```
> restart;
>
>
> OreAlgebraGaussianElimination := proc(A1, OA)
  local m, n, i, j, k, A, a, b, alpha;
    A := map(expand, A1);
    m, n := LinearAlgebra:-Dimensions(A);
    j := 1;
     for alpha to n while j < m do
       for i from j to m while A[i, alpha] = 0 do
      end do;
       if i > j and i ≤ m then
          A[j], A[i] := A[i], A[j];
       end if;
       for k  from i + 1 to m do
         if  A[k, alpha] ≠ 0 then
           a, b := Ore_algebra:-annihilators(A[j, alpha], A[k, alpha], OA)[ ] ;
           A[k] := zip( (el1, el2)→expand(Ore_algebra:-skew_product(a, el1, OA) + Ore_algebra:-
      skew_product(b, el2, OA) ), A[j], A[ k]);
         end if;
       end do:
       if i ≤ m then
         j := j + 1;
       end if
     end do;
     return copy(A);
  end proc:
> DiffOA := Ore_algebra:-skew_algebra(diff = [D1, x1], diff = [D2, x2]) :
>
```

$$A_1 := \begin{bmatrix} D_1 + D_2 & x_1 + x_2 & 0 \\ D_1 & x_2 & 1 \\ D_2 & x_1 & -1 \end{bmatrix} :$$

```
> OreAlgebraGaussianElimination(A1, DiffOA)
```

$$\begin{bmatrix} D_1 + D_2 & x_1 + x_2 & 0 \\ 0 & D_1 x_1 - x_2 D_2 & -D_1 - D_2 \\ 0 & 0 & 0 \end{bmatrix} \quad (1)$$

```
>
> ShiftOA := Ore_algebra:-skew_algebra(shift = [E1, n1], shift = [E2, n2]) :
>
```

$$A_2 := \begin{bmatrix} E_1 + E_2 & n_1 + n_2 & 0 \\ E_1 & n_2 & 1 \\ E_2 & n_1 & -1 \end{bmatrix} :$$

```
> OreAlgebraGaussianElimination(A2, ShiftOA)
```

$$\begin{bmatrix} E_1 + E_2 & n_1 + n_2 & 0 \\ 0 & E_1 n_1 - E_2 n_2 + E_1 - E_2 & -E_1 - E_2 \\ 0 & 0 & 0 \end{bmatrix} \quad (2)$$

```
> A3 := Matrix([[E1, n1, 0], [E1, 0, 1], [0, n1^2+1, E1-1]]):
> OreAlgebraGaussianElimination(A3, ShiftOA)
```

$$\begin{bmatrix} E_1 & n_1 & 0 \\ 0 & -n_1 & 1 \\ 0 & 0 & -E_1 n_1 - n_1^2 + n_1 - 1 \end{bmatrix} \tag{3}$$

```
> QShiftOA := Ore_algebra:-skew_algebra(qdilat = [Q1, x1, q],
                                        qdilat = [Q2, x2, q]):
> A4 := Matrix([[Q1 + 2*Q2, x1 + x2, 0], [Q1, x2, 1], [0, 1, 0]]):
>
> OreAlgebraGaussianElimination(A4, QShiftOA)
```

$$\begin{bmatrix} Q_1 + 2Q_2 & x_1 + x_2 & 0 \\ 0 & q\,Q_1 x_1 - 2\,q\,x_2 Q_2 & -Q_1 - 2Q_2 \\ 0 & 0 & -Q_1 - 2Q_2 \end{bmatrix} \tag{4}$$

```
>
```