

```
[> restart;
[> read "ldsformalsols.mpl";
```

### Example 3,7

```
> Syst2 := <<x^5|0>, <0|0>>.diff(y(x), x$2) +
<<0|0>, <0|1>>.diff(y(x), x) +
<<0|-x+2>, <x+2|0>>.y(x) = 0;
```

$$Syst2 := \begin{bmatrix} x^5 & 0 \\ 0 & 0 \end{bmatrix} \cdot \left( \frac{d^2}{dx^2} y(x) \right) + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \cdot \left( \frac{d}{dx} y(x) \right) + \begin{bmatrix} 0 & -x+2 \\ x+2 & 0 \end{bmatrix} \cdot y(x) = 0 \quad (1.1)$$

```
> infolevel[FormalSolution] := 3 :
```

```
> FormalSolution(Syst2, y(x));
```

```
ResolvingStep: resolving for [1 0]
```

```
ResolvingStep: 4 order resolving equation is constructed:
0.13e-1
```

```
ModuleApply: ResolvingSequence is constructed: 0.16e-1
```

```
ModuleApply: 2 exponential parts is found: 0.34e-1
```

```
ModuleApply: exponential part: [1 x = t]
```

```
ModuleApply: no regular solution: 0.20e-1
```

```
ModuleApply: exponential part: [exp(-6/t^2) x = (1/4)*t^3]
```

```
ModuleApply: 3 dimension regular solution space is found:
.167
```

```
ModuleApply: all time: .260
```

$$\left[ e^{-\frac{3}{2} \frac{4^{1/3}}{x^{2/3}}} \cdot \begin{bmatrix} -8 + O(x^{1/3}) \\ 4 \cdot 4^{2/3} x^{5/3} + O(x^2) \end{bmatrix}, e^{-\frac{3}{2} \frac{4^{1/3}(-1)^{2/3}}{x^{2/3}}} \cdot \begin{bmatrix} -8 + O(x^{1/3}) \\ -4 \cdot 4^{2/3} x^{5/3} (-1)^{1/3} + O(x^2) \end{bmatrix}, \right. \\ \left. e^{\frac{3}{2} \frac{4^{1/3}(-1)^{1/3}}{x^{2/3}}} \cdot \begin{bmatrix} -8 + O(x^{1/3}) \\ 4 \cdot 4^{2/3} x^{5/3} (-1)^{2/3} + O(x^2) \end{bmatrix} \right] \quad (1.2)$$

### Example 12

```
> Syst4 := <<-15*x+4 | 60*x^3-120*x^2>,
<60*x^4+120*x^3 | -15*x+4 >> . y(x) +
<<60*x^3 | 0 >,
<0 | 60*x^3>> . diff(y(x), x);
```

$$Syst4 := \begin{bmatrix} -15x+4 & 60x^3-120x^2 \\ 60x^4+120x^3 & -15x+4 \end{bmatrix} \cdot y(x) + \begin{bmatrix} 60x^3 & 0 \\ 0 & 60x^3 \end{bmatrix} \cdot \left( \frac{d}{dx} y(x) \right) \quad (2.1)$$

```
> infolevel[FormalSolution] := 3 :
```

```
> FormalSolution(Syst4, y(x));
```

```
ResolvingStep: resolving for [1 0]
```

```
ResolvingStep: 2 order resolving equation is constructed:
0.3e-2
```

ModuleApply: ResolvingSequence is constructed: 0.6e-2  
ModuleApply: 1 exponential parts is found: 0.17e-1  
ModuleApply: exponential part:  $[\exp(-(1/4)/t+(1/30)/t^2) \times =$   
 $t]$   
ModuleApply: 2 dimension regular solution space is found:  
0.23e-1  
ModuleApply: all time: 0.60e-1

$$\left[ e^{-\frac{1}{4x} + \frac{1}{30x^2}} \cdot \begin{bmatrix} \ln(x) (1 + O(x)) + O(x) \\ \ln(x) O(x) + \frac{1}{2} + O(x) \end{bmatrix}, e^{-\frac{1}{4x} + \frac{1}{30x^2}} \cdot \begin{bmatrix} \ln(x) O(x) + 1 + O(x) \\ \ln(x) O(x) + O(x) \end{bmatrix} \right] \quad (2.2)$$

## Example 13

### big system Syst13

```
> map(el → map(ell → if(ell = 0, 0, ""), op(1, el)).op(2, el), Syst13);
```

$$\begin{bmatrix} 0 & "" & 0 & 0 & 0 & "" \\ 0 & 0 & "" & 0 & 0 & 0 \\ 0 & 0 & 0 & "" & 0 & 0 \\ "" & "" & "" & "" & "" & "" \\ 0 & "" & 0 & 0 & 0 & 0 \\ 0 & 0 & "" & 0 & 0 & 0 \end{bmatrix} \cdot y(x) + \begin{bmatrix} 0 & "" & 0 & 0 & 0 & 0 \\ 0 & 0 & "" & 0 & 0 & 0 \\ 0 & 0 & 0 & "" & 0 & 0 \\ "" & "" & "" & "" & 0 & 0 \\ 0 & "" & 0 & 0 & 0 & 0 \\ 0 & 0 & "" & 0 & 0 & 0 \end{bmatrix} \cdot \left( \frac{d}{dx} y(x) \right) \quad (3.1)$$

$$+ \begin{bmatrix} 0 & "" & 0 & 0 & 0 & 0 \\ 0 & 0 & "" & 0 & 0 & 0 \\ 0 & 0 & 0 & "" & 0 & 0 \\ "" & "" & "" & "" & 0 & 0 \\ 0 & "" & 0 & 0 & 0 & 0 \\ 0 & 0 & "" & 0 & 0 & 0 \end{bmatrix} \cdot \left( \frac{d^2}{dx^2} y(x) \right) + \begin{bmatrix} 0 & 0 & "" & 0 & 0 & 0 \\ 0 & 0 & 0 & "" & 0 & 0 \\ "" & "" & "" & "" & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \cdot \left( \frac{d^3}{dx^3} y(x) \right)$$

```
> infolevel[FormalSolution] := 3;
```

```
> Res := FormalSolution(Syst13, y(x)) :
```

ResolvingStep: resolving for [1 0 0 0 0 0]

ResolvingStep: 12 order resolving equation is constructed:  
387.476

ResolvingSequence: reduction is evaluated: 33.187

ResolvingStep: resolving for [1 0 0 0 0 0]

ResolvingStep: 1 order resolving equation is constructed: 0.

ResolvingSequence: reduction is evaluated: 0.3e-2

ResolvingStep: resolving for [1 0 0 0 0]

ResolvingStep: 4 order resolving equation is constructed:  
1.003

ResolvingSequence: reduction is evaluated: 0.10e-1

ResolvingStep: resolving for [1]

ResolvingStep: 1 order resolving equation is constructed: 0.  
ModuleApply: ResolvingSequence is constructed: 421.863  
ModuleApply: 2 exponential parts is found: .110  
ModuleApply: exponential part: [1 x = t]  
ModuleApply: 8 dimension regular solution space is found:  
4.200  
ModuleApply: exponential part: [exp(-(621/1216)/t^2-  
(2601/92416)/t) x = t]  
ModuleApply: 2 dimension regular solution space is found:  
.530  
ModuleApply: 1 exponential parts is found: 0.4e-2  
ModuleApply: 1 exponential parts is found: 0.20e-1  
ModuleApply: 1 exponential parts is found: 0.  
ModuleApply: all time: 427.514

> nops(Res);

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(3.2)

> Res[2];

$$\left[ \left[ \frac{1}{2} \ln(x)^2 O(x^{13}) + \ln(x) \left( \frac{1299169936}{1277102439} + \frac{1295349272}{425700813} x + x^2 + O(x^8) \right) \right. \right. \quad (3.3)$$

$$+ \frac{18388975325074580878143509406948083905693}{2110610146799346933472299274883795255043}$$

$$- \frac{1980462899364504717265852}{1178241680298567417129837} x$$

$$+ \frac{11390142078090138715954796422472010749429}{703536715599782311157433091627931751681} x + O(x^3) \Big],$$

$$\left[ \frac{1}{2} \ln(x)^2 O(x^{13}) + \ln(x) O(x^8) + O(x^3) \right],$$

$$\left[ \frac{1}{2} \ln(x)^2 O(x^{13}) + \ln(x) O(x^8) + O(x^3) \right],$$

$$\left[ \frac{1}{2} \ln(x)^2 O(x^{13}) + \ln(x) O(x^8) - \frac{2074905030225760115562160}{392747226766189139043279} \right.$$

$$- \frac{29825082292747891383584}{7700926015023316451829} x - \frac{25883832939848298230648}{43638580751798793227031} x^2 + O(x^3) \Big],$$

$$\left[ \frac{1}{2} \ln(x)^2 O(x^{13}) + \ln(x) \left( \frac{1673413525620}{504171662863} x^3 - \frac{25320285776231343}{7165287672608956} x^4 \right. \right.$$

$$- \frac{106431922211512000623}{50916534201559241336} x^5 - \frac{3881974435056974308817903}{361812892036279968933616} x^6$$

$$+ \frac{19582044389280267528592871901}{2571042410809805459242275296} x^7 + O(x^8) \Big]$$

$$+ \frac{14132843995649186427042270222679}{1652655325325353116385598943237} + \frac{17845951186173552007282592372}{12558878070302430099186932583} x$$

$$+ O(x^3) \Big],$$

$$\left[ \frac{1}{2} \ln(x)^2 O(x^{13}) + \ln(x) O(x^8) + O(x^3) \right] \Big]$$

> Res[9..10]

$$\left[ \left( x^{\frac{506267}{14047232}} e^{-\frac{621}{1216}x^2 - \frac{2601}{92416}x} \right) \cdot \begin{bmatrix} O(x^3) \\ x^2 + O(x^3) \\ O(x^3) \\ O(x) \\ O(1) \end{bmatrix}, \left( x^{\frac{506267}{14047232}} e^{-\frac{621}{1216}x^2 - \frac{2601}{92416}x} \right) \cdot \begin{bmatrix} O(x^3) \\ O(x^3) \\ \frac{656708096}{399138435}x^2 + O(x^3) \\ O(x^3) \\ \frac{1057}{418} + O(x) \\ \frac{1}{x} + O(1) \end{bmatrix} \right] \tag{3.4}$$