

> restart;

>

Load TruncatedSeries2023.zip from

http://www.ccas.ru/ca/_media/truncatedseries2023.zip

This archive includes two files: maple.ind and maple.lib.

Put these files to some directory, for example to "/usr/userlib"

> libname := "/usr/userlib", libname :

> with(TruncatedSeries) :

>

> Matrix([[1 + O(x), x + O(x^2), x + O(x^2)],
[x + O(x^2), O(x), x + O(x^2)],
[O(x), x + O(x^2), 1 + x + O(x^2)]]).theta(y(x), x, 1) +
Matrix([[K 2 + x + O(x^2), O(1), x + O(x^2)],
[x + O(x^2), 1 + x + O(x^2), 1 + x + O(x^2)],
[x + O(x^2), x + O(x^2), K 2 + x + O(x^2)]]).y(x) = 0;

$$\begin{bmatrix} 1 + O(x) & x + O(x^2) & x + O(x^2) \\ x + O(x^2) & O(x) & x + O(x^2) \\ O(x) & x + O(x^2) & 1 + x + O(x^2) \end{bmatrix} \cdot \theta(y(x), x, 1) \quad (1)$$
$$+ \begin{bmatrix} K 2 + x + O(x^2) & O(1) & x + O(x^2) \\ x + O(x^2) & 1 + x + O(x^2) & 1 + x + O(x^2) \\ x + O(x^2) & x + O(x^2) & K 2 + x + O(x^2) \end{bmatrix} \cdot y(x) = 0$$

> TruncatedSeries:-LaurentSolution((1), y(x))

$$[[x^2_c_1 + O(x^3), O(x^3), O(x^3)]] \quad (2)$$

> Matrix([[1 + O(x), x + O(x^2), x + O(x^2)],
[x + O(x^2), O(x^2), x + O(x^2)],
[x + O(x^2), x + O(x^2), 1 + x + O(x^2)]]).theta(y(x), x, 1) +
Matrix([[K 2 + x + O(x^2), x + O(x^2), x + O(x^2)],
[x + O(x^2), 1 + x + O(x^2), 1 + x + O(x^2)],
[x + O(x^2), x + O(x^2), K 2 + x + O(x^2)]]).y(x) = 0;

$$\begin{bmatrix} 1 + O(x) & x + O(x^2) & x + O(x^2) \\ x + O(x^2) & O(x^2) & x + O(x^2) \\ x + O(x^2) & x + O(x^2) & 1 + x + O(x^2) \end{bmatrix} \cdot \theta(y(x), x, 1) \quad (3)$$
$$+ \begin{bmatrix} K 2 + x + O(x^2) & x + O(x^2) & x + O(x^2) \\ x + O(x^2) & 1 + x + O(x^2) & 1 + x + O(x^2) \\ x + O(x^2) & x + O(x^2) & K 2 + x + O(x^2) \end{bmatrix} \cdot y(x) = 0$$

> TruncatedSeries:-LaurentSolution((3), y(x));

$$[[x^2_c_1 + O(x^3), 2x^3_c_2 + x^2_c_2 + O(x^4), K 3x^3_c_1 K x^2_c_2 + O(x^4)], [x^2_c_1 + O(x^3), O(x^4), K 3x^3_c_1 + O(x^4)], [O(x^4), 2x^3_c_2 + x^2_c_2 + O(x^4), K x^2_c_2 + O(x^4)]] \quad (4)$$

>

> Matrix([[3*x + O(x^2), 7*x^2 + O(x^4)], [O(x^2), 17*x^2 + O(x^4)]]).theta(y(x), x, 2) +
Matrix([[K 1 + 2*x + O(x^2), 5*x^2 + x + O(x^4)], [O(x^2), 11*x^2 + O(x^4)]])

.theta(y(x), x, 1) +
 Matrix([[O(1), K 3*x^2 + x + O(x^4)], [1 + O(x^2), K 6*x^2 + O(x^4)]]) .y(x) = 0;

$$\begin{bmatrix} 3x + O(x^2) & 7x^2 + O(x^4) \\ O(x^2) & 17x^2 + O(x^4) \end{bmatrix} \cdot \theta(y(x), x, 2) + \begin{bmatrix} K 1 + 2x + O(x^2) & 5x^2 + x + O(x^4) \\ O(x^2) & 11x^2 + O(x^4) \end{bmatrix} \quad (5)$$

$$\cdot \theta(y(x), x, 1) + \begin{bmatrix} O(1) & K 3x^2 + x + O(x^4) \\ 1 + O(x^2) & K 6x^2 + O(x^4) \end{bmatrix} \cdot y(x) = 0$$

> TruncatedSeries:-LaurentSolution((5), y(x));

$$\left[\left[6x^2 _c_1 + O(x^3), \frac{-c_1}{x} + _c_1 + O(x) \right] \right] \quad (6)$$

> Matrix([[O(x^5), K 1 + O(x^5)], [1 + O(x^5), O(x^5)]]) .theta(y(x), x, 1) +
 Matrix([[O(x^5), O(1)], [2 + O(x^5), O(x^5)]]) .y(x) = 0;

$$\begin{bmatrix} O(x^5) & K 1 + O(x^5) \\ 1 + O(x^5) & O(x^5) \end{bmatrix} \cdot \theta(y(x), x, 1) + \begin{bmatrix} O(x^5) & O(1) \\ 2 + O(x^5) & O(x^5) \end{bmatrix} \cdot y(x) = 0 \quad (7)$$

The system (7) has no solutions

> TruncatedSeries:-LaurentSolution((7), y(x));

FAIL (8)

because there are its prolongations, the solutions of which have different initial terms

> TruncatedSeries:-LaurentSolution(eval((7), O(1) = 5 + O(x)), y(x))

$$\left[\left[O(x^{10}), x^5 _c_1 + O(x^6) \right] \right] \quad (9)$$

> TruncatedSeries:-LaurentSolution(eval((7), O(1) = 6 + O(x)), y(x))

$$\left[\left[O(x^{11}), x^6 _c_1 + O(x^7) \right] \right] \quad (10)$$