

```

> restart : read( "EGRRext.mpl" ) :
> with(EGRRext);
[EG, IsUnimodular]

```

```

> L1 := Matrix([[0, -1/x, 1, 0], [0, -x/2, x^2/2, 1]]);
L1 := 
$$\begin{bmatrix} 0 & -\frac{1}{x} & 1 & 0 \\ 0 & -\frac{x}{2} & \frac{x^2}{2} & 1 \end{bmatrix}$$


```

```

> st := time( ) : IsUnimodular(L1, x); time( ) - st;
true
0.047

```

```

> restart : read( "EGRRext.mpl" ) :
> with(EGRRext);
[EG, IsUnimodular]

```

```

> L1 := Matrix([[0, -1/x, 1, 0], [0, -x/2, x^2/2, 1]]);
L1 := 
$$\begin{bmatrix} 0 & -\frac{1}{x} & 1 & 0 \\ 0 & -\frac{x}{2} & \frac{x^2}{2} & 1 \end{bmatrix}$$


```

```

> st := time( ) : IsUnimodular(L1, x, 'InvL1'); time( ) - st;
true
0.063

```

```

> InvL1;

$$\left[ \left[ \begin{array}{cccc} -\frac{(x+1)^2}{2x} & \frac{1}{x} & 1 & 0 \\ 0 & 0 & -\frac{x^2}{2} & 1 \end{array} \right], 1, 0 \right]$$


```

```

> IsUnimodular(InvL1, x, 'InvInvL1');
true

```

```

> InvInvL1;

$$\left[ \left[ \begin{array}{cccc} 0 & -\frac{1}{x} & 1 & 0 \\ 0 & -\frac{x}{2} & \frac{x^2}{2} & 1 \end{array} \right], 1, 0 \right]$$


```

```

> restart : read( "EGRRext.mpl" );
> with(EGRRext);
[EG, IsUnimodular]

```

```

> L2 := Matrix([[0, 0, 0, -1/(x-1), 1, 0], [0, -x/2, x^2/2, 1, 0, 0]]);

```

$$L2 := \begin{bmatrix} 0 & 0 & 0 & -\frac{1}{x-1} & 1 & 0 \\ 0 & -\frac{x}{2} & \frac{x^2}{2} & 1 & 0 & 0 \end{bmatrix} \quad (11)$$

```
> st := time( ) : IsUnimodular(L2, x); time( ) - st;
      true
      0.063
```

```
> restart : read( "EGRRext.mpl");
> with(EGRRext);
      [EG, IsUnimodular]
```

```
> infolevel[EGRRext] := 3;
      infolevelEGRRext := 3
```

```
> L2 := Matrix([[0, 0, 0, -1/(x-1), 1, 0], [0, -x/2, x^2/2, 1, 0, 0]]);
      L2 := \begin{bmatrix} 0 & 0 & 0 & -\frac{1}{x-1} & 1 & 0 \\ 0 & -\frac{x}{2} & \frac{x^2}{2} & 1 & 0 & 0 \end{bmatrix} \quad (15)
```

```
> st := time( ) : IsUnimodular(L2, x, 'InvL2'); time( ) - st;
EG: reduction and shift in the row 2 its length is 71 its
maximal sum of degrees of numerator and denominator of entries
is 2
EG: reduction and shift in the row 2 its length is 55 its
maximal sum of degrees of numerator and denominator of entries
is 2
EG: reduction and shift in the row 1 its length is 42 its
maximal sum of degrees of numerator and denominator of entries
is 1
EG: reduction and shift in the row 1 its length is 49 its
maximal sum of degrees of numerator and denominator of entries
is 2
EG: reduction and shift in the row 2 its length is 54 its
maximal sum of degrees of numerator and denominator of entries
is 2
      true
      0.079
```

```
> InvL2;
      \left[ \left[ \begin{array}{cccccc} -\frac{x^2}{2(x-1)} & 0 & 1 & \frac{2\left(\frac{x}{2} - \frac{1}{2}\right)}{(x-1)^2} & 0 & 0 \\ 0 & 0 & -\frac{(x-1)^2}{2} & 0 & 0 & 1 \end{array} \right], 1, -1 \right] \quad (17)
```

```
> IsUnimodular(InvL2, x, 'InvInvL2');
EG: reduction and shift in the row 1 its length is 79 its
```

```

maximal sum of degrees of numerator and denominator of entries
is 3
EG: reduction and shift in the row 2 its length is 46 its
maximal sum of degrees of numerator and denominator of entries
is 2
EG: reduction and shift in the row 1 its length is 71 its
maximal sum of degrees of numerator and denominator of entries
is 3
EG: reduction and shift in the row 1 its length is 33 its
maximal sum of degrees of numerator and denominator of entries
is 0
EG: reduction and shift in the row 2 its length is 33 its
maximal sum of degrees of numerator and denominator of entries
is 0

```

*true* (18)

```
> InvInvL2;
```

$$\left[ \left[ \begin{array}{cccccc} 0 & 0 & 0 & -\frac{1}{x-1} & 1 & 0 \\ 0 & -\frac{x}{2} & \frac{x^2}{2} & 1 & 0 & 0 \end{array} \right], 2, 0 \right]$$

(19)

```
>
```

```
> restart : read( "EGRRext.mpl");
```

```
> with(EGRRext);
```

*[EG, IsUnimodular]* (20)

```
> infolevel[EGRRext] := 3;
```

*infolevel<sub>EGRRext</sub> := 3* (21)

```
> randomize(731487360);
```

*731487360* (22)

```
> r1 := 4 : m1 := 4 : M1 := Matrix(r1, r1*(m1 + 1)) : d1 := rand(1..2) : d2 := rand(0..1) :
  for i to r1 do for k to r1*(m1 + 1) do dn := d1( ) : dd := d2( ) : den := 0 : while den = 0
  do den := randpoly(x, degree = dd, coeffs = rand(-9..9)) end do : M1[i, k] := randpoly(x,
  degree = dn, coeffs = rand(-9..9)) / den; od; od:
```

```
> M := Matrix(2*r1, (m1 + 1)*2*r1) :
```

```
> for i to 2*r1 do M[i, m1*2*r1 + i] := 1 od:
```

```
> for k from 0 to m1 do A1 := M1[1..r1, (m1 - k)*r1 + 1..(m1 - k)*r1 + r1]; M[1..r1, (m1
  - k)*2*r1 + r1 + 1..(m1 - k)*2*r1 + 2*r1] := A1 od:
```

```
> eval(M);
```

$$\left[ \begin{array}{l} 8 \times 40 \text{ Matrix} \\ \text{Data Type: anything} \\ \text{Storage: rectangular} \\ \text{Order: Fortran\_order} \end{array} \right]$$

(23)

```
> st := time( ) : IsUnimodular(M, x, InvM1); time( ) - st;
```

```
EG: reduction and shift in the row 8 its length is 67 its
maximal sum of degrees of numerator and denominator of entries
is 0
```

```
EG: reduction and shift in the row 1 its length is 8057 its
```

maximal sum of degrees of numerator and denominator of entries is 27  
EG: reduction and shift in the row 2 its length is 33928 its maximal sum of degrees of numerator and denominator of entries is 60  
EG: reduction and shift in the row 3 its length is 103950 its maximal sum of degrees of numerator and denominator of entries is 95  
EG: reduction and shift in the row 4 its length is 235689 its maximal sum of degrees of numerator and denominator of entries is 142  
EG: reduction and shift in the row 7 its length is 67 its maximal sum of degrees of numerator and denominator of entries is 0  
EG: reduction and shift in the row 1 its length is 180812 its maximal sum of degrees of numerator and denominator of entries is 130  
EG: reduction and shift in the row 2 its length is 143690 its maximal sum of degrees of numerator and denominator of entries is 116  
EG: reduction and shift in the row 3 its length is 113098 its maximal sum of degrees of numerator and denominator of entries is 103  
EG: reduction and shift in the row 4 its length is 75448 its maximal sum of degrees of numerator and denominator of entries is 98  
EG: reduction and shift in the row 6 its length is 67 its maximal sum of degrees of numerator and denominator of entries is 0  
EG: reduction and shift in the row 1 its length is 48474 its maximal sum of degrees of numerator and denominator of entries is 84  
EG: reduction and shift in the row 2 its length is 27696 its maximal sum of degrees of numerator and denominator of entries is 66  
EG: reduction and shift in the row 3 its length is 15469 its maximal sum of degrees of numerator and denominator of entries is 48  
EG: reduction and shift in the row 4 its length is 6691 its maximal sum of degrees of numerator and denominator of entries is 38  
EG: reduction and shift in the row 5 its length is 67 its maximal sum of degrees of numerator and denominator of entries is 0  
EG: reduction and shift in the row 1 its length is 3078 its maximal sum of degrees of numerator and denominator of entries is 27  
EG: reduction and shift in the row 2 its length is 1464 its maximal sum of degrees of numerator and denominator of entries is 18  
EG: reduction and shift in the row 3 its length is 510 its maximal sum of degrees of numerator and denominator of entries is 8  
EG: reduction and shift in the row 4 its length is 162 its maximal sum of degrees of numerator and denominator of entries is 2

*true*  
11.578

(24)

> *InvM*;

$\left[ \left[ \begin{array}{l} 8 \times 40 \text{ Matrix} \\ \text{Data Type: anything} \\ \text{Storage: rectangular} \\ \text{Order: Fortran\_order} \end{array} \right], 4, 0 \right]$

(25)

> *IsUnimodular(InvM, x, 'InvInvM')*;

EG: reduction and shift in the row 8 its length is 67 its maximal sum of degrees of numerator and denominator of entries is 0

EG: reduction and shift in the row 1 its length is 8064 its maximal sum of degrees of numerator and denominator of entries is 27

EG: reduction and shift in the row 2 its length is 33971 its maximal sum of degrees of numerator and denominator of entries is 60

EG: reduction and shift in the row 3 its length is 104030 its maximal sum of degrees of numerator and denominator of entries is 95

EG: reduction and shift in the row 4 its length is 235942 its maximal sum of degrees of numerator and denominator of entries is 142

EG: reduction and shift in the row 7 its length is 67 its maximal sum of degrees of numerator and denominator of entries is 0

EG: reduction and shift in the row 1 its length is 181047 its maximal sum of degrees of numerator and denominator of entries is 130

EG: reduction and shift in the row 2 its length is 143881 its maximal sum of degrees of numerator and denominator of entries is 116

EG: reduction and shift in the row 3 its length is 113175 its maximal sum of degrees of numerator and denominator of entries is 103

EG: reduction and shift in the row 4 its length is 75448 its maximal sum of degrees of numerator and denominator of entries is 98

EG: reduction and shift in the row 6 its length is 67 its maximal sum of degrees of numerator and denominator of entries is 0

EG: reduction and shift in the row 1 its length is 48474 its maximal sum of degrees of numerator and denominator of entries is 84

EG: reduction and shift in the row 2 its length is 27696 its maximal sum of degrees of numerator and denominator of entries is 66

EG: reduction and shift in the row 3 its length is 15469 its maximal sum of degrees of numerator and denominator of entries is 48

EG: reduction and shift in the row 4 its length is 6691 its maximal sum of degrees of numerator and denominator of entries

```
is 38
EG: reduction and shift in the row 5 its length is 67 its
maximal sum of degrees of numerator and denominator of entries
is 0
EG: reduction and shift in the row 1 its length is 3078 its
maximal sum of degrees of numerator and denominator of entries
is 27
EG: reduction and shift in the row 2 its length is 1464 its
maximal sum of degrees of numerator and denominator of entries
is 18
EG: reduction and shift in the row 3 its length is 510 its
maximal sum of degrees of numerator and denominator of entries
is 8
EG: reduction and shift in the row 4 its length is 162 its
maximal sum of degrees of numerator and denominator of entries
is 2
```

*true*

**(26)**

```
=
>
> lprint(simplify( M - InvInvM[1]));
Matrix(8, 40, {}, datatype = anything, storage = rectangular,
order = Fortran_order, shape = [])
=
>
>
```