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> restart;
>
> 
$$\begin{bmatrix} x^5 & 0 \\ 0 & 0 \end{bmatrix} \cdot \left( \frac{d^2}{dx^2} y(x) \right) + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \cdot \left( \frac{d}{dx} y(x) \right) + \begin{bmatrix} 0 & -x+2 \\ x+2 & 0 \end{bmatrix} \cdot y(x) = 0 :$$

> syst :=  $\left\{ x^5 \left( \frac{d^2}{dx^2} y1(x) \right) + (-x+2) y2(x) = 0, \right.$ 

$$\left. \frac{d}{dx} y2(x) + (x+2) y1(x) = 0 \right\} :$$

http://www.ccas.ru/ca/resolvingsequence
> read "resolvingsequence.mpl";
> L := RS:-ResolvingSequence(syst, {y1(x), y2(x)}, OreTools:-SetOreRing(x,'differential'));
L :=  $\left[ (x^5 - 2x^4 - 8x^3 + 16x^2 + 16x - 32) \left( \frac{d}{dx} y1(x) \right) + (8x^6 - 18x^5 - 60x^4 \right.$  (1)

$$+ 160x^3) \left( \frac{d^2}{dx^2} y1(x) \right) + (7x^7 - 16x^6 - 36x^5 + 80x^4) \left( \frac{d^3}{dx^3} y1(x) \right) + (x^8 - 2x^7$$


$$- 4x^6 + 8x^5) \left( \frac{d^4}{dx^4} y1(x) \right) = 0 \right]$$

> DEtools:-formal_sol(L[1], y1(x), t, 'order'=1);

$$\left[ [1 + O(t), t=x], \left[ e^{-\frac{6}{t^2}} (1 + O(t)), \frac{1}{4} t^3 = x \right] \right]$$
 (2)
http://www.ccas.ru/ca/formalsolution
> read "ldsformalsols.mpl";
> FormalSolution(syst, [y1(x), y2(x)], t, 'order'=2);

$$\left[ e^{-\frac{6}{t^2}} \cdot \begin{bmatrix} -8 + O(t^2) \\ t^5 + O(t^7) \end{bmatrix}, \frac{1}{4} t^3 = x \right]$$
 (3)
>
>
>
> S2 := EG $\left( \begin{bmatrix} x^5 & 0 \\ 0 & 0 \end{bmatrix} \cdot \left( \frac{d^2}{dx^2} y(x) \right) + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \cdot \left( \frac{d}{dx} y(x) \right) + \begin{bmatrix} 0 & -x+2 \\ x+2 & 0 \end{bmatrix} \cdot y(x) = 0, \right.$ 

$$\left. y(x), OreTools:-SetOreRing(x,'differential') \right)$$

S2 :=  $\begin{bmatrix} x^5 & 0 \\ 0 & x+2 \end{bmatrix} \cdot \left( \frac{d^2}{dx^2} y(x) \right) + \begin{bmatrix} 0 & 0 \\ (x+2)^2 & -1 \end{bmatrix} \cdot \left( \frac{d}{dx} y(x) \right) + \begin{bmatrix} 0 & -x+2 \\ 0 & 0 \end{bmatrix} \cdot y(x)$  (4)
= 0
> FormalSolution(S2, y(x), t, 'order'=5);

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(5)

$$\left[\begin{bmatrix} 1 + O(t^5) \\ O(t^6) \end{bmatrix}, t=x, e^{-\frac{6}{t^2}} \cdot \begin{bmatrix} -8 - \frac{35}{9} t^2 + \frac{1}{3} t^3 - \frac{223}{1296} t^4 + O(t^5) \\ t^5 + \frac{5}{72} t^7 + \frac{1}{12} t^8 - \frac{197}{10368} t^9 + O(t^{10}) \end{bmatrix}, \frac{1}{4} t^3 = x \right]$$

>

>

> #Sdetermine(syst, {y1(x), y2(x)}, OreTools:-SetOreRing(x,'differential'));

> L := ResolvingSequence(syst, [y1(x), y2(x)], OreTools:-SetOreRing(x,'differential')) :

>

> B := ResolvingMatrix() : m := 2 : i := Indicator() [1] : n := 2 :

B, Y := op(1, ResolvingMatrix() [1]), op(2, ResolvingMatrix() []) :

Y[i] ≠ 0; L[1];

Rhs := Matrix(m·n, 1, [Y[i], diff(Y[i], x), diff(Y[i], x\$2), diff(Y[i], x\$3)]) :

ResolvingMatrix() [1] = Rhs;

op(2, lhs('%')) = '((op(1, lhs('%')))⁻¹, rhs(%)) ;

value(%) :

lhs(%)[1..2] = rhs(%)[1..2];

$$\begin{aligned} & y1(x) \neq 0 \\ & (x^5 - 2x^4 - 8x^3 + 16x^2 + 16x - 32) \left(\frac{d}{dx} y1(x) \right) + (8x^6 - 18x^5 - 60x^4 \\ & + 160x^3) \left(\frac{d^2}{dx^2} y1(x) \right) + (7x^7 - 16x^6 - 36x^5 + 80x^4) \left(\frac{d^3}{dx^3} y1(x) \right) + (x^8 - 2x^7 \\ & - 4x^6 + 8x^5) \left(\frac{d^4}{dx^4} y1(x) \right) = 0 \end{aligned}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & \frac{x-2}{x^5} & 0 & 0 \\ 0 & -\frac{2(2x-5)}{x^6} & 0 & \frac{x-2}{x^5} \end{bmatrix} \cdot \begin{bmatrix} y1(x) \\ y2(x) \\ \frac{d}{dx} y1(x) \\ \frac{d}{dx} y2(x) \end{bmatrix} = \begin{bmatrix} y1(x) \\ \frac{d}{dx} y1(x) \\ \frac{d^2}{dx^2} y1(x) \\ \frac{d^3}{dx^3} y1(x) \end{bmatrix}$$

$$\begin{bmatrix} y1(x) \\ y2(x) \\ \frac{d}{dx} y1(x) \\ \frac{d}{dx} y2(x) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{x^5}{x-2} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{2(2x-5)x^4}{(x-2)^2} & \frac{x^5}{x-2} \end{bmatrix} \cdot \begin{bmatrix} y1(x) \\ \frac{d}{dx} y1(x) \\ \frac{d^2}{dx^2} y1(x) \\ \frac{d^3}{dx^3} y1(x) \end{bmatrix}$$

$$\begin{bmatrix} y1(x) \\ y2(x) \end{bmatrix} = \begin{bmatrix} y1(x) \\ \frac{x^5 \left(\frac{d^2}{dx^2} y1(x) \right)}{x-2} \end{bmatrix} \quad (6)$$

>

> Indicator();
(ResolvingMatrix());
ResolvingDependence();
ResolvingSystems();

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & \frac{x-2}{x^5} & 0 & 0 \\ 0 & -\frac{2(2x-5)}{x^6} & 0 & \frac{x-2}{x^5} \end{bmatrix} \cdot \begin{bmatrix} y1(x) \\ y2(x) \\ \frac{d}{dx} y1(x) \\ \frac{d}{dx} y2(x) \end{bmatrix}$$

$$\left[\left\{ x^5 \left(\frac{d^2}{dx^2} y1(x) \right) + (-x+2) y2(x) = 0, \frac{d}{dx} y2(x) + (x+2) y1(x) = 0 \right\} \right] \quad (7)$$

>

>

> RS:-ResolvingEquation(syst, {y1(x), y2(x)}, OreTools:-SetOreRing(x,'differential'));

$$\begin{aligned} & (x^5 - 2x^4 - 8x^3 + 16x^2 + 16x - 32) \left(\frac{d}{dx} y1(x) \right) + (8x^6 - 18x^5 - 60x^4 \\ & + 160x^3) \left(\frac{d^2}{dx^2} y1(x) \right) + (7x^7 - 16x^6 - 36x^5 + 80x^4) \left(\frac{d^3}{dx^3} y1(x) \right) + (x^8 - 2x^7 \\ & - 4x^6 + 8x^5) \left(\frac{d^4}{dx^4} y1(x) \right) = 0 \end{aligned} \quad (8)$$

> RS:-ResolvingMatrix();

$$\begin{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & \frac{x-2}{x^5} & 0 & 0 \\ 0 & -\frac{2(2x-5)}{x^6} & 0 & \frac{x-2}{x^5} \end{bmatrix} \end{bmatrix} \quad (9)$$

> RS:-CyclicVector(syst, {y1(x), y2(x)}, OreTools:-SetOreRing(x,'differential'));

$$(3x^{11} + 12x^{10} + 62x^9 - 248x^8 - 152x^7 - 368x^6 - 120x^5 + 1584x^4 + 1216x^3 + 128x^2) \quad (10)$$

$$\begin{aligned}
 &+ 128x - 256) \left(\frac{d}{dx} u(x) \right) + (-2x^{12} - 12x^{11} - 56x^{10} + 160x^9 + 504x^8 - 24x^7 \\
 &- 112x^6 - 48x^5 - 160x^4 + 1280x^3) \left(\frac{d^2}{dx^2} u(x) \right) + (x^{13} + 8x^{12} + 42x^{11} - 16x^{10} \\
 &- 360x^9 - 720x^8 + 56x^7 - 128x^6 - 288x^5 + 640x^4) \left(\frac{d^3}{dx^3} u(x) \right) + (x^{14} + 4x^{13} \\
 &+ 14x^{12} - 4x^{11} - 72x^{10} - 120x^9 + 8x^8 - 16x^7 - 32x^6 + 64x^5) \left(\frac{d^4}{dx^4} u(x) \right) = 0
 \end{aligned}$$

> RS:-ResolvingMatrix();

$$\left[\begin{bmatrix} 4 & 2 & 0 & 0 \\ 0 & 0 & 4 & 2 \\ 0 & \frac{4(x-2)}{x^5} & -2x-4 & \frac{2}{x+2} \\ 0 & -\frac{2(x^3+4x-20)}{x^6} & -4 & \frac{4(x-2)}{x^5} \end{bmatrix} \right]$$

(11)