Slode[Liouvillian_series_sol] - formal power series solutions with Liouvillian coefficients for a linear ODE

Calling Sequence

Liouvillian_series_sol(**ode**, **var**,**opts**)
Liouvillian_series_sol(**LODEstr**,**opts**)

Parameters

`ode`	-	linear ODE with polynomial coefficients
`var`	-	dependent variable, for example **y(x)**
`opts`	-	optional arguments of the form **keyword=value**
`LODEstr`	-	LODEstruct data structure

Description

- The **Liouvillian_series_sol** command returns one formal power series solution or a set of formal power series solutions of the given linear ordinary differential equation with polynomial coefficients. The ODE must be homogeneous.
- If **ode** is an expression, then it is equated to zero.
- The routine returns an error message if the differential equation **ode** does not satisfy the following conditions.
  - **ode** must be homogeneous and linear in **var**
  - **ode** must have polynomial coefficients in the independent variable of **var**, for example, x
  - The coefficients of **ode** must be either rational numbers or depend rationally on one or more parameters.
- A homogeneous linear ordinary differential equation with coefficients that are polynomials in x has a linear space of formal power series solutions $\sum_{n=0}^{\infty} v(n)\, P_n(x)$ where $P_n(x)$ is one of $(x-a)^n$, $\frac{(x-a)^n}{n!}$, $\frac{1}{x^n}$, or $\frac{1}{x^n\, n!}$, a is the expansion point, and the sequence $v(n)$ satisfies a homogeneous linear recurrence.
- The routine selects such formal power series solutions where $v(n)$ is a Liouvillian sequence for all sufficiently large n.

Options

- **x=a** or **'point'=a**

  Specifies the expansion point **a**. The default is $a=0$. It can be an algebraic number, depending rationally on some parameters, or ∞.
  If this option is given, then the command returns one formal power series solution at **a** with Liouvillian coefficients if it exists; otherwise, it returns **NULL**. If **a** is not given, it returns a set of one formal power series solution with Liouvillian coefficients for the **a=0**.

• **'free'=C**

Specifies a base name **C** to use for free variables **C[0]**, **C[1]**, etc. The default is the global name **_C**. Note that the number of free variables may be less than the order of the given equation if the expansion point is singular.

• **'indices'=[n,k]**

Specifies names for dummy variables. The default value are the global names **_n** and **_k**.The name **n** is used as the summation index in the power series. The name **k** is used as the product index.

Examples

```
> restart;
```

'mylib' must be a path to the file *Slode.mla*

```
> libname := mylib, libname:
> with(Slode):
> ode := diff(y(x),x)*x^3+diff(y(x),x)-2*x^4*y(x)-2*x*y(x)-3*x^2*y(x);
```

$$ode := \left(\frac{d}{dx}\,y(x)\right)x^3+\frac{d}{dx}\,y(x)-2\,x^4\,y(x)-2\,x\,y(x)-3\,x^2\,y(x) \quad (5.1)$$

```
> Liouvillian_series_sol(ode,y(x));
```

$$\left\{\_C_5+\_C_5\left(\sum_{\_n=2}^{\infty}\left(\left\{\begin{array}{ll}\dfrac{1}{\Gamma\left(\frac{1}{2}\,\_n+1\right)} & irem(\_n,2)=0\\[2ex] \dfrac{1}{\Gamma\left(\frac{1}{2}\,\_n-\frac{1}{2}\right)} & irem(\_n,2)=1\end{array}\right.\right)x^{\_n}\right)\right\} \quad (5.2)$$

```
> ode := diff(y(x),x)*(x+1)^3+diff(y(x),x)-2*(x+1)^4*y(x)-2*(x+1)*y(x)-3*(x+1)^2*y(x);
```

$$ode := \left(\frac{d}{dx}\,y(x)\right)(x+1)^3+\frac{d}{dx}\,y(x)-2\,(x+1)^4\,y(x)-2\,(x+1)\,y(x)-3\,(x+1)^2\,y(x) \quad (5.3)$$

```
> Liouvillian_series_sol(ode,y(x));
```

$$\{\,\} \quad (5.4)$$

```
> Liouvillian_series_sol(ode,y(x), 'indices'=['n','k'], 'free'='_c', 'point'=-1);
```

(5.5)

$$\_c_5 + \_c_5 \left(\sum_{n=2}^{\infty} \left(\begin{cases} \dfrac{1}{\Gamma\left(\frac{1}{2}\, n + 1\right)} & irem(n, 2) = 0 \\ \dfrac{1}{\Gamma\left(\frac{1}{2}\, n - \frac{1}{2}\right)} & irem(n, 2) = 1 \end{cases} \right) (x + 1)^n \right) \tag{5.5}$$

See Also

LODEstruct, LREtools[LiouvillianSolution], Slode, Slode[candidate_points], Slode[hypergeom_series_sol], Slode[rational_series_sol]