```
> restart;
> libname := "maple.lib", libname;
```
$$libname := \text{"maple.lib"}, \text{"/opt/maple2019/lib"} \tag{1}$$

```
> with(TruncatedSeries);
```
$$[LaurentSolution, RegularSolution] \tag{2}$$

```
> eq1 := (x + O(x^2)) * (theta(y(x), x, 1)) + (-x + O(x^2)) * y(x);
```
$$eq1 := \left(x + O(x^2)\right)\theta(y(x), x, 1) + \left(-x + O(x^2)\right)y(x) \tag{3}$$

```
> LaurentSolution(eq1, y(x));
```
$$\left[x\,\_c_1 + O(x^2)\right] \tag{4}$$

```
> LaurentSolution(eq1, y(x), `degree`=2);
```
$$\left[x\,\_c_1 + x^2\left(-U_{[0,\,2]}\,\_c_1 - U_{[1,\,2]}\,\_c_1\right) + O(x^3)\right] \tag{5}$$

```
> eq2 := (x + O(x^3)) * (theta(y(x), x, 1)) + (-x + x^3/2 + O(x^4)) * y(x);
```
$$eq2 := \left(x + O(x^3)\right)\theta(y(x), x, 1) + \left(-x + \frac{x^3}{2} + O(x^4)\right)y(x) \tag{6}$$

```
> LaurentSolution(eq2, y(x));
```
$$\left[x\,\_c_1 + O(x^3)\right] \tag{7}$$

```
> eq3 := (x + x^2/2 + O(x^3)) * theta(y(x), x, 1) + (-x-x^2-x^3/2 + O(x^4)) * y(x);
```
$$eq3 := \left(x + \frac{x^2}{2} + O(x^3)\right)\theta(y(x), x, 1) + \left(-x - x^2 - \frac{x^3}{2} + O(x^4)\right)y(x) \tag{8}$$

```
> LaurentSolution(eq3, y(x));
```
$$\left[x\,\_c_1 + \frac{x^2\,\_c_1}{2} + O(x^3)\right] \tag{9}$$

```
> eq4 := (-1 + x + O(x^2)) * theta(y(x), x, 2) + (-2 + O(x^2)) * theta(y(x), x, 1) + (x + O(x^2)) * y(x);
```
$$eq4 := \left(-1 + x + O(x^2)\right)\theta(y(x), x, 2) + \left(-2 + O(x^2)\right)\theta(y(x), x, 1) + \left(x + O(x^2)\right)y(x) \tag{10}$$

```
> LaurentSolution(eq4, y(x));
```
$$\left[\_c_1 + \frac{x\,\_c_1}{3} + O(x^2)\right] \tag{11}$$

```
> LaurentSolution(eq4, y(x), `degree`=3);
```
$$\left[\_c_1 + \frac{x\,\_c_1}{3} + x^2\left(\frac{1}{12}\,\_c_1 + \frac{1}{8}\,U_{[0,\,2]}\,\_c_1\right) + x^3\left(\frac{1}{36}\,\_c_1 + \frac{23}{360}\,U_{[0,\,2]}\,\_c_1 + \frac{1}{45}\,U_{[1,\,2]}\,\_c_1\right.\right. \tag{12}$$
$$\left.\left. + \frac{1}{45}\,\_c_1\,U_{[2,\,2]} + \frac{1}{15}\,U_{[0,\,3]}\,\_c_1\right) + O(x^4)\right]$$

```
> eq5 := (-1 + x + x^2 + O(x^3)) * theta(y(x), x, 2) + (-2 + O(x^3)) * theta(y(x), x, 1) + (x + 6*x^2
    + O(x^4)) * y(x);
```
$$eq5 := \left(-1 + x + x^2 + O(x^3)\right)\theta(y(x), x, 2) + \left(-2 + O(x^3)\right)\theta(y(x), x, 1) + \left(x + 6x^2 + O(x^4)\right)y(x) \tag{13}$$

```
> LaurentSolution(eq5, y(x));
```
$$\left[\frac{\_c_1}{x^2} - \frac{5\,\_c_1}{x} + \_c_2 + O(x), \_c_2 + \frac{x\,\_c_2}{3} + \frac{5\,x^2\,\_c_2}{6} + \frac{13\,x^3\,\_c_2}{30} + O(x^4)\right] \tag{14}$$

```
> eq6 := (-1 + x + x^2 + O(x^3)) * theta(y(x), x, 2) + (-2 + x^2 + O(x^3)) * theta(y(x), x, 1) + (x + 6*x
    ^2 + O(x^4)) * y(x);
```
$$eq6 := \left(-1 + x + x^2 + O(x^3)\right)\theta(y(x), x, 2) + \left(-2 + x^2 + O(x^3)\right)\theta(y(x), x, 1) + \left(x + 6x^2 \right. \tag{15}$$
$$\left. + O(x^4)\right)y(x)$$

```
> LaurentSolution(eq6, y(x));
```
$$\left[\_c_1 + \frac{x\,\_c_1}{3} + \frac{5\,x^2\,\_c_1}{6} + \frac{41\,x^3\,\_c_1}{90} + O(x^4)\right] \tag{16}$$

```
> eq7 := (1 + O(x)) * (theta(y(x), x, 1)) + (x^4 + O(x^5)) * y(x);
```
$$eq7 := \left(1 + O(x)\right)\theta(y(x), x, 1) + \left(x^4 + O(x^5)\right)y(x) \tag{17}$$

> *LaurentSolution*(*eq7*, *y*(*x*));

$$\left[\_c_1 - \frac{x^4 \_c_1}{4} + O(x^5) \right] \tag{18}$$

> *eq8* := (1 + O(*x*)) * (theta(*y*(*x*), *x*, 1)) + O(*x*) * *y*(*x*);

$$eq8 := (1 + O(x)) \, \theta(y(x), x, 1) + O(x) \, y(x) \tag{19}$$

> *LaurentSolution*(*eq8*, *y*(*x*));

$$\left[\_c_1 + O(x) \right] \tag{20}$$

> *eq9* := (2 + O(*x*)) * (theta(*y*(*x*), *x*, 1)) + (1 + O(*x*)) * *y*(*x*);

$$eq9 := (2 + O(x)) \, \theta(y(x), x, 1) + (1 + O(x)) \, y(x) \tag{21}$$

> *LaurentSolution*(*eq9*, *y*(*x*));

$$[\ ] \tag{22}$$

> *eq10* := (-*x* + *x*^2 + *x*^3 + O(*x*^4)) * (*diff*(*y*(*x*), *x*, *x*)) + (-3 + *x* + O(*x*^2)) * (*diff*(*y*(*x*), *x*)) + O(*x*^3) * *y*(*x*);

$$eq10 := \left(-x + x^2 + x^3 + O(x^4)\right) \left(\frac{d^2}{dx^2} y(x) \right) + \left(-3 + x + O(x^2)\right) \left(\frac{d}{dx} y(x) \right) + O(x^3) \, y(x) \tag{23}$$

> *LaurentSolution*(*eq10*, *y*(*x*));

$$\left[\_c_1 + O(x^4) \right] \tag{24}$$

> *eq11* := (*x*^2 + O(*x*^3)) * *diff*(*y*(*x*), *x*, *x*) + O(*x*) * *diff*(*y*(*x*), *x*) + (1 + O(*x*)) * *y*(*x*);

$$eq11 := \left(x^2 + O(x^3)\right) \left(\frac{d^2}{dx^2} y(x) \right) + O(x) \left(\frac{d}{dx} y(x) \right) + (1 + O(x)) \, y(x) \tag{25}$$

> *LaurentSolution*(*eq11*, *y*(*x*));

$$FAIL \tag{26}$$

>

> *eq12* := (-1 + *x* + *x*^2 + O(*x*^3)) * theta(*y*(*x*), *x*, 2) + (-2 + O(*x*^2)) * theta(*y*(*x*), *x*, 1) + (O(*x*^4)) * *y*(*x*);

$$eq12 := \left(-1 + x + x^2 + O(x^3)\right) \, \theta(y(x), x, 2) + \left(-2 + O(x^2)\right) \, \theta(y(x), x, 1) + O(x^4) \, y(x) \tag{27}$$

> *RegularSolution*(*eq12*, *y*(*x*));

$$\left[\_c_1 + O(x^4) \right] \tag{28}$$

> *eq13* := (-1 + *x* + *x*^2 + O(*x*^3)) * theta(*y*(*x*), *x*, 2) + (-2 + *x*^2 + O(*x*^3)) * theta(*y*(*x*), *x*, 1) + O(*x*^4) * *y*(*x*);

$$eq13 := \left(-1 + x + x^2 + O(x^3)\right) \, \theta(y(x), x, 2) + \left(-2 + x^2 + O(x^3)\right) \, \theta(y(x), x, 1) + O(x^4) \, y(x) \tag{29}$$

> *RegularSolution*(*eq13*, *y*(*x*));

$$\left[-\frac{\_c_1}{x^2} + \frac{4 \_c_1}{x} + \_c_2 + O(x) + \ln(x) \left(\_c_1 + O(x^4) \right), \_c_2 + O(x^4) \right] \tag{30}$$

> *RegularSolution*(*eq13*, *y*(*x*), 'output'='literal');

$$\ln(x) \left(\_c_1 + \frac{x^4 \, U_{[0,4]} \_c_1}{24} + O(x^5) \right) - \frac{\_c_1}{x^2} + \frac{4 \_c_1}{x} + \_c_2 + x \left(\frac{2}{3} \_c_1 U_{[1,3]} - \frac{4}{3} \_c_1 U_{[2,3]} \right) + x^2 \left(\right. \tag{31}$$

$$\left. -\frac{5}{12} \_c_1 U_{[1,3]} + \frac{1}{3} \_c_1 U_{[2,3]} - \frac{1}{8} U_{[0,4]} \_c_1 + \frac{1}{4} \_c_1 U_{[1,4]} - \frac{1}{2} \_c_1 U_{[2,4]} + \frac{1}{8} \_c_1 \right)$$

$$+ x^3 \left(\frac{2}{45} \_c_1 U_{[1,3]} - \frac{4}{45} \_c_1 U_{[2,3]} + \frac{7}{30} U_{[0,4]} \_c_1 - \frac{1}{5} \_c_1 U_{[1,4]} + \frac{2}{15} \_c_1 U_{[2,4]} + \frac{1}{30} \_c_1 \right.$$

$$\left. - \frac{1}{15} \_c_1 U_{[0,5]} + \frac{2}{15} \_c_1 U_{[1,5]} - \frac{4}{15} \_c_1 U_{[2,5]} \right) + x^4 \left(-\frac{7}{80} \_c_1 U_{[1,3]} + \frac{1}{20} \_c_1 U_{[2,3]} \right.$$

$$+ \frac{7}{180} U_{[0,4]} \_c_1 + \frac{7}{240} \_c_1 U_{[1,4]} - \frac{3}{40} \_c_1 U_{[2,4]} + \frac{7}{160} \_c_1 + \frac{17}{120} \_c_1 U_{[0,5]} - \frac{7}{60} \_c_1 U_{[1,5]}$$

$$+ \frac{1}{15} \_c_1 U_{[2,5]} - \frac{1}{36} U_{[2,3]} \_c_1 U_{[1,3]} - \frac{1}{18} \_c_1 U_{[2,3]}^2 + \frac{1}{36} \_c_1 U_{[1,3]}^2 + \frac{1}{24} U_{[0,4]} \_c_2$$

$$\left. - \frac{1}{24} \_c_1 U_{[0,6]} + \frac{1}{12} \_c_1 U_{[1,6]} - \frac{1}{6} \_c_1 U_{[2,6]} \right) + O(x^5)$$

`> RegularSolution(eq13, y(x), `degree`=2);`

$$\left[-\frac{\_c_1}{x^2} + \frac{4\,\_c_1}{x} + \_c_2 + x\left(\frac{2}{3}\,\_c_1\,U_{[1,3]} - \frac{4}{3}\,\_c_1\,U_{[2,3]}\right) + x^2\left(-\frac{5}{12}\,\_c_1\,U_{[1,3]} + \frac{1}{3}\,\_c_1\,U_{[2,3]} \right.\right. \tag{32}$$
$$\left. -\frac{1}{8}\,U_{[0,4]}\,\_c_1 + \frac{1}{4}\,\_c_1\,U_{[1,4]} - \frac{1}{2}\,\_c_1\,U_{[2,4]} + \frac{1}{8}\,\_c_1\right) + O(x^3) + \ln(x)\left(\_c_1 + O(x^3)\right), \_c_2$$
$$\left. + O(x^3)\right]$$

`> RegularSolution(eq13, y(x), 'output'='literal', 'degree'=2);`

$$-\frac{\_c_1}{x^2} + \frac{4\,\_c_1}{x} + \_c_2 + x\left(\frac{2}{3}\,\_c_1\,U_{[1,3]} - \frac{4}{3}\,\_c_1\,U_{[2,3]}\right) + x^2\left(-\frac{5}{12}\,\_c_1\,U_{[1,3]} + \frac{1}{3}\,\_c_1\,U_{[2,3]}\right. \tag{33}$$
$$\left. -\frac{1}{8}\,U_{[0,4]}\,\_c_1 + \frac{1}{4}\,\_c_1\,U_{[1,4]} - \frac{1}{2}\,\_c_1\,U_{[2,4]} + \frac{1}{8}\,\_c_1\right) + O(x^3) + \ln(x)\left(\_c_1 + O(x^3)\right)$$

`> eq14 := (-1 + x + x^2 + O(x^3)) * theta(y(x), x, 2) + (-2 + O(x^3)) * theta(y(x), x, 1) + O(x^4) * y(x);`

$$eq14 := \left(-1 + x + x^2 + O(x^3)\right)\theta(y(x), x, 2) + \left(-2 + O(x^3)\right)\theta(y(x), x, 1) + O(x^4)\,y(x) \tag{34}$$

`> RegularSolution(eq14, y(x));`

$$\left[\frac{\_c_1}{x^2} - \frac{4\,\_c_1}{x} + \_c_2 + O(x),\ \_c_2 + O(x^4)\right] \tag{35}$$

`> eq15 := (1 + x^2 + O(x^3)) * theta(y(x), x, 3) + (4 - x + (1/2)*x^2 + O(x^3)) * theta(y(x), x, 2) + (4 - 2*x + x^2 + O(x^3)) * theta(y(x), x, 1) + O(x^3) * y(x);`

$$eq15 := \left(1 + x^2 + O(x^3)\right)\theta(y(x), x, 3) + \left(4 - x + \frac{x^2}{2} + O(x^3)\right)\theta(y(x), x, 2) + \left(4 - 2x + x^2 \right. \tag{36}$$
$$\left. + O(x^3)\right)\theta(y(x), x, 1) + O(x^3)\,y(x)$$

`> RegularSolution(eq15, y(x));`

$$\left[\frac{\frac{21\,\_c_1}{16} + \frac{\_c_2}{2}}{x^2} + \frac{\_c_1}{x} + \_c_3 + O(x) + \ln(x)\left(\frac{\_c_1}{2x^2} + \_c_2 + O(x)\right) + \ln(x)^2\left(\frac{\_c_1}{2} + O(x^3)\right), \frac{\_c_2}{2x^2}\right. \tag{37}$$
$$\left. + \_c_3 + O(x) + \ln(x)\left(\_c_2 + O(x^3)\right), \_c_3 + O(x^3)\right]$$

`> eq16 := (-1 + x + O(x^3)) * theta(y(x), x, 2) + (-1 - x - (3/2)*x^2 + O(x^3)) * theta(y(x), x, 1) + (3/4 + (1/4)*x + (3/4)*x^2 + O(x^3)) * y(x);`

$$eq16 := \left(-1 + x + O(x^3)\right)\theta(y(x), x, 2) + \left(-1 - x - \frac{3x^2}{2} + O(x^3)\right)\theta(y(x), x, 1) + \left(\frac{3}{4} + \frac{x}{4} + \frac{3x^2}{4}\right. \tag{38}$$
$$\left. + O(x^3)\right)y(x)$$

`> RegularSolution(eq16, y(x));`

$$\left[\sqrt{x}\left(-\frac{2\,\_c_1}{x^2} + \frac{8\,\_c_1}{x} + \_c_2 + O(x) + \ln(x)\left(\_c_1 + O(x^3)\right)\right), \sqrt{x}\left(\_c_2 + O(x^3)\right)\right] \tag{39}$$

`> eq17 := (1 + O(x^2)) * theta(y(x), x, 3) + (1 + 2*x + O(x^2)) * theta(y(x), x, 2) + (2 + x + O(x^2)) * theta(y(x), x, 1) + (2 - x + O(x^2)) * y(x);`

$$eq17 := \left(1 + O(x^2)\right)\theta(y(x), x, 3) + \left(1 + 2x + O(x^2)\right)\theta(y(x), x, 2) + \left(2 + x + O(x^2)\right)\theta(y(x), x, 1) \tag{40}$$
$$+ \left(2 - x + O(x^2)\right)y(x)$$

`> RegularSolution(eq17, y(x));`

$$\left[\frac{\_c_1}{x} + O(x) + x^{RootOf(\_Z^2 + 2,\, index=1)}\left(\_c_2 - \frac{x\left(20 + 23\,RootOf(\_Z^2 + 2,\, index=1)\right)\_c_2}{54} + O(x^2)\right)\right. \tag{41}$$
$$\left. + x^{RootOf(\_Z^2 + 2,\, index=2)}\left(\_c_3 - \frac{x\left(20 + 23\,RootOf(\_Z^2 + 2,\, index=2)\right)\_c_3}{54} + O(x^2)\right)\right]$$

> $eq18 := (-x + x^2 + x^3 + O(x^4)) * (diff(y(x), x, x)) + (-3 + x + 2*x^2 + O(x^3)) * (diff(y(x), x)) + O(x^3) * y(x);$

$$eq18 := \left(-x + x^2 + x^3 + O(x^4)\right)\left(\frac{d^2}{dx^2}\, y(x)\right) + \left(-3 + x + 2\,x^2 + O(x^3)\right)\left(\frac{d}{dx}\, y(x)\right) + O(x^3)\, y(x) \tag{42}$$

> $RegularSolution(eq18, y(x));$

$$\left[-\frac{\_c_1}{x^2} + \frac{4\,\_c_1}{x} + \_c_2 + O(x) + \ln(x)\left(\_c_1 + O(x^4)\right),\ \_c_2 + O(x^4)\right] \tag{43}$$

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