```
> restart;
```

non-strongly non-singular matrix, 2×2, 1 marked element

```
> P := Matrix([[1 + T[1,1]*10^(-1), -3333*10^(-3)], [-3*10^(-1), 99*10^(-2)]]):
```

```
> Dt := LinearAlgebra:-Determinant(P);
```

$$Dt := -\frac{99}{10000} + \frac{99\,T_{1,1}}{1000} \tag{1.1}$$

```
> S := indets(Dt)
```

$$S := \{T_{1,1}\} \tag{1.2}$$

```
> solve(Dt);
  % >= 0 and % <= 1
```

$$\frac{1}{10}$$

$$true \tag{1.3}$$

```
> P := Matrix([[1 + T[1,1]*10^(-2), -3333*10^(-3)], [-3*10^(-1), 99*10^(-2)]]):
>
```

```
> Dt := LinearAlgebra:-Determinant(P);
```

$$Dt := -\frac{99}{10000} + \frac{99\,T_{1,1}}{10000} \tag{1.4}$$

```
> S := indets(Dt)
```

$$S := \{T_{1,1}\} \tag{1.5}$$

```
> solve(Dt);
  % >= 0 and % <= 1
```

$$1$$

$$true \tag{1.6}$$

non-strongly non-singular matrix, 2×2, 3 marked element

> $P := \begin{bmatrix} 1 & -3333 \cdot 10^{-3} - T_{1,2} \cdot 10^{-3} \\ -3 \cdot 10^{-1} - T_{2,1} \cdot 10^{-2} & 99 \cdot 10^{-2} + T_{2,2} \cdot 10^{-2} \end{bmatrix}$:

> $Dt := LinearAlgebra:-Determinant(P);$

$$Dt := -\frac{99}{10000} + \frac{1}{100} T_{2,2} - \frac{3333}{100000} T_{2,1} - \frac{3}{10000} T_{1,2} - \frac{1}{100000} T_{1,2} T_{2,1} \quad (2.1)$$

> $S := [indets(Dt)[\,]]$

$$S := [T_{1,2}, T_{2,1}, T_{2,2}] \quad (2.2)$$

> $exists(S, And(Dt = 0, seq([i \geq 0, i \leq 1][\,], i = S)))$

$$\exists \left([T_{1,2}, T_{2,1}, T_{2,2}], -\frac{99}{10000} + \frac{1}{100} T_{2,2} - \frac{3333}{100000} T_{2,1} - \frac{3}{10000} T_{1,2} - \frac{1}{100000} T_{1,2} T_{2,1} = 0 \wedge 0 \leq T_{1,2} \wedge T_{1,2} \leq 1 \wedge 0 \leq T_{2,1} \wedge T_{2,1} \leq 1 \wedge 0 \leq T_{2,2} \wedge T_{2,2} \leq 1 \right) \quad (2.3)$$

> $QuantifierElimination:-QuantifierEliminate(\%);$

$$true \quad (2.4)$$

>

```
1  StronglyNonSingular := proc(A)
2       local Dt, S, slv, condidtion, st, i, res;
3       Dt := LinearAlgebra:-Determinant(A);
4       S := [indets(Dt)[]];
5       if nops(S) = 0 then
6            res := Dt=0;
7       elif nops(S) = 1 then
8            slv := solve(Dt);
9            res := slv >= 0 and slv <= 1;
10      else
11           condidtion := exists(S, And(Dt = 0, seq([0 <= i, i <= 1][
12           st := time();
13           res := QuantifierElimination:-QuantifierEliminate(condidt
14           userinfo(1, procname, "Time of QuantifierEliminate: ", ti
15      end if;
```

> $infolevel[StronglyNonSingular] := 1$:

>

non-strongly non-singular matrix, 2×2, 1 marked element

```
> P := Matrix([[1 + T[1,1]*10^(-1), -3333*10^(-3)], [-3*10^(-1), 99*10^(-2)]]):
```

```
> StronglyNonSingular(P);
```

false **(3.1)**

```
> P := Matrix([[1 + T[1,1]*10^(-2), -3333*10^(-3)], [-3*10^(-1), 99*10^(-2)]]):
```

```
>
```

```
> StronglyNonSingular(P);
```

false **(3.2)**

strongly non-singular matrix, 2×2

```
> P := Matrix([[14*10^(-2) + T[1,1]*10^(-2), -78*10^(-1) - T[1,2]*10^(-1)], [9954*10^(-2) + T[2,1]*10^(-2), 46*10^(-2) + T[2,2]*10^(-2)]]):
```

```
> StronglyNonSingular(P);
```

```
StronglyNonSingular: Time of QuantifierEliminate:  19.349
```

true **(4.1)**

strongly non-singular matrix, 5×5, 1 marked element

```
> LinearAlgebra:-RandomMatrix(5, 5, generator = (rand(-999..999))):
```

$$\begin{bmatrix} 140 & -782 & 250 & -913 & -324 \\ 995 & 46 & 701 & 901 & -573 \\ 804 & -306 & -375 & 984 & 257 \\ -764 & -881 & 238 & -185 & -297 \\ -203 & 689 & -194 & -618 & -48 \end{bmatrix} \quad \textbf{(5.1)}$$

```
> LinearAlgebra:-RandomMatrix(5, 5, generator = (rand(1..2))):
```

$$\begin{bmatrix} 1 & 1 & 1 & 2 & 1 \\ 2 & 2 & 2 & 1 & 2 \\ 2 & 2 & 1 & 1 & 2 \\ 1 & 2 & 1 & 2 & 1 \\ 2 & 2 & 2 & 2 & 1 \end{bmatrix} \quad (5.2)$$

```
> P := [[14 + T[1,1]*10^(-1), -782*10^(-1), 25, -913*10^(-2), -324*10^(-1)],
        [995*10^(-2), 46*10^(-2), 701*10^(-2), 901*10^(-1), -573*10^(-2)],
        [804*10^(-2), -306*10^(-2), -375*10^(-1), 984*10^(-2), 257*10^(-1)],
        [-764*10^(-2), -881*10^(-2), 238*10^(-2), -185*10^(-2), -297*10^(-2)],
        [-203*10^(-2), 689*10^(-2), -194*10^(-2), -618*10^(-2), -48*10^(-1)]]:
```

```
> StronglyNonSingular(P)
```

true (5.3)

strongly non-singular matrix, 5×5, 2 marked elements

```
> P := [[14 + T[1,1]*10^(-1), -782*10^(-1), 25, -913*10^(-2), -324*10^(-1)],
        [995*10^(-2), 46*10^(-2), 701*10^(-2), 901*10^(-1), -573*10^(-2)],
        [804*10^(-2), -306*10^(-2) - T[3,2]*10^(-2), -375*10^(-1), 984*10^(-2), 257*10^(-1)],
        [-764*10^(-2), -881*10^(-2), 238*10^(-2), -185*10^(-2), -297*10^(-2)],
        [-203*10^(-2), 689*10^(-2), -194*10^(-2), -618*10^(-2), -48*10^(-1)]]:
```

```
> StronglyNonSingular(P)
StronglyNonSingular: Time of QuantifierEliminate:  .273
```

true (6.1)

strongly non-singular matrix, 5×5, 3 marked elements

```
> P := [[14 + T[1,1]*10^(-1), -782*10^(-1), 25, -913*10^(-2), -324*10^(-1)],
        [995*10^(-2), 46*10^(-2), 701*10^(-2), 901*10^(-1) + T[2,4]*10^(-1), -573*10^(-2)],
        [804*10^(-2), -306*10^(-2) - T[3,2]*10^(-2), -375*10^(-1), 984*10^(-2), 257*10^(-1)],
```

$$\left[-764\cdot10^{-2}, -881\cdot10^{-2}, 238\cdot10^{-2}, -185\cdot10^{-2}, -297\cdot10^{-2}\right],$$
$$\left[-203\cdot10^{-2}, 689\cdot10^{-2}, -194\cdot10^{-2}, -618\cdot10^{-2}, -48\cdot10^{-1}\right]]:$$

> *StronglyNonSingular*(P)

StronglyNonSingular: Time of QuantifierEliminate: 7.612

true **(7.1)**

strongly non-singular matrix, 5×5, 4 marked elements

> $P := [[14 + T_{1,1}\cdot10^{-1}, -782\cdot10^{-1}, 25, -913\cdot10^{-2}, -324\cdot10^{-1}],$
$[995\cdot10^{-2}, 46\cdot10^{-2}, 701\cdot10^{-2}, 901\cdot10^{-1} + T_{2,4}\cdot10^{-1}, -573\cdot10^{-2}],$
$[804\cdot10^{-2}, -306\cdot10^{-2} - T_{3,2}\cdot10^{-2}, -375\cdot10^{-1}, 984\cdot10^{-2}, 257\cdot10^{-1}],$
$[-764\cdot10^{-2}, -881\cdot10^{-2}, 238\cdot10^{-2}, -185\cdot10^{-2}, -297\cdot10^{-2}],$
$[-203\cdot10^{-2}, 689\cdot10^{-2}, -194\cdot10^{-2} - T_{5,3}\cdot10^{-2}, -618\cdot10^{-2}, -48\cdot10^{-1}]]:$

> *StronglyNonSingular*(P)

StronglyNonSingular: Time of QuantifierEliminate: 697.698

true **(8.1)**

non-strongly non-singular matrix, 5×5, 1 marked element

> $P := \begin{bmatrix} 14 & -782\cdot10^{-1} & 25 & -2798\cdot10^{-2} & -1399\cdot10^{-1} \\ 995\cdot10^{-2} & 46\cdot10^{-2} & 701\cdot10^{-2} & 83\cdot10^{-2} & 415\cdot10^{-2} \\ 804\cdot10^{-2} & -306\cdot10^{-2} & -375\cdot10^{-1} & 2739\cdot10^{-2} & 13695\cdot10^{-2} \\ -764\cdot10^{-2} & -881\cdot10^{-2} & 238\cdot10^{-2} & -126\cdot10^{-1} - T_{4,4}\cdot10^{-1} & -6305\cdot10^{-2} \\ -203\cdot10^{-2} & 689\cdot10^{-2} & -194\cdot10^{-2} & -138\cdot10^{-1} & -69 \end{bmatrix}:$

> *StronglyNonSingular*(P)

false **(9.1)**

non-strongly non-singular matrix, 5×5, 2 marked

element

```
> P
      := [[14 + T[1,1]·10^-1, -782·10^-1, 25, -2798·10^-2, -1399·10^-1],
      [995·10^-2, 46·10^-2, 701·10^-2, 83·10^-2, 415·10^-2],
      [804·10^-2, -306·10^-2, -375·10^-1, 2739·10^-2, 13695·10^-2],
      [-764·10^-2, -881·10^-2, 238·10^-2, -126·10^-1 - T[4,4]·10^-1, -6305·10^-2],
      [-203·10^-2, 689·10^-2, -194·10^-2, -138·10^-1, -69]]:
>
> StronglyNonSingular(P)
```

StronglyNonSingular: Time of QuantifierEliminate: .416

false **(10.1)**

non–strongly non–singular matrix, 5×5, 3 marked element

```
> P := [[14 + T[1,1]·10^-1, -782·10^-1, 25, -2798·10^-2, -1399·10^-1],
      [995·10^-2, 46·10^-2, 701·10^-2, 83·10^-2, 415·10^-2],
      [804·10^-2, -306·10^-2 - T[3,2]·10^-2, -375·10^-1, 2739·10^-2, 13695·10^-2],
      [-764·10^-2, -881·10^-2, 238·10^-2, -126·10^-1 - T[4,4]·10^-1, -6305·10^-2],
      [-203·10^-2, 689·10^-2, -194·10^-2, -138·10^-1, -69]]:
>
> StronglyNonSingular(P)
```

StronglyNonSingular: Time of QuantifierEliminate: .373

false **(11.1)**

non–strongly non–singular matrix, 5×5, 4 marked element

```
> P := [[14 + T[1,1]·10^-1, -782·10^-1, 25, -2798·10^-2, -1399·10^-1],
      [995·10^-2, 46·10^-2, 701·10^-2, 83·10^-2, 415·10^-2],
      [804·10^-2, -306·10^-2 - T[3,2]·10^-2, -375·10^-1, 2739·10^-2, 13695·10^-2],
      [-764·10^-2, -881·10^-2, 238·10^-2, -126·10^-1 - T[4,4]·10^-1, -6305·10^-2],
      [-203·10^-2, 689·10^-2, -194·10^-2 - T[5,3]·10^-2, -138·10^-1, -69]]:
>
> StronglyNonSingular(P)
```

StronglyNonSingular: Time of QuantifierEliminate: .440

false (12.1)

non-strongly non-singular matrix, 5×5, 5 marked element

```
> P := [[14 + T[1,1]·10^-1, -782·10^-1, 25, -2798·10^-2, -1399·10^-1],
        [995·10^-2, 46·10^-2, 701·10^-2, 83·10^-2, 415·10^-2 + T[2,5]·10^-2],
        [804·10^-2, -306·10^-2 - T[3,2]·10^-2, -375·10^-1, 2739·10^-2, 13695·10^-2],
        [-764·10^-2, -881·10^-2, 238·10^-2, -126·10^-1 - T[4,4]·10^-1, -6305·10^-2],
        [-203·10^-2, 689·10^-2, -194·10^-2 - T[5,3]·10^-2, -138·10^-1, -69]]:
>
> StronglyNonSingular(P)
StronglyNonSingular: Time of QuantifierEliminate:  15.299
```

false (13.1)

non-strongly non-singular matrix, 5×5, 6 marked element

```
> P := [[14 + T[1,1]·10^-1, -782·10^-1, 25 + T[1,3]·10^-1, -2798·10^-2, -1399·10^-1],
        [995·10^-2, 46·10^-2, 701·10^-2, 83·10^-2, 415·10^-2 + T[2,5]·10^-2],
        [804·10^-2, -306·10^-2 - T[3,2]·10^-2, -375·10^-1, 2739·10^-2, 13695·10^-2],
        [-764·10^-2, -881·10^-2, 238·10^-2, -126·10^-1 - T[4,4]·10^-1, -6305·10^-2],
        [-203·10^-2, 689·10^-2, -194·10^-2 - T[5,3]·10^-2, -138·10^-1, -69]]:
>
> StronglyNonSingular(P)
StronglyNonSingular: Time of QuantifierEliminate:  353.682
```

false (14.1)

non-strongly non-singular matrix, 5×5, 7 marked element

```
> P := [[14 + T[1,1]·10^-1, -782·10^-1, 25 + T[1,3]·10^-1, -2798·10^-2, -1399·10^-1],
        [995·10^-2, 46·10^-2 + T[2,3]·10^-2, 701·10^-2, 83·10^-2, 415·10^-2 + T[2,5]·10^-2],
        [804·10^-2, -306·10^-2 - T[3,2]·10^-2, -375·10^-1, 2739·10^-2, 13695·10^-2],
```

$$\left[-764\cdot10^{-2}, -881\cdot10^{-2}, 238\cdot10^{-2}, -126\cdot10^{-1} - T_{4,4}\cdot10^{-1}, -6305\cdot10^{-2}\right],$$
$$\left[-203\cdot10^{-2}, 689\cdot10^{-2}, -194\cdot10^{-2} - T_{5,3}\cdot10^{-2}, -138\cdot10^{-1}, -69\right]]:$$

> $StronglyNonSingular(P)$

StronglyNonSingular: Time of QuantifierEliminate: 7415.417

false **(15.1)**

```
BlockTriangular := proc(P)
    local A, l, w, m, k, k1, i, i1, blocks;
    A := copy(P);
    blocks :=[LinearAlgebra:-Dimensions(A)];
    l := blocks[2]; w := 0;
    blocks :=[blocks[1]+1..blocks[1]+1,blocks[2]+1..blocks[2]+1], b
    for m from blocks[-1][1] by -1 to 1 while k <> 0 do
        k := 0; i1 := 0;
        for i from m by -1 to 1 do
            k1 := nops(select(`=`, convert(LinearAlgebra:-Row(A, i)
            if k < k1 then
                k := k1; i1 := i;
            end if;
        end do;
        if 0 < i1 then
            if i1 <> m then
                A := <LinearAlgebra:-Row(A, [1 .. i1 - 1, m, i1 + 1
            end if;
            for i1 from l by -1 to 1 while A[m, i1] <> 0 do end do;
            for i from i1 - 1 by -1 to 1 do
                if A[m, i] <> 0 then
                    A := <A[1 .. -1, 1 .. i - 1] | A[1 .. -1, i + 1
                    i1 := i1 - 1;
                end if;
            end do;
        end if;
        w := w + 1;
        l := k;
        if l = blocks[-1][2] then
            blocks := [op(1, blocks[1][1])-1.. op(1, blocks[1][1])-
        elif l <  blocks[-1][2] and l >= blocks[-1][1] - w then
            blocks := [blocks[-1][1]-w+1 .. op(1, blocks[1][1])-1,
        end if;
    end do;
    if k = 0 and op(1, blocks[1][1]) > 1 then
```

sparse strongly non-singular matrix, 5×5

```
> P := [[14 + T[1,1]·10^-1, 0, 25 + T[1,3]·10^-1, -913·10^-2 - T[1,4]·10^-2, 0],
    [995·10^-2 + T[2,1]·10^-2, 46·10^-2 + T[2,2]·10^-2, 701·10^-2 + T[2,3]·10^-2, 901·10^-1
    + T[2,4]·10^-1, -573·10^-2 - T[2,5]·10^-2],
    [804·10^-2 + T[3,1]·10^-2, 0, 0, 984·10^-2 + T[3,4]·10^-2, 0],
    [-764·10^-2 - T[4,1]·10^-2, 0, 0, -185·10^-2 - T[4,4]·10^-2, 0],
    [-203·10^-2 - T[5,1]·10^-2, 689·10^-2 + T[5,2]·10^-2, -194·10^-2 - T[5,3]·10^-2, -618
    ·10^-2 - T[5,4]·10^-2, -48·10^-1 - T[5,5]·10^-1]]:
```

```
> StronglyNonSingular(P);
StronglyNonSingular: Time of QuantifierEliminate:  2394.840
                                  true                                  (16.1)
```

```
> st := time( ):
  B, w := BlockTriangular(P);
   time( ) - st;
```

$$B, w := \left[\left[\frac{689}{100} + \frac{1}{100}\,T_{5,2}, -\frac{24}{5} - \frac{1}{10}\,T_{5,5}, -\frac{97}{50} - \frac{1}{100}\,T_{5,3}, -\frac{203}{100} - \frac{1}{100}\,T_{5,1}, -\frac{309}{50} - \frac{1}{100}\,T_{5,4}\right], \left[\frac{23}{50} + \frac{1}{100}\,T_{2,2}, -\frac{573}{100} - \frac{1}{100}\,T_{2,5}, \frac{701}{100} + \frac{1}{100}\,T_{2,3}, \frac{199}{20} + \frac{1}{100}\,T_{2,1}, \frac{901}{10} + \frac{1}{10}\,T_{2,4}\right], \left[0, 0, 25 + \frac{1}{10}\,T_{1,3}, 14 + \frac{1}{10}\,T_{1,1}, -\frac{913}{100} - \frac{1}{100}\,T_{1,4}\right], \left[0, 0, 0, \frac{201}{25} + \frac{1}{100}\,T_{3,1}, \frac{246}{25} + \frac{1}{100}\,T_{3,4}\right], \left[0, 0, 0, -\frac{191}{25} - \frac{1}{100}\,T_{4,1}, -\frac{37}{20} - \frac{1}{100}\,T_{4,4}\right]\right], [[1..2, 1..2], [3..3, 3..3], [4..5, 4..5]]$$

```
                                  0.027                                 (16.2)
```

```
> StronglyNonSingular(B)
StronglyNonSingular: Time of QuantifierEliminate:  2248.785
                                  true                                  (16.3)
```

```
> StronglyNonSingular(B[w[1][ ]])
StronglyNonSingular: Time of QuantifierEliminate:  17.759
                                  true                                  (16.4)
```

```
> StronglyNonSingular(B[w[3][ ]]);
StronglyNonSingular: Time of QuantifierEliminate:  27.199
                                  true                                  (16.5)
```