```
> restart;
> read "lrshypergeomsols.mpl";
```

A basis of hypergeometric solutions for a homogeneous system

> $S := \begin{bmatrix} -\dfrac{5x^2-1}{x^2-5x+6} & 0 \\ -\dfrac{5x^2-1}{x^2-5x+6} & 0 \end{bmatrix} \cdot y(x+2) + \begin{bmatrix} 0 & \dfrac{x^3+x^2-10x+8}{x-3} \\ 0 & -\dfrac{x^3+x^2-10x+8}{x-3} \end{bmatrix} \cdot y(x+1) + \begin{bmatrix} 5x^2+20x+19 & -x^3-x^2+5x-3 \\ 5x^2+20x+19 & x^3+x^2-5x+3 \end{bmatrix} \cdot y(x) = 0:$

For the homogeneous system, the independent solutions are output in a list as [Sol_1,..., Sol_n]

> $Outpt := LRS\text{:-}HypergeometricSolution(\mathrm{S}, \mathrm{y}(\mathrm{x}));$

$$Outpt := \left[\begin{bmatrix} \Gamma(x-3)\left(x^2-\frac{1}{5}\right) \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ \dfrac{\Gamma(x-3)(x-2)}{x+3} \end{bmatrix}, \begin{bmatrix} (-1)^x\,\Gamma(x-3)\left(x^2-\frac{1}{5}\right) \\ 0 \end{bmatrix} \right] \tag{1.1}$$

The general solution of the system, where C[1], C[2], C[3] are arbitrary constants:

> $Outpt[1]\cdot C[1] + Outpt[2]\cdot C[2] + Outpt[3]\cdot C[3]$

$$\begin{bmatrix} C_1\,\Gamma(x-3)\left(x^2-\frac{1}{5}\right) + C_3\,(-1)^x\,\Gamma(x-3)\left(x^2-\frac{1}{5}\right) \\ \dfrac{C_2\,\Gamma(x-3)(x-2)}{x+3} \end{bmatrix} \tag{1.2}$$

>

>

A general hypergeometric solution for an inhomogeneous system

> $S := \begin{bmatrix} -\dfrac{5x^2-1}{x^2-5x+6} & 0 \\ -\dfrac{5x^2-1}{x^2-5x+6} & 0 \end{bmatrix} . \; y(x+2) + \begin{bmatrix} 0 & \dfrac{x^3+x^2-10x+8}{x-3} \\ 0 & -\dfrac{x^3+x^2-10x+8}{x-3} \end{bmatrix} . \; y(x+1)$

$$+\begin{bmatrix} 5x^2+20x+19 & -x^3-x^2+5x-3 \\ 5x^2+20x+19 & x^3+x^2-5x+3 \end{bmatrix} \cdot y(x)$$

$$=\left[\left[-\frac{4\left(5x^3+15x^2-7x-29\right)\Gamma(x+1)}{(x-2)(x-3)} - \frac{x^6-8x^5-9x^4+97x^3+16x^2-255x+82}{(x-2)(x-3)}\right], \left[-\frac{4\left(5x^3+15x^2-7x-29\right)\Gamma(x+1)}{(x-2)(x-3)} + \frac{x^6+2x^5-19x^4-15x^3+46x^2-25x+86}{(x-2)(x-3)}\right]\right]:$$

For the inhomogeneous system, the output is a list containing the list of independent solutions of the homogeneous system in the first element, and a particular solution in the second

```
> Outpt := LRS:-HypergeometricSolution(S, y(x));
```

$$Outpt := \left[\left[\begin{bmatrix} (-1)^x\Gamma(x-3)\left(x^2-\frac{1}{5}\right) \\ 0 \end{bmatrix}, \begin{bmatrix} \frac{\Gamma(x+1)\left(x^2-\frac{1}{5}\right)}{(x-1)x(x-2)(x-3)} \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ \frac{\Gamma(x+1)}{(x-1)x(x-3)(x+3)} \end{bmatrix}\right], \begin{bmatrix} \frac{\Gamma(x+1)\left(x^4-6x^3-409x^2-6x+84\right)}{(x-1)x(x-2)(x-3)}+x \\ x+2 \end{bmatrix}\right] \quad \textbf{(2.1)}$$

The general solution:

```
> Outpt[1][1]·C[1] + Outpt[1][2]·C[2] + Outpt[1][3]·C[3] + Outpt[2]
```

$$\left[\left[C_1(-1)^x\Gamma(x-3)\left(x^2-\frac{1}{5}\right) + \frac{C_2\Gamma(x+1)\left(x^2-\frac{1}{5}\right)}{(x-1)x(x-2)(x-3)} + \frac{\Gamma(x+1)\left(x^4-6x^3-409x^2-6x+84\right)}{(x-1)x(x-2)(x-3)}+x\right], \left[\frac{C_3\Gamma(x+1)}{(x-1)x(x-3)(x+3)}+x+2\right]\right] \quad \textbf{(2.2)}$$

```
>
>
```

No hypergeometric solutions for a homogeneous system

$$> S := \begin{bmatrix} x & 1 \\ x^2+x & 0 \end{bmatrix} \cdot y(x+1) + \begin{bmatrix} -1 & 0 \\ -2x^2-4x & x^2+3x+2 \end{bmatrix} \cdot y(x) = 0 :$$

The null basis of hypergeometric solutions, the output is the empty list:

```
> LRS:-HypergeometricSolution(S, y(x));
```

$$[\ ] \quad \textbf{(3.1)}$$

\>

\>

A particular hypergeometric solution for an inhomogeneous system

\> $S := \begin{bmatrix} x & 1 \\ x^2+x & 0 \end{bmatrix} \cdot y(x+1) + \begin{bmatrix} -1 & 0 \\ -2x^2-4x & x^2+3x+2 \end{bmatrix} \cdot y(x) = \begin{bmatrix} (x^2+x-1)\,x! - (-1)^x x \\ (x^3-3x)\,x! + (-4x^2-8x-2)\,(-1)^x \end{bmatrix}$:

The output is a list containing the empty list in the first element, and a particular solution in the second

\> LRS:-$HypergeometricSolution$(S, y(x));

$$\left[[\ ], \begin{bmatrix} x! + (-1)^x \\ -(-1)^x \end{bmatrix} \right] \qquad (4.1)$$

With the optional argumen "output=partsol", the output is a particular solution.

\> LRS:-$HypergeometricSolution$(S, y(x), $output = partsol$);

$$\begin{bmatrix} x! + (-1)^x \\ -(-1)^x \end{bmatrix} \qquad (4.2)$$

\>

No hypergeometric soutions for an inhomogeneous system

\> $S := \begin{bmatrix} x & 1 \\ x^2+x & 0 \end{bmatrix} \cdot y(x+1) + \begin{bmatrix} -1 & 0 \\ -2x^2-4x & x^2+3x+2 \end{bmatrix} \cdot y(x) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$:

The output is NULL

\> LRS:-$HypergeometricSolution$(S, y(x));

\>