> restart;

>

Load lfs.zip from http://www.ccas.ru/ca/_media/lfs.zip
This archive includes two files: lfs.ind and lfs.lib.
Put these files to some directory, for example to ''/usr/userlib''

> libname := "/usr/userlib/lfs.lib", libname :

>

The differential case

> $S := \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \cdot \left(\frac{d^2}{dx^2} y(x) \right) + \begin{bmatrix} 1 & 0 \\ 0 & x \end{bmatrix} \cdot y(x) = \begin{bmatrix} \frac{1}{4} \frac{4x^2-1}{x^{3/2}} \\ \frac{1}{4} \frac{4x^3-x^2-4}{x^{3/2}(x^2+4)} \end{bmatrix}$:

A particular solution for the system S:

> Solution := LFS:-HypergeometricSolution(S, y(x), output = partsol);

$$Solution := \begin{bmatrix} \sqrt{x} \\ \frac{\sqrt{x}}{x^2+4} \end{bmatrix} \quad (1.1)$$

Check the solution by substitute:

> simplify(eval(S, y(x) = Solution))

$$\begin{bmatrix} \frac{4x^2-1}{4x^{3/2}} \\ \frac{4x^3-x^2-4}{x^{3/2}(4x^2+16)} \end{bmatrix} = \begin{bmatrix} \frac{4x^2-1}{4x^{3/2}} \\ \frac{4x^3-x^2-4}{x^{3/2}(4x^2+16)} \end{bmatrix} \quad (1.2)$$

>

The correspondent system for rational solutions given by substitute y(x) = sqrt(x) F(x) to the system S:

> $S1 := \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \cdot \left(\frac{d^2}{dx^2} F(x) \right) + \begin{bmatrix} \frac{1}{x} & 0 \\ \frac{1}{x} & 0 \end{bmatrix} \cdot \left(\frac{d}{dx} F(x) \right) + \begin{bmatrix} 1 - \frac{1}{4x^2} & 0 \\ -\frac{1}{4x^2} & x \end{bmatrix} . F(x) = \begin{bmatrix} \frac{4x^2-1}{4x^2} \\ \frac{4x^3-x^2-4}{4x^2(x^2+4)} \end{bmatrix}$:

A particular rational solution for the system S1:

> Rational_Solution := LFS:-RationalSolution(S1, F(x), output = partsol);

$$Rational\_Solution := \begin{bmatrix} 1 \\ \frac{1}{x^2+4} \end{bmatrix} \quad (1.3)$$

Check the solution by substitute:

> map(el→map(normal, el), eval(S1, F(x) = Rational_Solution))

$$\begin{bmatrix} \frac{4x^2-1}{4x^2} \\ \frac{4x^3-x^2-4}{4x^2(x^2+4)} \end{bmatrix} = \begin{bmatrix} \frac{4x^2-1}{4x^2} \\ \frac{4x^3-x^2-4}{4x^2(x^2+4)} \end{bmatrix} \quad (1.4)$$

>

>

The difference case

> $S2 := \begin{bmatrix} n & 1 \\ n^2+n & 0 \end{bmatrix} \cdot y(n+1) + \begin{bmatrix} -1 & 0 \\ -2n^2-4n & n^2+3n+2 \end{bmatrix} \cdot y(n)$
$= \begin{bmatrix} (n^2+n-1)\,n! - (-1)^n n \\ (n^3-3n)\,n! + (-4n^2-8n-2)\,(-1)^n \end{bmatrix}$:

>

A particular solution for the system S2:

> *Solution* := *LFS:-HypergeometricSolution*(S2, y(n), output = partsol);

$$Solution := \begin{bmatrix} n! + (-1)^n \\ -(-1)^n \end{bmatrix} \quad (2.1)$$

Check the solution by substitute:

> *simplify*(*eval*((*lhs* − *rhs*)(*S2*), {*y*(*n*) = *Solution*, *y*(*n* + 1) = *eval*(*Solution*, *n* = *n* + 1)}))

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (2.2)$$

>

The correspondent system for rational solutions given by substitute y(n) = n! F(n) to the system S2:

> $S3 := \begin{bmatrix} (n+1)\,n & n+1 \\ (n+1)\,(n^2+n) & 0 \end{bmatrix} \cdot F(n+1) + \begin{bmatrix} -1 & 0 \\ -2n^2-4n & n^2+3n+2 \end{bmatrix} \cdot F(n)$
$= \begin{bmatrix} n^2+n-1 \\ n\,(n^2-3) \end{bmatrix}$:

A particular rational solution for the system S3:

> *Rational_Solution* := *LFS:-RationalSolution*(*S3*, *F*(*n*), output = partsol);

$$Rational\_Solution := \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad (2.3)$$

Check the solution by substitute:

> *map*(*el*→*map*(*expand*, *el*), *eval*(*S3*, {*F*(*n*) = *Rational_Solution*, *F*(*n* + 1) = *eval*(*Rational_Solution*, *n* = *n* + 1)}))

$$\begin{bmatrix} n^2+n-1 \\ n^3-3n \end{bmatrix} = \begin{bmatrix} n^2+n-1 \\ n^3-3n \end{bmatrix} \quad (2.4)$$

>

>

>

The q-difference case

> $S4 := \begin{bmatrix} -\frac{q^3-x}{q^2} & \frac{q}{x} \\ \frac{q^6-q^3x^2-x^2}{q\,x^2} & \frac{q^3-x^2-x}{x} \end{bmatrix} \cdot y(q\,x) + \begin{bmatrix} -\frac{x}{q} & -1 \\ -\frac{q^5-q^2x-x}{x} & -\frac{q^3-x}{q} \end{bmatrix} \cdot y(x)$
$= \begin{bmatrix} -\frac{x}{q^4} \\ q^{\frac{k(k+1)}{2}} + \frac{q^4-q^3x-q\,x^2+x^2}{q^4\,x} \end{bmatrix}:$

A particular solution for the system S4:

> $Solution := LFS\text{:-}HypergeometricSolution(\mathrm{S4}, \mathrm{y(x)}, \mathrm{x} = \mathrm{q}^k, \mathrm{output} = \mathrm{partsol});$

$$Solution := \begin{bmatrix} \frac{x}{q^6} \\ -\frac{q\,q^{\frac{k(k+1)}{2}}}{x^2} \end{bmatrix} \quad (3.1)$$

Check the solution by substitute:

> $map(simplify@expand, eval(eval((lhs-rhs)(S4), \{y(x) = Solution, y(x\cdot q) = eval(Solution, \{k = k+1, x = x\cdot q\})\}), x = q^k));$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (3.2)$$

A particular rational solution for the system S4 with rational right-hand sides

> $\mathrm{lhs(S4)} = \begin{bmatrix} -\frac{x}{q^4} \\ \frac{q^4-q^3x-q\,x^2+x^2}{q^4\,x} \end{bmatrix};$

$LFS\text{:-}RationalSolution(\%, \mathrm{y(x)}, \mathrm{x} = \mathrm{q}^k, \mathrm{output} = \mathrm{partsol});$

$$\begin{bmatrix} -\frac{q^3-x}{q^2} & \frac{q}{x} \\ \frac{q^6-q^3x^2-x^2}{q\,x^2} & \frac{q^3-x^2-x}{x} \end{bmatrix} \cdot y(q\,x) + \begin{bmatrix} -\frac{x}{q} & -1 \\ -\frac{q^5-q^2x-x}{x} & -\frac{q^3-x}{q} \end{bmatrix} \cdot y(x)$$
$$= \begin{bmatrix} -\frac{x}{q^4} \\ \frac{q^4-q^3x-q\,x^2+x^2}{q^4\,x} \end{bmatrix}$$

$$\begin{bmatrix} \frac{x}{q^6} \\ 0 \end{bmatrix} \quad (3.3)$$

The correspondent system for rational solutions given by substitute y(x) = q^(1/2*k*(k+1))/x*R(x) to the system S4:

> $S5 := \begin{bmatrix} -\frac{x}{q} & -1 \\ -\frac{q^5 - q^2 x - x}{x} & -\frac{q^3 - x}{q} \end{bmatrix} . F(x) + \begin{bmatrix} -\frac{x(q^3 - x)}{q^2} & q \\ \frac{q^6 - q^3 x^2 - x^2}{x q} & q^3 - x^2 - x \end{bmatrix} . F(q x) = \begin{bmatrix} 0 \\ x \end{bmatrix}$:

A particular rational solution for the system S5:

> $Rational\_Solution := LFS\text{:-}RationalSolution\left(S5, F(x), x = q^k, output = partsol\right);$

$$Rational\_Solution := \begin{bmatrix} 0 \\ -\frac{q}{x} \end{bmatrix} \quad \mathbf{(3.4)}$$

Check the solution by substitute:

> $simplify(eval((lhs - rhs)(S5), \{F(x) = Rational\_Solution, F(x \cdot q) = eval(Rational\_Solution, x = x \cdot q)\}))$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \mathbf{(3.5)}$$

>

>

The q-difference case, q=2

> $S6 := \begin{bmatrix} \frac{x}{4} - 2 & \frac{2}{x} \\ \frac{-9x^2 + 64}{2x^2} & \frac{-x^2 - x + 8}{x} \end{bmatrix} \cdot y(2x) + \begin{bmatrix} -\frac{x}{2} & -1 \\ -\frac{-5x + 32}{x} & \frac{x}{2} - 4 \end{bmatrix} \cdot y(x) = \begin{bmatrix} -\frac{x}{16} \\ 2^{\frac{k(k+1)}{2}} + \frac{-x^2 - 8x + 16}{16x} \end{bmatrix}$:

A particular solution for the system S6:

> $Solution := LFS\text{:-}HypergeometricSolution\left(S6, y(x), x = 2^k, output = partsol\right);$

$$Solution := \begin{bmatrix} \frac{x}{64} \\ -\frac{2\, 2^{\frac{k(k+1)}{2}}}{x^2} \end{bmatrix} \quad \mathbf{(4.1)}$$

>

Check the solution by substitute:

> $eval\left(eval(S6, \{y(x) = Solution, y(x \cdot 2) = eval(Solution, \{k = k+1, x = x \cdot 2\})\}), x = 2^k\right);$
$seq(eval((lhs - rhs)(\%), k = ki), ki = 0..10);$

$$\left[\left[\frac{2^k \left(\frac{2^k}{4} - 2\right)}{32} - \frac{2^{\frac{(k+1)(k+2)}{2}}}{(2^k)^3} - \frac{(2^k)^2}{128} + \frac{2\, 2^{\frac{k(k+1)}{2}}}{(2^k)^2}\right],\right.$$

$$\left[\frac{-9(2^k)^2 + 64}{64\, 2^k} - \frac{\left(-(2^k)^2 - 2^k + 8\right) 2^{\frac{(k+1)(k+2)}{2}}}{2 (2^k)^3} + \frac{5\, 2^k}{64} - \frac{1}{2}\right.$$

$$-\frac{2\left(\frac{2^k}{2}-4\right)2^{\frac{k(k+1)}{2}}}{\left(2^k\right)^2}\Bigg]\Bigg] = \begin{bmatrix} -\frac{2^k}{16} \\ 2^{\frac{k(k+1)}{2}} + \frac{-\left(2^k\right)^2-8\,2^k+16}{16\,2^k} \end{bmatrix}$$

$$\begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}0\\0\end{bmatrix} \quad \textbf{(4.2)}$$

A particular rational solution for the system S6 with rational right-hand sides

> $\mathrm{lhs}(S6) = \begin{bmatrix} -\frac{x}{16} \\ \frac{-x^2-8x+16}{16x} \end{bmatrix};$

$LFS{:}\text{-}RationalSolution(\%, \mathrm{y}(\mathrm{x}), \mathrm{x} = 2^k, \mathrm{output} = \mathrm{partsol});$

$$\begin{bmatrix} \frac{x}{4}-2 & \frac{2}{x} \\ \frac{-9x^2+64}{2x^2} & \frac{-x^2-x+8}{x} \end{bmatrix} \cdot y(2x) + \begin{bmatrix} -\frac{x}{2} & -1 \\ -\frac{-5x+32}{x} & \frac{x}{2}-4 \end{bmatrix} \cdot y(x)$$

$$= \begin{bmatrix} -\frac{x}{16} \\ \frac{-x^2-8x+16}{16x} \end{bmatrix}$$

$$\begin{bmatrix} \frac{x}{64} \\ 0 \end{bmatrix} \quad \textbf{(4.3)}$$

The correspondent system for rational solutions given by substitute y(x) = 2^(1/2*k*(k+1))/x*R(x) to the system S6:

> $S7 := \begin{bmatrix} -\frac{x}{2} & -1 \\ -\frac{-5x+32}{x} & \frac{x}{2}-4 \end{bmatrix} \cdot F(x) + \begin{bmatrix} -\frac{x(-x+8)}{4} & 2 \\ \frac{-9x^2+64}{2x} & -x^2-x+8 \end{bmatrix} \cdot F(2x) = \begin{bmatrix} 0 \\ x \end{bmatrix}:$

A particular rational solution for the system S7:

> $Rational\_Solution := LFS{:}\text{-}RationalSolution(\mathrm{S7}, \mathrm{F}(\mathrm{x}), \mathrm{x} = 2^k, \mathrm{output} = \mathrm{partsol});$

$$Rational\_Solution := \begin{bmatrix} 0 \\ -\frac{2}{x} \end{bmatrix} \quad \textbf{(4.4)}$$

Check the solution by substitute:

> $simplify(eval((lhs - rhs)(S7), \{F(x) = Rational\_Solution, F(x \cdot 2) = eval(Rational\_Solution, x = x \cdot 2)\}))$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \textbf{(4.5)}$$

>

>

>