>

The difference case, no solutions

```
> restart;
>
```

Load lfs.zip from http://www.ccas.ru/ca/_media/lfs.zip
This archive includes two files: lfs.ind and lfs.lib.
Put these files to some directory, for example to ''/usr/userlib''

```
> libname := "/usr/userlib/lfs.lib", libname :
>
> eq1 := Matrix([[x^2 + 102 x + 101, x^3 + 104 x^2 + 305 x + 202],
        [x^2-x-2, x^3 + x^2-4 x-4]]) . y(x + 2) +
     Matrix([[-x^2-99 x + 202, -x + 2],
        [x-2, (x-2) / (x + 101)]]) . y(x + 1) +
     Matrix([[-x-101, -(x + 101) / (x + 100)],
        [-x, -x/ (x + 100)]]) . y(x) = 0;
```

$$eq1 := \begin{bmatrix} x^2+102x+101 & x^3+104x^2+305x+202 \\ x^2-x-2 & x^3+x^2-4x-4 \end{bmatrix} \cdot y(x+2) + \begin{bmatrix} -x^2-99x+202 & -x+2 \\ x-2 & \frac{x-2}{x+101} \end{bmatrix} \cdot y(x+1) + \begin{bmatrix} -x-101 & -\frac{x+101}{x+100} \\ -x & -\frac{x}{x+100} \end{bmatrix} \cdot y(x) = 0 \quad (1.1)$$

```
>
```

There is no rational solutions, the empty list is returned.
For this system, the indicial polynomial has no roots. The algorithm with checkpoints stops early.

```
> st := time() : LFS:-RationalSolution(eq1, y(x), earlyterminate = true); time() - st;
```

[ ]

0.067 (1.2)

```
> restart;
> libname := "/usr/userlib/lfs.lib", libname :
>
> eq1 := Matrix([[x^2 + 102 x + 101, x^3 + 104 x^2 + 305 x + 202],
        [x^2-x-2, x^3 + x^2-4 x-4]]) . y(x + 2) +
     Matrix([[-x^2-99 x + 202, -x + 2],
        [x-2, (x-2) / (x + 101)]]) . y(x + 1) +
     Matrix([[-x-101, -(x + 101) / (x + 100)],
        [-x, -x/ (x + 100)]]) . y(x) = 0 :
>
```

The algorithm without checkpoints found a universal denominator U(x) (its degree is equal 205), made the substitution y(x)=z(x)/U(x) in the given system, fond the indicial polynomial for the new system, and stopped because the indicial polynomial has no roots

```
> st := time() : LFS:-RationalSolution(eq1, y(x), earlyterminate = false); time() - st;
```

[ ]

4.019 (1.3)

```
> degree(eval(LFS:-System(eq1, y(x)))[universal_denominator], x)
```

205 (1.4)

```
>
```

The differential case, no solutions

```
> restart;
> libname := "/usr/userlib/lfs.lib", libname :
>
> eq2 :=
    Matrix([[2, 0],
        [0, x^2 + x]]) . diff(y(x), x$2) +
    Matrix([[1, 1],
        [x, x^2 + 5 x + 2]]) . diff(y(x), x) +
    Matrix([[-1, 1],
        [x + 1, 2 x + 4]]) . y(x) = 0;
```

$$eq2 := \begin{bmatrix} 2 & 0 \\ 0 & x^2+x \end{bmatrix} \cdot \left(\frac{d^2}{dx^2}\, y(x)\right) + \begin{bmatrix} 1 & 1 \\ x & x^2+5x+2 \end{bmatrix} \cdot \left(\frac{d}{dx}\, y(x)\right) + \begin{bmatrix} -1 & 1 \\ x+1 & 2x+4 \end{bmatrix} \cdot y(x) = 0 \qquad (2.1)$$

```
>
```

There is no rational solutions.
The indicial polynomial for this system has the integer root $n^*=-3$, then the algorithm with checkpoints found a universal denominator U(x) (deg U(x) = 2)and after that stopped because $n^* +\deg U(x) < 0$

```
> st := time( ) : LFS:-RationalSolution(eq2, y(x), earlyterminate = true); time( ) − st;
```

[ ]

0.175 **(2.2)**

```
> restart;
> libname := "/usr/userlib/lfs.lib", libname :
>
> eq2 :=
    Matrix([[2, 0],
        [0, x^2 + x]]) . diff(y(x), x$2) +
    Matrix([[1, 1],
        [x, x^2 + 5 x + 2]]) . diff(y(x), x) +
    Matrix([[-1, 1],
        [x + 1, 2 x + 4]]) . y(x) = 0 :
>
```

The algorithm without checkpoints takes near the same time:

```
> st := time( ) : LFS:-RationalSolution(eq2, y(x), earlyterminate = false); time( ) − st;
```

[ ]

0.100 **(2.3)**

```
> LFS:-System(eq2, y(x))[universal_denominator]
```

$x\,(x+1)$ **(2.4)**

```
>
```

The difference case, there are solutions

```
> restart;
> libname := "/usr/userlib/lfs.lib", libname :
>
> eq3 :=
    Matrix([[0, 0],
        [x^3 + 5 x^2 + 9 x + 5,
         x^3 + 5 x^2 + 9 x + 5]]) · y(x + 2) +
    Matrix([[2 x^2 - 2, 2(x^2 - 1) / (x + 101)],
        [x^3 - x^2 - x + 1,
```

```
(x^3 - x^2 - x + 1) / (x + 101)]]) · y(x + 1) +
Matrix([[-2 x^2 + 2 x, -2 x (x - 1) / (x + 100)],
       [-2 x^3 + x^2 - 2 x - 1,
      -(x^4 + 102 x^3 + 99 x^2 + 102 x + 100) /
                  (x + 100)]]) · y(x);
```

$$eq3 := \begin{bmatrix} 0 & 0 \\ x^3+5x^2+9x+5 & x^3+5x^2+9x+5 \end{bmatrix} \cdot y(x+2) + \begin{bmatrix} 2x^2-2 & \frac{2(x^2-1)}{x+101} \\ x^3-x^2-x+1 & \frac{x^3-x^2-x+1}{x+101} \end{bmatrix} \cdot y(x+1) + \begin{bmatrix} -2x^2+2x & -\frac{2x(x-1)}{x+100} \\ -2x^3+x^2-2x-1 & -\frac{x^4+102x^3+99x^2+102x+100}{x+100} \end{bmatrix} \cdot y(x) \quad \textbf{(3.1)}$$

```
>
```

The algorithms with and without checkpoints takes not large difference of the time if there are rational solution

```
> st := time( ) : LFS:-RationalSolution(eq3, y(x), earlyterminate = true); time( ) - st;
```

$$\left[\left[\begin{array}{c} -\frac{1}{(x^2+1)(x+99)} \\ \frac{x+100}{(x^2+1)(x+99)} \end{array}\right], \left[\begin{array}{c} -\frac{x^3+100x^2-59600x+100}{x(x^2+1)(x+99)} \\ \frac{x^3-59501x^2-5960099x+100}{x(x^2+1)(x+99)} \end{array}\right]\right]$$

$$84.679 \quad \textbf{(3.2)}$$

```
> restart;
> libname := "/usr/userlib/lfs.lib", libname :
>
> eq3 :=
    Matrix([[0, 0],
           [x^3 + 5 x^2 + 9 x + 5,
            x^3 + 5 x^2 + 9 x + 5]]) · y(x + 2) +
    Matrix([[2 x^2 - 2, 2(x^2 - 1) / (x + 101)],
           [x^3 - x^2 - x + 1,
         (x^3 - x^2 - x + 1) / (x + 101)]]) · y(x + 1) +
    Matrix([[-2 x^2 + 2 x, -2 x (x - 1) / (x + 100)],
           [-2 x^3 + x^2 - 2 x - 1,
          -(x^4 + 102 x^3 + 99 x^2 + 102 x + 100) /
                      (x + 100)]]) · y(x) :
>
> st := time( ) : LFS:-RationalSolution(eq3, y(x), earlyterminate = false); time( ) - st;
```

$$\left[\left[\begin{array}{c} -\frac{1}{(x^2+1)(x+99)} \\ \frac{x+100}{(x^2+1)(x+99)} \end{array}\right], \left[\begin{array}{c} -\frac{x^3+100x^2-59600x+100}{x(x^2+1)(x+99)} \\ \frac{x^3-59501x^2-5960099x+100}{x(x^2+1)(x+99)} \end{array}\right]\right]$$

$$74.040 \quad \textbf{(3.3)}$$

```
> degree(eval(LFS:-System(eq3, y(x)))[universal_denominator], x)
```

(3.4)