3\,\mathrm{RootOf}(\_Z^2+\_Z+1,\,index=2)}{x^{1/3}}} y_{reg,2}\left(x^{1/3}\right) + e^{-\frac{3}{x^{1/3}}} y_{reg,3}\left(x^{1/3}\right) + y_{irr}(x) \right] \tag{1.3}$$

```
> 
```

```
> Example3_1 := (-x^2 + O(x^4)) θ(y(x), x, 5) + (x^3 + O(x^4)) θ(y(x), x, 4) + (x + O(x^2)) θ(y(x), x, 3)
      + (O(x^5)) θ(y(x), x, 2) + (O(x^5)) θ(y(x), x, 1) + (-1 + O(x)) y(x) :
```

```
> FormalSolution(Example3_1, y(x));
```

$$\left[e^{-\frac{3\,\mathrm{RootOf}(\_Z^2+\_Z+1,\,index=1)}{x^{1/3}}} x^{2/3}\left(\_c_1 + \left(\frac{7}{9} + \frac{7\,\mathrm{RootOf}(\_Z^2+\_Z+1,\,index=1)}{9}\right)\_c_1\, x^{1/3} + O\left(x^{2/3}\right)\right) + e^{-\frac{3\,\mathrm{RootOf}(\_Z^2+\_Z+1,\,index=2)}{x^{1/3}}} x^{2/3}\left(\_c_2 + \left(\frac{7}{9} + \frac{7\,\mathrm{RootOf}(\_Z^2+\_Z+1,\,index=2)}{9}\right)\_c_2\, x^{1/3} + O\left(x^{2/3}\right)\right) + e^{-\frac{3}{x^{1/3}}} x^{2/3}\left(\_c_3 - \frac{7\,\_c_3\, x^{1/3}}{9} + O\left(x^{2/3}\right)\right) + e^{-\frac{2}{\sqrt{x}}} x^{5/4}\left(\_c_4 + O\left(\sqrt{x}\right)\right) + e^{\frac{2}{\sqrt{x}}} x^{5/4}\left(\_c_5 + O\left(\sqrt{x}\right)\right) \right] \tag{1.4}$$

```
> 
```

```
> Example4 := (-x^3 + O(x^4)) θ(y(x), x, 5) + (x^3 + O(x^4)) θ(y(x), x, 4) + (x + O(x^3)) θ(y(x), x, 3)
      + (O(x^3)) θ(y(x), x, 2) + (O(1)) θ(y(x), x, 1) + (-1 + O(x)) y(x) :
```

```
> FormalSolution(Example4, y(x));
```

$$\left[y_1(x) + y_{irr,1}(x) + y_{irr,2}(x) + e^{-\frac{1}{x}} y_{reg,1}(x) + e^{\frac{1}{x}} y_{reg,2}(x) \right] \tag{1.5}$$

```
> 
```

```
> Example4_1 := (-x^3 + O(x^4)) θ(y(x), x, 5) + (x^3 + O(x^4)) θ(y(x), x, 4) + (x + O(x^3)) θ(y(x), x, 3)
      + (x^3 + O(x^4)) θ(y(x), x, 2) + (1 + O(x)) θ(y(x), x, 1) + (-1 + O(x)) y(x) :
```

```
> TruncatedSeries:-RegularSolution(Example4_1, y(x))
```

$$\left[x\left(\_c_1 + O(x)\right) \right] \tag{1.6}$$

```
> FormalSolution(Example4_1, y(x));
```

$$\left[x\left(\_c_1 + O(x)\right) + e^{-\frac{2\,\mathrm{RootOf}(\_Z^2+1,\,index=1)}{\sqrt{x}}} x^{1/4}\left(\_c_2 + O\left(\sqrt{x}\right)\right) + e^{-\frac{2\,\mathrm{RootOf}(\_Z^2+1,\,index=2)}{\sqrt{x}}} x^{1/4}\left(\_c_3 + O\left(\sqrt{x}\right)\right) + e^{-\frac{1}{x}} y_{reg,1}(x) + e^{\frac{1}{x}} y_{reg,2}(x) \right] \tag{1.7}$$

```
> 
> 
```

Section 4

```
> Example5 := O(x^2) * theta(y(x), x, 2) + (x^2 + O(x^3)) * theta(y(x), x, 1) + O(x^3) * y(x) :
  FormalSolution(Example5, y(x));
```

$$\left[y_{reg}(x) \right] \tag{2.1}$$

```
> Example5_1 := (x^2 + O(x^3)) θ(y(x), x, 2) + (x^2 + O(x^3)) θ(y(x), x, 1) + (x^3 + O(x^4)) y(x) :
> FormalSolution(Example5_1, y(x));
```

$$\left[\_c_1 - \frac{\_c_1\, x}{2} + O\left(x^2\right) + y_{reg}(x) \right] \tag{2.2}$$

```
> Example5_2 := (-x^2 + O(x^3)) θ(y(x), x, 2) + (x^2 + O(x^3)) θ(y(x), x, 1) + (x^3 + O(x^4)) y(x):
> FormalSolution(Example5_2, y(x));
```

$$\left[x \left(\frac{\_c_1}{x} + \_c_2 + O(x) + \ln(x) \left(\_c_1 + O(x) \right) \right), x \left(\_c_2 + O(x) \right) \right] \tag{2.3}$$

Section 6

```
> (x + O(x^3)) θ(y(x), x, 2) + (x^2 + O(x^3)) θ(y(x), x, 1) + (2 + O(x^2)) y(x):
> FormalSolution(%, y(x));
```

$$\left[e^{-\frac{2\, RootOf\left(\_Z^2 + 2, index = 1\right)}{\sqrt{x}}} x^{1/4} \left(\_c_1 + \frac{RootOf\left(\_Z^2 + 2, index = 1\right) \_c_1 \sqrt{x}}{32} - \frac{521\, \_c_1\, x}{1024} + O\left(x^{3/2}\right) \right) + e^{-\frac{2\, RootOf\left(\_Z^2 + 2, index = 2\right)}{\sqrt{x}}} x^{1/4} \left(\_c_2 + \frac{RootOf\left(\_Z^2 + 2, index = 2\right) \_c_2 \sqrt{x}}{32} - \frac{521\, \_c_2\, x}{1024} + O\left(x^{3/2}\right) \right) \right] \tag{3.1}$$

```
> (x^2 + x^5 + O(x^6)) θ(y(x), x, 2) + (2 x + x^4 + O(x^5)) θ(y(x), x, 1) + (1 - x + x^3 + O(x^4)) y(x):
  FormalSolution(%, y(x))
```

$$\left[e^{\frac{1}{x}} \left(\_c_2 + \left(-\_c_2 + 2\, \_c_1 \right) x + O\left(x^2\right) + \ln(x) \left(\_c_1 - \_c_1\, x + O\left(x^2\right) \right) \right), e^{\frac{1}{x}} \left(\_c_2 - \_c_2\, x + O\left(x^2\right) \right), e^{\frac{1}{x}} \left(2\, \_c_1\, x + O\left(x^2\right) + \ln(x) \left(\_c_1 - \_c_1\, x + O\left(x^2\right) \right) \right) \right] \tag{3.2}$$

```
> (O(x^4)) θ(y(x), x, 2) + (O(x)) θ(y(x), x, 1) + O(1) y(x):
> FormalSolution(%, y(x));
```

FAIL (3.3)

No result

```
> (O(x^4)) θ(y(x), x, 2) + (O(x)) θ(y(x), x, 1) + (2 + O(x^2)) y(x):
> FormalSolution(%, y(x));
```

FAIL (3.4)

First term of one exponential part

```
> (O(x^4)) θ(y(x), x, 2) + (3 x + O(x^2)) θ(y(x), x, 1) + (2 + O(x^2)) y(x):
> FormalSolution(%, y(x));
```

$$\left[e^{\frac{2}{3x}} y_{reg}(x) \right] \tag{3.5}$$

One exponential part and its regular part

```
> (O(x^4)) θ(y(x), x, 2) + (3 x + O(x^3)) θ(y(x), x, 1) + (2 + O(x^2)) y(x):
> FormalSolution(%, y(x));
```

$$\left[e^{\frac{2}{3x}} \left(\_c_1 + O(x) \right) \right] \tag{3.6}$$

>

First terms of both exponential parts (no regular parts)

> $\left(4x^4 + O(x^5)\right) \theta(y(x), x, 2) + \left(3x + O(x^3)\right) \theta(y(x), x, 1) + \left(2 + O(x^2)\right) y(x) :$

> $FormalSolution(\%, y(x));$

$$\left[e^{\frac{2}{3x}} \left(\_c_1 + O(x) \right) + e^{\frac{1}{4x^3}} y_1(x) \right] \tag{3.7}$$

>

> $\left(4x^4 + O(x^8)\right) \theta(y(x), x, 2) + \left(3x + O(x^4)\right) \theta(y(x), x, 1) + \left(2 + O(x^2)\right) y(x) :$

> $FormalSolution(\%, y(x));$

$$\left[e^{\frac{2}{3x}} \left(\_c_1 + O(x) \right) + e^{\frac{1}{4x^3} - \frac{2}{3x}} y_{reg}(x) \right] \tag{3.8}$$

>

Both exponential parts with their regular parts

> $\left(4x^4 + O(x^8)\right) \theta(y(x), x, 2) + \left(3x + O(x^5)\right) \theta(y(x), x, 1) + \left(2 + O(x^2)\right) y(x) :$

> $FormalSolution(\%, y(x));$

$$\left[e^{\frac{2}{3x}} \left(\_c_1 + O(x) \right) + e^{\frac{1}{4x^3} - \frac{2}{3x}} x^3 \left(\_c_2 + O(x) \right) \right] \tag{3.9}$$

>

Both exponential parts with more terms of their regular parts

> $\left(4x^4 + O(x^9)\right) \theta(y(x), x, 2) + \left(3x + O(x^6)\right) \theta(y(x), x, 1) + \left(2 + O(x^4)\right) y(x) :$

> $FormalSolution(\%, y(x));$

$$\left[e^{\frac{2}{3x}} \left(\_c_1 - \frac{16 \_c_1 x}{27} - \frac{196 \_c_1 x^2}{729} + O(x^3) \right) + e^{\frac{1}{4x^3} - \frac{2}{3x}} x^3 \left(\_c_2 + \frac{16 \_c_2 x}{27} + O(x^2) \right) \right] \tag{3.10}$$

>

>