```
> restart;
> read "lrshypergeomsolscasc2015.mpl";
>
```

Example 1

$$> A1 := \begin{bmatrix} \frac{x-1}{x} & 0 & -\frac{x-1}{x+1} & 0 \\ 1 & 0 & \frac{2}{x+1} & -x \\ -1 & 1 & x-1 & 1 \\ -\frac{x+2}{x} & \frac{x+1}{x} & \frac{x^2-x-1}{(x+1)\cdot x} & \frac{x+x^2+1}{x} \end{bmatrix}:$$

\>

The procedure returns a basis of hypergeom solutions space for Y(x+1)=A1.Y(x).
One-dimention hypergeometric solutions space is found

$> st := time(\,):$
$Res := LRS\text{:-}HypergeometricSolution(A1, x);$
$time(\,) - st;$

$$\left[\begin{bmatrix} 0 \\ -\Gamma(x) \\ 0 \\ \Gamma(x) \end{bmatrix}\right]:$$

$$Res := \left[\begin{bmatrix} 0 \\ -\Gamma(x) \\ 0 \\ \Gamma(x) \end{bmatrix}\right]$$

$$0.320 \qquad \textbf{(1.1)}$$

Check result (must be zero-vector.):

$> seq(map(simplify, eval(Res[i], x = x+1) - A1.Res[i]), i = 1..nops(Res));$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad \textbf{(1.2)}$$

\>

Compute the y1-resolving sequence:

$> L, B := LRS\text{:-}Resolving(A1, x, y):$
$L[1]; L[2];$

$$(-x^6 - x^5 + 4x^4 + 3x^3 - 3x^2 - 2x)\, y(x) + (2x^6 + 6x^5 + 2x^4 - 6x^3 - 6x^2 - 4x)\, y(x+1) + (-x^6 - 6x^5 - 11x^4 - 2x^3 + 8x^2 + 8x + 4)\, y(x+2) + (x^5 + 4x^4 + x^3 - 6x^2)\, y(x+3)$$

$$-y(x)\, x + y(x+1) \quad \textbf{(1.3)}$$

Compute an equivalent defference equation by the cyclic-method:

```
> infolevel[LRS] := 3 :
  L, B := LRS:-CyclicVector(A1, x, y) :
  L; B;
```

```
CyclicVector: check if  [-6 -1 -1 -9] is a cyclic vector
CyclicVector: 4 order resolving equation is constructed:
0.33e-1
```

$$[(343x^{16} + 6076x^{15} + 49308x^{14} + 234748x^{13} + 684903x^{12} + 1095233x^{11} + 177770x^{10} - 3145154x^9 - 6608380x^8 - 5975068x^7 - 1019999x^6 + 4530659x^5 + 6265455x^4 + 3183906x^3 + 520200x^2)\, y(x) + (-686x^{16} - 13867x^{15} - 130711x^{14} - 755460x^{13} - 2951533x^{12} - 8112599x^{11} - 15795356x^{10} - 21393549x^9 - 19234665x^8 - 8979850x^7 + 6618927x^6 + 24640619x^5 + 34156784x^4 + 26007746x^3 + 11068840x^2 + 2199360x)\, y(x+1) + (343x^{16} + 8477x^{15} + 95613x^{14} + 658430x^{13} + 3102863x^{12} + 10572268x^{11} + 26738910x^{10} + 50720193x^9 + 72866501x^8 + 81497117x^7 + 72910088x^6 + 45373790x^5 - 2882504x^4 - 49477911x^3 - 57134114x^2 - 30393864x - 6739200)\, y(x+2) + (-686x^{15} - 14210x^{14} - 136689x^{13} - 814106x^{12} - 3353898x^{11} - 10046276x^{10} - 22285791x^9 - 36737792x^8 - 45828310x^7 - 47976801x^6 - 49835116x^5 - 42455703x^4 - 7176998x^3 + 29009408x^2 + 27231168x + 8380800)\, y(x+3) + (343x^{14} + 6076x^{13} + 49896x^{12} + 254915x^{11} + 899651x^{10} + 2276489x^9 + 4164348x^8 + 5503364x^7 + 5755407x^6 + 6897982x^5 + 8885611x^4 + 5263006x^3 - 2605216x^2 - 4469472x - 1670400)\, y(x+4)]$$

$$\Bigg[\Big[\big[-6, -1, -1, -9\big], \quad \textbf{(1.4)}$$

$$\left[\frac{3(x+8)}{x}, -\frac{10x+9}{x}, -\frac{x^3+3x^2-2x-9}{(x+1)x}, -\frac{8x^2+10x+9}{x}\right],$$

$$\left[\frac{9x^4+57x^3+161x^2+188x+54}{(x+1)(x+2)x}, -\frac{9x^4+56x^3+128x^2+126x+54}{(x+1)(x+2)x}, -\frac{x^6+14x^5+43x^4+66x^3+25x^2-104x-54}{(x+1)^2(x+2)x}, -\frac{8x^5+41x^4+96x^3+144x^2+126x+54}{(x+1)(x+2)x}\right],$$

$$\left[\frac{9x^7+133x^6+816x^5+2767x^4+5565x^3+6386x^2+3780x+938}{(x+2)^2(x+3)(x+1)x},\right.$$

$$-\frac{9 x^7 + 125 x^6 + 727 x^5 + 2338 x^4 + 4496 x^3 + 5095 x^2 + 3210 x + 938}{(x+2)^2 (x+3) (x+1) x},$$

$$-\frac{1}{(x+2)^2 (x+3) (x+1)^2 x} (x^9 + 28 x^8 + 225 x^7 + 895 x^6 + 2028 x^5 + 2461 x^4 + 762 x^3 - 2032 x^2 - 2656 x - 938),$$

$$-\frac{8 x^8 + 97 x^7 + 517 x^6 + 1631 x^5 + 3474 x^4 + 5232 x^3 + 5287 x^2 + 3210 x + 938}{(x+2)^2 (x+3) (x+1) x}]]$$

>

Example 2

```
> A2 := Matrix([
[(x^3 + 4*x^2 + 4*x-2)/((x + 4)*(x + 2)*(x + 1)), (x^2 + 3*x + 1)/((x + 2)*(x
    + 1)), (x + 1)/(x + 4), (2*x + 4)/(x + 4)],
[ -(x^3 + 4*x^2 + 4*x-2)/((x + 4)*(x + 2)*(x + 1)), 1/((x + 2)*(x + 1)), -(x
    + 1)/(x + 4),  -x/(x + 4)],
[-x*(2*x^2 + 8*x + 9)/((x + 4)*(x + 2)*(x + 1)), -(x^2 + 3*x + 1)/((x + 2)
    *(x + 1)), -(2*x + 2)/(x + 4), -2*x/(x + 4)],
[(x + 1)/(x + 4), 0, (x + 1)/(x + 4), x/(x + 4)]]);
```

$$A2 := \begin{bmatrix} \frac{x^3+4x^2+4x-2}{(x+4)(x+2)(x+1)} & \frac{x^2+3x+1}{(x+2)(x+1)} & \frac{x+1}{x+4} & \frac{2x+4}{x+4} \\ -\frac{x^3+4x^2+4x-2}{(x+4)(x+2)(x+1)} & \frac{1}{(x+2)(x+1)} & -\frac{x+1}{x+4} & -\frac{x}{x+4} \\ -\frac{x(2x^2+8x+9)}{(x+4)(x+2)(x+1)} & -\frac{x^2+3x+1}{(x+2)(x+1)} & -\frac{2x+2}{x+4} & -\frac{2x}{x+4} \\ \frac{x+1}{x+4} & 0 & \frac{x+1}{x+4} & \frac{x}{x+4} \end{bmatrix} \qquad \textbf{(2.1)}$$

Four-dimention hypergeometric solutions space is found

```
> st := time( ) :
  Res := LRS:-HypergeometricSolution(A2, x);
  time( ) - st;
```

$$Res := \left[\left[\begin{array}{c} \frac{(-1)^x (2x+5)}{(x+3)(x+2)} \\ -\frac{(-1)^x (2x+5)}{(x+3)(x+2)} \\ -\frac{(6x^2+23x+19)(-1)^x}{(x+1)(x+2)(x+3)} \\ \frac{(-1)^x (2x+5)}{(x+3)(x+2)} \end{array}\right], \left[\begin{array}{c} \frac{1}{(x+3)(x+2)} \\ -\frac{1}{(x+3)(x+2)} \\ -\frac{x-1}{(x+1)(x+2)(x+3)} \\ \frac{1}{(x+3)(x+2)} \end{array}\right], \left[\begin{array}{c} -\frac{x}{\Gamma(x+2)} \\ -\frac{1}{\Gamma(x+2)} \\ \frac{x}{\Gamma(x+2)} \\ 0 \end{array}\right], \left[\begin{array}{c} -\frac{(x+2)(-1)^x}{\Gamma(x+2)} \\ \frac{(-1)^x}{\Gamma(x+2)} \\ \frac{(x+2)(-1)^x}{\Gamma(x+2)} \\ 0 \end{array}\right]\right]$$

0.490 **(2.2)**

Check result (must be zero-vectors.):

> $seq(map(simplify, eval(Res[i], x=x+1) - A2.Res[i]), i=1..nops(Res));$

$$\left[\begin{array}{c}0\\0\\0\\0\end{array}\right], \left[\begin{array}{c}0\\0\\0\\0\end{array}\right], \left[\begin{array}{c}0\\0\\0\\0\end{array}\right], \left[\begin{array}{c}0\\0\\0\\0\end{array}\right] \quad \textbf{(2.3)}$$

>

>

Example 3

> A3 := Matrix(4, 4, {(1, 2) = (x^6 + 11 * x^5 + 41 * x^4 + 65 * x^3 + 50 * x^2 - 36) / ((x + 3) * (x + 6) * (x + 1) * (x + 5)), (3, 4) = - (3 * (x + 3)) / (x + 5), (4, 4) = 1, (2, 4) = 0, (4, 1) = 0, (1, 4) = - (6 * (x + 2)) / (x + 5), (2, 2) = x^2 / ((x + 6) * (x + 2)), (1, 1) = (x - 1) * (x + 2) / (x * (x + 5)), (4, 3) = 0, (3, 1) = - (x - 1) / (x * (x + 5)), (2, 1) = 0, (3, 2) = (x^5 + 7 * x^4 + 11 * x^3 + 4 * x^2 - 5 * x + 6) / ((x + 6) * (x + 1) * (x + 5)), (2, 3) = 0, (3, 3) = 1, (1, 3) = 0, (4, 2) = -x^2 / (x + 6)});

(3.1)

$$A3 := \begin{bmatrix} \frac{(x-1)(x+2)}{x(x+5)} & \frac{x^6+11x^5+41x^4+65x^3+50x^2-36}{(x+3)(x+6)(x+1)(x+5)} & 0 & -\frac{6(x+2)}{x+5} \\ 0 & \frac{x^2}{(x+6)(x+2)} & 0 & 0 \\ -\frac{x-1}{x(x+5)} & \frac{x^5+7x^4+11x^3+4x^2-5x+6}{(x+6)(x+1)(x+5)} & 1 & -\frac{3(x+3)}{x+5} \\ 0 & -\frac{x^2}{x+6} & 0 & 1 \end{bmatrix} \quad \textbf{(3.1)}$$

Only rational solutions are found:

```
> st := time( ) :
  Res := LRS:-HypergeometricSolution(A3, x);
  time( ) - st;
  nops(Res) = 2;
```

$$Res := \left[\begin{bmatrix} -\frac{370080}{(x-1)(x+2)(x+4)(x+3)} \\ 0 \\ \frac{x^5+10x^4+35x^3+50x^2-92496x-74016}{x(x+1)(x+2)(x+4)(x+3)} \\ 0 \end{bmatrix}, \begin{bmatrix} \frac{4}{(x-1)(x+2)(x+4)(x+3)} \\ 0 \\ \frac{1}{5}\,\frac{5x+4}{x(x+1)(x+2)(x+4)(x+3)} \\ 0 \end{bmatrix} \right]$$

0.276

$$2 = 2 \quad \textbf{(3.2)}$$

Check result (must be zero-vectors.):

```
> seq(map(simplify, eval(Res[i], x = x + 1) - A3.Res[i]), i = 1 ..nops(Res));
>
```

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \textbf{(3.3)}$$

Example (16 ×16)-matrix

```
>
```

input A16

Two-dimentional solutions space is found:

```
> st := time( ) :
  Res := LRS:-HypergeometricSolution(A16, x);
  time( ) - st;
  nops(Res) = 2;
```

$$Res := \left[\left[\begin{array}{c} \textit{1 .. 16 Vector}_{column} \\ \textit{Data Type: anything} \\ \textit{Storage: rectangular} \\ \textit{Order: Fortran\_order} \end{array} \right], \left[\begin{array}{c} \textit{1 .. 16 Vector}_{column} \\ \textit{Data Type: anything} \\ \textit{Storage: rectangular} \\ \textit{Order: Fortran\_order} \end{array} \right] \right]$$

308.946

$$2 = 2 \tag{4.1}$$

\>

\>

All solutions are rational:

```
> convert(Res[1], list);
  convert(Res[2], list);
```

$$\left[\frac{1}{9} \frac{x (15 x+7)}{(15 x+4) (9 x^2+9 x+4)}, \frac{1}{3} \frac{3 x^2+5 x+2}{(15 x+4) (9 x^2+9 x+4)}, \right.$$

$$-\frac{2}{9} \frac{3 x+2}{(15 x+4) x (9 x^2+9 x+4)}, -\frac{1}{3} \frac{3 x+2}{(15 x+4) x (9 x^2+9 x+4)},$$

$$-\frac{1}{18} \frac{x (270 x^4+1377 x^3+1728 x^2+413 x-48)}{(15 x+4) (9 x^2+9 x+4) (19+15 x)},$$

$$-\frac{1}{18} \frac{675 x^4+1800 x^3+1527 x^2+370 x-32}{(15 x+4) (9 x^2+9 x+4) (19+15 x)},$$

$$\frac{1}{9} \frac{135 x^3+486 x^2+585 x+214}{(15 x+4) (9 x^2+9 x+4) (19+15 x)},$$

$$\frac{1}{18} \frac{675 x^3+2070 x^2+1869 x+562}{(19+15 x) (9 x^2+9 x+4) (x+1) (15 x+4)},$$

$$\frac{1}{54} \frac{x (2025 x^5+5265 x^4+3465 x^3-1197 x^2-1718 x-360)}{(15 x+4) (9 x^2+9 x+4) (19+15 x)},$$

$$\frac{1}{9} \frac{135 x^5+261 x^4-21 x^3-321 x^2-214 x-40}{(15 x+4) (9 x^2+9 x+4) (19+15 x)},$$

$$-\frac{1}{54} \frac{2025 x^4+6480 x^3+7569 x^2+3762 x+664}{(15 x+4) (9 x^2+9 x+4) (19+15 x)},$$

$$-\frac{1}{6} \frac{90 x^3+309 x^2+187 x+44}{(15 x+4) (9 x^2+9 x+4) (19+15 x)},$$

$$-\frac{1}{9} \frac{x (x-1) (9 x^4+39 x^3+61 x^2+41 x+10)}{(9 x^2+9 x+4) (19+15 x)},$$

$$\frac{2}{27}\,\frac{9x^4+39x^3+61x^2+41x+10}{(9x^2+9x+4)(19+15x)},\ \frac{1}{9}\,\frac{9x^4+39x^3+61x^2+41x+10}{(9x^2+9x+4)(19+15x)},$$
$$\left.\frac{1}{27}\,\frac{9x^2+12x-5}{(9x^2+9x+4)(19+15x)}\right]$$
$$\left[\frac{1}{270}, 0, 0, 0, 0, \frac{1}{270}, 0, 0, 0, 0, \frac{1}{270}, 0, 0, 0, 0, \frac{1}{270}\right] \quad \textbf{(4.2)}$$

Check result (must be zero-vectors.):

> *seq(convert(map(simplify, eval(Res[i], x = x + 1) − A16.Res[i]), list), i = 1 ..nops(Res));*

>

$$[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0] \quad \textbf{(4.3)}$$

>

Example (16 ×16)-matrix with hypergeometric solutions

> *A16_h := (19 + 15 x) (3 x + 2)·A16;*

$$A16\_h := \begin{bmatrix} \textit{16 x 16 Matrix} \\ \textit{Data Type: anything} \\ \textit{Storage: rectangular} \\ \textit{Order: Fortran\_order} \end{bmatrix} \quad \textbf{(5.1)}$$

Two-dimentional solutions space is found:

> *st := time( ) :*
Res := LRS:-HypergeometricSolution(A16_h, x);
time( ) − st;
nops(Res) = 2;

$$Res := \left[\begin{bmatrix} \textit{1 .. 16 Vector}_{column} \\ \textit{Data Type: anything} \\ \textit{Storage: rectangular} \\ \textit{Order: Fortran\_order} \end{bmatrix}, \begin{bmatrix} \textit{1 .. 16 Vector}_{column} \\ \textit{Data Type: anything} \\ \textit{Storage: rectangular} \\ \textit{Order: Fortran\_order} \end{bmatrix}\right]$$

280.984

$$2 = 2 \quad \textbf{(5.2)}$$

>

All solutions are hypergeometric:

> *convert(Res[1], list);*
convert(Res[2], list);

$$\left[\frac{1}{270}\,\frac{\Gamma\left(x+\frac{2}{3}\right)\Gamma\left(\frac{19}{15}+x\right)45^x\,(135x^3+261x^2+138x+16)}{(15x+4)(9x^2+9x+4)},\right.$$
$$\frac{1}{15}\,\frac{(3x+2)\,\Gamma\left(x+\frac{2}{3}\right)\Gamma\left(\frac{19}{15}+x\right)45^x\,(x+1)}{(15x+4)(9x^2+9x+4)},$$

$$-\frac{2}{45}\,\frac{45^{x}\,\Gamma\left(\frac{19}{15}+x\right)\Gamma\left(x+\frac{2}{3}\right)(3x+2)}{x\,(15x+4)\,(9x^{2}+9x+4)},$$

$$-\frac{1}{15}\,\frac{45^{x}\,\Gamma\left(\frac{19}{15}+x\right)\Gamma\left(x+\frac{2}{3}\right)(3x+2)}{x\,(15x+4)\,(9x^{2}+9x+4)},$$

$$-\frac{1}{90}\,\frac{x\,(270x^{4}+1377x^{3}+1728x^{2}+413x-48)\,45^{x}\,\Gamma\left(\frac{19}{15}+x\right)\Gamma\left(x+\frac{2}{3}\right)}{(15x+4)\,(19+15x)\,(9x^{2}+9x+4)},$$

$$-\frac{1}{135}\,\frac{(135x^{3}-54x^{2}-477x-200)\,45^{x}\,\Gamma\left(\frac{19}{15}+x\right)\Gamma\left(x+\frac{2}{3}\right)}{(15x+4)\,(9x^{2}+9x+4)\,(19+15x)},$$

$$\frac{1}{45}\,\frac{(45x^{2}+132x+107)\,45^{x}\,\Gamma\left(\frac{19}{15}+x\right)\Gamma\left(x+\frac{2}{3}\right)(3x+2)}{(15x+4)\,(19+15x)\,(9x^{2}+9x+4)},$$

$$\frac{1}{90}\,\frac{(675x^{3}+2070x^{2}+1869x+562)\,45^{x}\,\Gamma\left(\frac{19}{15}+x\right)\Gamma\left(x+\frac{2}{3}\right)}{(15x+4)\,(x+1)\,(19+15x)\,(9x^{2}+9x+4)},$$

$$\frac{1}{270}\,\frac{1}{(15x+4)\,(19+15x)\,(9x^{2}+9x+4)}\Big(x\,(675x^{4}+1305x^{3}+285x^{2}$$

$$-589x-180)\,45^{x}\,\Gamma\left(\frac{19}{15}+x\right)\Gamma\left(x+\frac{2}{3}\right)(3x+2)\Big),$$

$$\frac{1}{45}\,\frac{(45x^{4}+57x^{3}-45x^{2}-77x-20)\,45^{x}\,\Gamma\left(\frac{19}{15}+x\right)\Gamma\left(x+\frac{2}{3}\right)(3x+2)}{(15x+4)\,(19+15x)\,(9x^{2}+9x+4)},$$

$$-\frac{1}{45}\,\frac{(225x^{3}+480x^{2}+283x+60)\,45^{x}\,\Gamma\left(\frac{19}{15}+x\right)\Gamma\left(x+\frac{2}{3}\right)}{(15x+4)\,(9x^{2}+9x+4)\,(19+15x)},$$

$$-\frac{1}{30}\,\frac{(90x^{3}+309x^{2}+187x+44)\,45^{x}\,\Gamma\left(\frac{19}{15}+x\right)\Gamma\left(x+\frac{2}{3}\right)}{(15x+4)\,(19+15x)\,(9x^{2}+9x+4)},$$

$$-\frac{1}{45}\,\frac{x\,(x^{3}+x^{2}-x-1)\,45^{x}\,\Gamma\left(\frac{19}{15}+x\right)\Gamma\left(x+\frac{2}{3}\right)(3x+5)\,(3x+2)}{(9x^{2}+9x+4)\,(19+15x)},$$

$$\frac{2}{135}\,\frac{(x^{2}+2x+1)\,45^{x}\,\Gamma\left(\frac{19}{15}+x\right)\Gamma\left(x+\frac{2}{3}\right)(3x+5)\,(3x+2)}{(19+15x)\,(9x^{2}+9x+4)},$$

$$\frac{1}{45}\,\frac{(x^{2}+2x+1)\,45^{x}\,\Gamma\left(\frac{19}{15}+x\right)\Gamma\left(x+\frac{2}{3}\right)(3x+5)\,(3x+2)}{(19+15x)\,(9x^{2}+9x+4)},$$

$$\frac{1}{90}\,\frac{\left(45\,x^3+108\,x^2+85\,x+22\right)\,45^x\,\Gamma\!\left(\frac{19}{15}+x\right)\,\Gamma\!\left(x+\frac{2}{3}\right)}{\left(9\,x^2+9\,x+4\right)\,\left(19+15\,x\right)}\Bigg]$$

$$\left[\frac{1}{4050}\,\Gamma\!\left(x+\frac{2}{3}\right)\Gamma\!\left(\frac{19}{15}+x\right)45^x, 0, 0, 0, 0, \frac{1}{4050}\,\Gamma\!\left(x+\frac{2}{3}\right)\Gamma\!\left(\frac{19}{15}+x\right)45^x, 0, 0, 0, 0, \frac{1}{4050}\,\Gamma\!\left(x+\frac{2}{3}\right)\Gamma\!\left(\frac{19}{15}+x\right)45^x, 0, 0, 0, 0, \frac{1}{4050}\,\Gamma\!\left(x+\frac{2}{3}\right)\Gamma\!\left(\frac{19}{15}+x\right)45^x\right] \tag{5.3}$$

Check result (must be zero-vectors.):

> $seq(convert(map(simplify, eval(Res[i], x=x+1) - A16\_h.Res[i]), list), i=1..nops(Res));$

>

$$[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0] \tag{5.4}$$

Details of the process and the CPU time of some steps:

> $infolevel[LRS] := 3 :$
$LRS\text{:-}HypergeometricSolution(A16\_h, x);$

```
Resolving: resolving for  [0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0] : 
0.3e-2
Resolving: 15 order resolving equation is constructed: 9.297
Resolving: Ã is constructed: 1.767
Resolving: resolving for  [1] : 0.
Resolving: 1 order resolving equation is constructed: 0.3e-2
HypergeometricSolution: 1 dimension hypergeometric solution 
space is found: 316.517
ByRationalSolution: 2 dimension rational solution space are 
found:  .687
HypergeometricSolution: 1 dimension hypergeometric solution 
space is found: 0.13e-1
HypergeometricSolution: all time: 328.340
```

$$\left[\left[\begin{array}{c} 1\,..\,16\ Vector_{column} \\ Data\ Type:\ anything \\ Storage:\ rectangular \\ Order:\ Fortran\_order \end{array}\right], \left[\begin{array}{c} 1\,..\,16\ Vector_{column} \\ Data\ Type:\ anything \\ Storage:\ rectangular \\ Order:\ Fortran\_order \end{array}\right]\right] \tag{5.5}$$

>

Zero and nonzero elements of the matrix:

> $tmp := map(el \rightarrow \text{`if`}(el \neq 0, \text{""}, 0), A16\_h) :$
$tmp[1..8, 1..8], tmp[1..8, 9..16];$
$tmp[9..16, 1..8], tmp[9..16, 9..16];$

$$
\begin{bmatrix}
0 & \text{""} & 0 & 0 & 0 & \text{""} & 0 & 0 \\
0 & 0 & \text{""} & 0 & 0 & 0 & \text{""} & 0 \\
0 & 0 & 0 & \text{""} & 0 & 0 & 0 & \text{""} \\
\text{""} & \text{""} & \text{""} & \text{""} & \text{""} & \text{""} & \text{""} & \text{""} \\
0 & \text{""} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \text{""} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \text{""} & 0 & 0 & 0 & 0 \\
\text{""} & \text{""} & \text{""} & \text{""} & 0 & 0 & 0 & 0
\end{bmatrix},
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \text{""} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \text{""} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \text{""} & 0 & 0 & 0 & 0 \\
\text{""} & \text{""} & \text{""} & \text{""} & 0 & 0 & 0 & 0
\end{bmatrix}
$$

$$
\begin{bmatrix}
0 & \text{""} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \text{""} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \text{""} & 0 & 0 & 0 & 0 \\
\text{""} & \text{""} & \text{""} & \text{""} & 0 & 0 & 0 & 0 \\
0 & \text{""} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \text{""} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \text{""} & 0 & 0 & 0 & 0 \\
\text{""} & \text{""} & \text{""} & \text{""} & 0 & 0 & 0 & 0
\end{bmatrix},
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & \text{""} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \text{""} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \text{""} \\
0 & 0 & 0 & 0 & \text{""} & \text{""} & \text{""} & \text{""} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\quad \textbf{(5.6)}
$$

>

>