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Calculations of Quantum Corrections in Supersymmetric Theories Using Computer Algebra Methods

Calculations in quantum field theory

Calculations within the framework of quantum field theory (QFT) are of great importance in modern high-energy physics. Well-known experiments at the Large Hadron Collider (LHC) and other large accelerator projects are described exclusively using its methods. The most common approach to such calculations is the Feynman diagram technique. In essence, this is a graphical interpretation of expressions obtained using perturbation theory. Each diagram encodes a certain expression that contributes to some physical quantity.
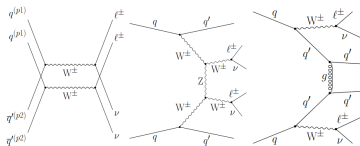
The standard scheme of calculations is that first draw all the diagrams corresponding to the desired process in a given theory. Then the expressions that they encode are evaluated. In the end, integrals (usually divergent) over the momentum space are obtained, which are taken using various regularization methods. Depending on the theory, some of these steps may be easier or more difficult. Often, such calculations are extremely difficult to carry out manually, so special programs are used.

Software for QFT calculations

Attempts to create software for calculations within the framework of quantum perturbation theory have been made for more than fifty years

> J. A. Campbell and A. C. Hearn, J. Comput. Phys. **5** (1970), 280-327
> Гердт В. П., Тарасов О. В. и Ширков Д. В., УФН. **130** (1980), 113–147.

Software:

- Programs for graph generation (QGRAPH, FeynArts):

  > P. Nogueira, J. Comput. Phys. **105** (1993), 279-289
  > T. Hahn, Comput. Phys. Commun. **140** (2001), 418-431

- Programs designed to calculate various processes, usually within the framework of the Standard Model(madevent, Herwig++, SHERPA, aMC@NLO):

  > F. Maltoni and T. Stelzer, JHEP **02** (2003), 027
  > M. Bahr, *et al.* Eur. Phys. J. C **58** (2008), 639-707
  > T. Gleisberg, *et al.*, JHEP **02** (2009), 007
  > J. Alwall, *et al.*, JHEP **07** (2014), 079

- Computer algebra systems (Maple, Mathematica, Schoonschip, FORM):

  > M. J. G. Veltman and D. N. Williams, arXiv:hep-ph/9306228 [hep-ph]].
  > B. Ruijl, T. Ueda and J. Vermaseren, [arXiv:1707.06453 [hep-ph]].

Software for QFT calculations

There are some programs that calculate Feynman momentum integrals in the D-dimension, which are formed after construction of the amplitudes. Mostly, the following approach to their calculation is implemented. Firstly, tensor integrals are reduced to scalar integrals by standard methods, and then using such techniques as integration by parts in D-dimension and the application of Lorentz invariance scalar integrals are reduced to a small number of typical master integrals.

K. G. Chetyrkin and F. V. Tkachov, Nucl. Phys. B **192** (1981), 159-204

- Software for master integrals reduction (FIRE, LiteRed):

  R. N. Lee, J. Phys. Conf. Ser. **523** (2014), 012059
  A. V. Smirnov and F. S. Chuharev, Comput. Phys. Commun. **247** (2020), 106877

- Programs for calculating master integrals (FIESTA, SecDec):

  A. V. Smirnov, Comput. Phys. Commun. **204** (2016), 189-199
  S. Borowka, *et al.*, Comput. Phys. Commun. **196** (2015), 470-491

Software for QFT calculations

We should also mention the package for Schoonschip Mincer

S. G. Gorishnii, S. A. Larin, L. R. Surguladze and F. V. Tkachov, Comput. Phys. Commun. **55** (1989), 381-408

which was firstly used to calculate the four-loop β-function using dimensional regularization

S. G. Gorishnii, A. L. Kataev, S. A. Larin and L. R. Surguladze, Phys. Lett. B **256** (1991), 81-86

- Software packages that combine the generation of diagrams and amplitudes, operations with them and taking integrals (HepLib, `tapir`):

F. Feng, Y. F. Xie, Q. C. Zhou and S. R. Tang, Comput. Phys. Commun. **265** (2021), 107982
M. Gerlach, F. Herren and M. Lang, Comput. Phys. Commun. **282** (2023), 108544

- Programs for working with superfields in the superspace (SUSYCAL, SusyMath):

T. Kreuzberger, W. Kummer and M. Schweda, Comput. Phys. Commun. **58** (1990), 89-104
A. F. Ferrari, Comput. Phys. Commun. **176** (2007), 334-346

One of the most interesting theories in high energy physics are supersymmetric theories. They introduce special transformations that transform particles with integer spin (bosons) into particles with half-integer spin (fermions). Such theories have various interesting properties and give many physical predictions.

It is convenient to introduce superspace. $\mathcal{N} = 1$ superspace is a space with the coordinates (ct, x, y, z, θ), where θ is a Majorana spinor. Spinors in four dimensions are transformed according to a special spinor law with respect to the Lorentz group. Due to using of the superspace supersymmetry is explicit, even at the quantum level. The spinor indices are raised and lowered using charge conjugation matrices:

$$\theta^a \equiv \theta_b C^{ba}; \qquad \theta_a = \theta^b C_{ab}. \tag{1}$$

The supersymmetric covariant derivative is usually introduced as follows:

$$\bar{D}_{\dot{a}} = \frac{\partial}{\partial \bar{\theta}^{\dot{a}}} - i(\gamma^{\mu})_{\dot{a}}{}^b \theta_b \, \partial_{\mu}. \tag{2}$$

$\mathcal{N} = 1$ superfields

The usual fields in this approach are components of superfields. So, the gauge field is a component of the real superfield $V(x^\mu, \theta)$, spinor, and scalar superfields are components of chiral or antichiral fields ($\phi(x^\mu, \theta)$ and $\phi^*(x^\mu, \theta)$, respectively), which, by definition, satisfy the conditions:

$$\bar{D}_{\dot{a}}\Phi = 0, \quad D_a\Phi^* = 0 \tag{3}$$

For example superfield $\Phi(x^\mu, \theta)$ can be introduces as follows:

$$\Phi(x^\mu, \theta) = \phi(y^\mu) + \bar{\theta}(1 + \gamma_5)\psi(y^\mu) + \frac{1}{2}\bar{\theta}(1 + \gamma_5)\theta f(y^\mu) \tag{4}$$

where $y^\mu = x^\mu + \frac{i}{2}\bar{\theta}\gamma^\mu\gamma_5\theta$ In addition, when we construct supersymmetric actions, integration is introduced with respect to θ variables. In our notation , it can be defined as follows:

$$\int d^2\bar{\theta} = \frac{1}{2}\bar{D}^2 = \frac{1}{2}\bar{D}^{\dot{a}}\bar{D}_{\dot{a}}, \quad \int d^2\theta = -\frac{1}{2}D^2 = -\frac{1}{2}D^a D_a$$

$$\int d^4\theta = \int d^2\bar{\theta}d^2\theta \tag{5}$$

Perturbation theory in $\mathcal{N} = 1$ superspace formalism

There are 2 invariants under supersymmetry transformations, which can be written as follows:

$$S_1 = \int d^4\theta d^4x\, \mathbb{V}, \quad S_2 = \int d^2\theta d^4x\, \Phi + \text{к.с.} \tag{6}$$

where $\mathbb{V}$ is real superfield, and Φ is a chiral superfield. They are used to construct action of the theory.

It is most convenient to carry out quantization by the functional integral method. The main element of this approach, from which various quantities can be obtained, is the generating functional Z, which is constructed as follows:

$$Z = e^{iS_{int}(\frac{1}{i}\frac{\delta}{\delta j}, \frac{1}{i}\frac{\delta}{\delta J})} \int \mathcal{D}(\text{superfields})\, e^{i(S^{(2)} + S_{\text{sources}})} = e^{iS_{int}(\frac{1}{i}\frac{\delta}{\delta j}, \frac{1}{i}\frac{\delta}{\delta J})} Z_0 \tag{7}$$

where $S^{(2)}$ is a contribution to the action quadratic in fields, and S_{int} is a sum of contributions of degree higher than 2, $S_{\text{sources}} = j\phi + JV$ and $\mathcal{D}(\text{superfields})$ is a measure of the functional integration. In the end all sources must be set to zero. The Gaussian integral Z_0 is taken using standard methods. The interaction term series expansion is interpreted graphically using Feynman diagrams. However we will consider the effective action. It is obtained by Legendre transformation:

$$\Gamma = -i \ln Z[\text{Sources}] - S_{\text{sources}}|_{\text{sources} \longrightarrow \text{superfields}},$$

In fact, Γ removes all disconnected diagrams and one-particle reducible diagrams from Z. These are diagrams that can be divided by cutting a single internal line.

Computer program for supergarph calculations

Explicit calculations in the framework of $N = 1$ superspace are rather complicated, so special C++ program was created to deal with them.

At the present moment the programm can deal with $N = 1$ SQED with N_f flavours and can calculate two-point Green function of matter fields. The program makes such steps of calculation:

1. Generation of all supergraphs in desired order of perturbation theory using given vertexes and propagators.

2. Evaluation of so-called D-algebra, using standart procedure of removing supersymmetric covariant derivatives and taking integral over superspace.

3. Removing objects with spinor indices by taking γ-matrix traces etc.

4. Reducing of remaining impulse intregrals, by collecting terms using some integral transformations.

At present moment impulse integrals must be taken by hand.

Main objects

1 Superspacial delta function:

Expression	Code
$\delta_{13}^8(k)$	`d_{2}{13}`
$\delta_{93}^8(p+k+l)$	`d_{30}{93}`
$\delta_{23}^8(-k)$	`d_{-2}{23}`

2 Superfields:

Expression	Code
$\tilde{\phi}_1^*(k)$	`F1#_1^1{2}`
$\phi_9(p)$	`F_9^1{5}`
$V_3(-(k+p))$	`V_3^1{-10}`

3 Operators:

Expression	Code
$\bar{D}_1^{\dot{a}} D_1^2 \delta_{13}^8(k)$	`D#_1{^1}(D_1(d_{2}{13}))`
$D_1^2 \bar{D}_2^2 \delta_{12}^8(k+l+p)$	`D_1(D#_2(d_{30}{12}))`

4 Momenta, constants:

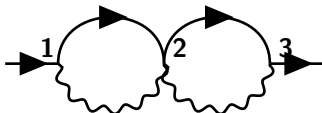| Expression	Code
k^2	`I{2^2}`
l_μ	`I{3_1}`
$1/R_k$	`K5{2}`

Example

$$Z_0(J,j) = \exp\{\frac{i}{2} \int d^8x J \left[-\frac{2}{R\partial^4} \right] J + i \int d^8x \left(j \frac{1}{\partial^2} j^* + \tilde{j} \frac{1}{\partial^2} \tilde{j}^* \right) \}.$$

$$S_{int} = \frac{1}{4} \int d^4x \, d^4\theta \, \phi^* e^{2Ve_0} \phi = \frac{e_0}{2} \int d^8x \, \phi^* V \phi + \frac{e_0^2}{2} \int d^8x \, \phi^* V^2 \phi + ...$$

$$\Gamma \sim -i\frac{e_0}{2} \int d^8x_1 \, \phi_1^* \frac{1}{i} \frac{\delta}{\delta J_1} \frac{1}{i} \frac{\delta}{\delta j_1} \frac{e_0^2}{2} \int d^8x_2 \, \frac{1}{i} \frac{\delta}{\delta j_2^*} \frac{1}{i} \frac{\delta}{\delta J_2} \frac{1}{i} \frac{\delta}{\delta J_2} \frac{1}{i} \frac{\delta}{\delta j_2}$$

$$\times \frac{e_0}{2} \int d^8x_3 \, \frac{1}{i} \frac{\delta}{\delta j_3^*} \frac{1}{i} \frac{\delta}{\delta J_3} \phi_3 Z_0 \bigg|_{J,j=0}$$

$$\Gamma \sim \frac{-ie_0^4}{8} \int d^8x_1 \, d^8x_2 \, d^8x_3 \, \phi_1^* \phi_3 \overbrace{\frac{\delta}{\delta J_1} \frac{\delta}{\delta j_1} \frac{\delta}{\delta j_2^*} \frac{\delta}{\delta J_2}}^{} \overbrace{\frac{\delta}{\delta J_2} \frac{\delta}{\delta j_2} \frac{\delta}{\delta j_3^*} \frac{\delta}{\delta J_3}}^{} Z_0 \bigg|_{J,j=0}$$

Stage 1: Exponent expansion

```
-1/8*i*e^4*F#_1^1{-1}*J_1^1{-7}*j_1_1{-11}*j#_2^1{7}*J_2^1{-7}
*F_2_1{1}*j#_3^1{1001}*J_3^1{-7}*J_3^1{-11}*j_3_1{-13}
```

$$\Gamma \sim -i\frac{e_0}{2} \int d^8x_1 \, \phi_1^* \frac{1}{i}\frac{\delta}{\delta J_1} \frac{1}{i}\frac{\delta}{\delta j_1} \frac{e_0^2}{2} \int d^8x_2 \, \frac{1}{i}\frac{\delta}{\delta j_2^*} \frac{1}{i}\frac{\delta}{\delta J_2} \frac{1}{i}\frac{\delta}{\delta J_2} \frac{1}{i}\frac{\delta}{\delta j_2}$$

$$\times \frac{e_0}{2} \int d^8x_3 \, \frac{1}{i}\frac{\delta}{\delta j_3^*} \frac{1}{i}\frac{\delta}{\delta J_3} \phi_3 Z_0 \bigg|_{J,j=0}$$

Stage 2: Pairing of derivatives

```
-1/4*i*e^4*F#_1^1{-1}*F_2_1{1}*J_1^1{-7}*J_3^1{-7}*j_1_1{-11}
*j#_3^1{1001}*j#_2^1{7}*j_3_1{-13}*J_2^1{-7}*J_3^1{-11}
```

$$\Gamma \sim \frac{-ie_0^4}{8} \int d^8x_1 \, d^8x_2 \, d^8x_3 \, \phi_1^* \phi_3 \frac{\delta}{\delta J_1} \frac{\delta}{\delta j_1} \frac{\delta}{\delta j_2^*} \frac{\delta}{\delta J_2} \frac{\delta}{\delta J_2} \frac{\delta}{\delta j_2} \frac{\delta}{\delta j_3^*} \frac{\delta}{\delta J_3} Z_0 \bigg|_{J,j=0}$$

Coefficient changes because of combinatorial factor.

Stage 3: One-particle irreducibility check
At this point it is necessary to check if the diagram is one-particle reducible or not and delete reducible ones.
Stage 4: Propagator insertion Each pair produces so-called propagators that also consists of superspacial delta function, that acts just like simple delta-function but in superspace:

$$Z_0(J,j) = \exp\{\frac{i}{2}\int d^8x J\left[-\frac{2}{R\partial^4}\right] J + i\int d^8x \left(j\frac{1}{\partial^2}j^* + \tilde{j}\frac{1}{\partial^2}\tilde{j^*}\right)\}.$$

$$\frac{\delta}{\delta J_1}\frac{\delta}{\delta J_2}Z_0(J,j) = -\frac{2}{R\partial^2}\delta_{12}^8 \rightarrow \texttt{-2*e\^{}0*I\{1\^{}-2\}*K4\{1\}*d\_\{1\}\{12\}}$$

$$\frac{\delta}{\delta j_1}\frac{\delta}{\delta j_2^*}Z_0(J,j) = \frac{\bar{D}_1^2 D_2^2}{4\partial^2}\delta_{12}^8 \rightarrow \texttt{1/4*e\^{}0*I\{1\^{}-2\}*D\#\_1(D\_2(d\_\{1\}\{12\}))}$$

```
1/16*e^4*F#_1^1{-1}*F_2_2{1}*d_{-3}{13}*d_{2}{23}*D_3(D#_1(d_{3}{13}))
*D_2(D#_3(d_{-2}{23}))*K4{3}*I{3^-2}*K4{2}*I{2^-2}*I{3^-2}*I{2^-2}
```

Momentum representation

Here we need to move to momentum representation $\partial^2 \to K^2$.

$$\delta_{12}^4 = \int \frac{d^4 k}{(2\pi)^4} \exp(ik(x_1 - x_2))$$

$$\partial^2 \delta_{12}^4 = \int \frac{d^4 k}{(2\pi)^4} \partial^2 \exp(ik(x_1 - x_2)) = -\int \frac{d^4 k}{(2\pi)^4} k^2 \exp(ik(x_1 - x_2))$$

K is euclidean. So we made Wick transformation $K_4 = iK_0$:

$$K^2 = K_1^2 + K_2^2 + K_3^2 + K_4^2 = K_1^2 + K_2^2 + K_3^2 - K_0^2$$

Each momentum is given by a prime number, and the sum of the momenta corresponds to their product, so each number uniquely sets the sum. For example: we have momenta K, L and Q. Let us assign K-number 2, L-number 3, Q-number 5. Then, for example, $K + L$ will be 6, and $K + L + Q$ will be 30. This makes comparison and other operations easier.

"D-algebra"

The results takes form:

```
1/16*e^4*F#_1^1{-1}*F_2_2{1}*d_{-3}{13}*d_{2}{23}*D_3
(D#_1(d_{3}{13}))*D_2(D#_3(d_{-2}{23}))*K4{3}*I{3^-2}*K4{2}
*I{2^-2}*I{3^-2}*I{2^-2}
```

Then according to "D-Algebra" rules we commute supersymmetric covriate derivatives:

$$\delta_{12}^8 D^2 \bar{D}^2 \delta_{12}^8 = 4\delta_{12}^8 \delta^4(x_1 - x_2)$$
$$\{D_a, \bar{D}_{\dot{b}}\} = 2i(\gamma^\mu)_{a\dot{b}}\partial_\mu, \quad \{D_a, D_b\} = \{\bar{D}_{\dot{a}}, \bar{D}_{\dot{b}}\} = 0$$
$$[D_a, \bar{D}^2] = 4i(\gamma^\mu)_a^{\dot{b}} \bar{D}_{\dot{b}}\partial_\mu, \quad [D_a, D^2] = 0,$$
$$[\bar{D}^2, D^2] = 4i(\gamma^\mu)^{\dot{b}a}[D_a, \bar{D}_{\dot{b}}]\partial_\mu,$$
$$\bar{D}^2 D^2 \bar{D}^2 = -16\partial^2 \bar{D}^2, \quad D^2 \bar{D}^2 D^2 = -16\partial^2 D^2.$$

Then we obtain

```
1/4*e^4*F#_1^1{-1}*F_1_2{1}*K4{3}*I{3^-2}*K4{2}*I{2^-2}*I{2^-2}
*I{2^-2}*I{6^-2}*[gamma_5_4{1}]*I{2_1}*[gamma^5^4{3}]*I{2_3}

-1/4*e^4*F#_1^1{-1}*F_1_2{1}*K4{3}*I{3^-2}*K4{2}*I{2^-2}*I{2^-2}
*I{2^-2}*I{6^-2}*[gamma_5^4{2}]*I{2_2}*[gamma^5_4{5}]*I{2_5}
```

Expression simplification

```
-1*e^4*F#_1^1{-1}*F_1_2{1}*K4{3}*I{3^-2}*K4{2}*I{2^-2}
*I{2^-2}*I{2^-2}*I{6^-2}*I{2_1}*I{2_1}
```

In this case, everything is elementary, because: I{2_1}*I{2_1} is essentially:

$$K^\mu K_\mu = K^2$$

So the expression becomes:

```
-1*e^4*F#_1^1{-1}*F_1_2{1}*K4{3}*I{3^-2}*K4{2}*I{2^-4}*I{6^-2}
```

In the analytic form:

$$-e_0^4 \int \frac{d^4K}{(2\pi)^4} \frac{d^4L}{(2\pi)^4} d^4\theta \, \phi_\alpha^*(0,\theta) \phi_\alpha(0,\theta) \frac{1}{R_K K^4 R_L L^2 (K+L)^2}.$$

In the general case, the following contribution may arise, in which it is necessary to pick out perfect squares:

$$(Q+K+L)^\mu(Q+K)_\mu = (Q+K)^2 + L^\mu(Q+K)_\mu$$
$$= (Q+K)^2 + \frac{1}{2}(Q+K+L)^2 - \frac{1}{2}L^2 - \frac{1}{2}(Q+K)^2 \qquad (8)$$

The program neglects some parts as integration. Such expression is final form that program produces. At the end it try to collect terms using some types of integral transformations. In the lowest orders it is usually enough but in higher ones sometimes collecting of terms is not full, and a lot of work to be done by hand. Making collecting of terms more efficient is an open problem.

$\mathcal{N} = 1$ SQED with N_f flavors, regularized by higher derivatives

In order to regularize the theory by higher derivatives, it is necessary to add the higher derivative term to the action:

$$S_{\text{reg}} = \frac{1}{4e_0^2}\text{Re} \int d^4x \, d^2\theta \, W^a R(\partial^2/\Lambda^2) W_a$$
$$+ \sum_{f=1}^{N_f} \frac{1}{4} \int d^4x \, d^4\theta \left(\phi_f^* e^{2V} \phi_f + \widetilde{\phi}_f^* e^{-2V} \widetilde{\phi}_f \right),$$

where $R(\partial^2/\Lambda^2)$ is a regulator, e.g. $R = 1 + \partial^{2n}/\Lambda^{2n}$.
Another similar regulator function appears in the gauge fixing term

$$S_{\text{gf}} = -\frac{1}{32\xi_0 e_0^2} \int d^4x \, d^4\theta \, D^2 V K(\partial^2/\Lambda^2) \bar{D}^2 V,$$

where ξ_0 is the bare gauge parameter. The minimal (Feynman) gauge corresponds to $\xi_0 = 1$ and $R(x) = K(x)$. However, we will make calculations for an arbitrary ξ_0 and $K(x) \neq R(x)$.

Propagators

$$P\left(\phi_x, \phi_y^*\right) = \frac{\bar{D}_x^2 D_y^2}{4\partial^2}\delta_{xy}^8 \rightarrow \text{1/4*e^0*I\{1^-2\}*D\#\_1(D\_2(d\_\{1\}\{12\}));}$$

$$P\left(\tilde{\phi}_x, \tilde{\phi}_y^*\right) = \frac{\bar{D}_x^2 D_y^2}{4\partial^2}\delta_{xy}^8 \rightarrow \text{1/4*e^0*I\{1^-2\}*D\#\_1(D\_2(d\_\{1\}\{12\}));}$$

$$P\left(\Phi_x, \tilde{\Phi}_y\right) = \frac{M\bar{D}^2}{\partial^2 + M^2}\delta_{xy}^8 \rightarrow \text{1*e^0*K3\{1\}*D\#\_1(d\_\{1\}\{12\});}$$

$$P\left(\Phi_x^*, \tilde{\Phi}_y^*\right) = \frac{MD^2}{\partial^2 + M^2}\delta_{xy}^8 \rightarrow \text{1*e^0*K3\{1\}*D\_1(d\_\{1\}\{12\});}$$

$$P\left(\Phi_x, \Phi_y^*\right) = \frac{\bar{D}_x^2 D_y^2}{4\left(\partial^2 + M^2\right)}\delta_{xy}^8 \rightarrow \text{1/4*e^0*K2\{1\}*D\#\_1(D\_2(d\_\{1\}\{12\})));}$$

$$P\left(\tilde{\Phi}_x, \tilde{\Phi}_y^*\right) = \frac{\bar{D}_x^2 D_y^2}{4\left(\partial^2 + M^2\right)}\delta_{xy}^8 \rightarrow \text{1/4*e^0*K2\{1\}*D\#\_1(D\_2(d\_\{1\}\{12\})))}$$

$$P_1\left(V_x, V_y\right) = -\frac{2\xi_0}{K\partial^2}\delta_{xy}^8 \rightarrow \text{-2*e^0*I\{1^-2\}*K4\{1\}*d\_\{1\}\{12\};}$$

$$P_2\left(V_x, V_y\right) = \left(\frac{\xi_0}{K} - \frac{1}{R}\right)\frac{D^a \bar{D}^2 D_a}{4\partial^4}\delta_{xy}^8 \rightarrow \text{1/4*e^0*[K4\{1\}-K5\{1\}]*I\{1^-4\}}$$

\text{*D\#_1\{^1\}(D_1(D\#_1\{_1\}(d_\{1\}\{12\})))).}

```
Type:
F#_1^1{-1}*F_1^1{1}
Option:
SQED
Loops:
1
Order:
2
Propagators:
V_1^1{1}*V_2^1{1}
-1/4*i*e^0*[K4{1}-K5{1}]*I{1^-4}*D#_1{^1}(D_1(D#_1{_1}(d_{1}{12})))
2*i*e^0*I{1^-2}*K4{1}*d_{1}{12}
F#_1^1{1}*F_2^1{1}
-1/4*i*e^0*I{1^-2}*D_1(D#_2(d_{1}{12}))
Vertexes:
1/2*i*e*F#_1^1{6}*V_1^1{-3}*F_1_1{-2}
1/2*i*e^2*F#_1^1{30}*V_1^1{-3}*V_1^1{-5}*F_1_1{-2}
```
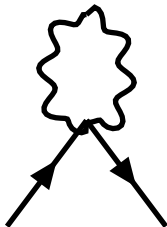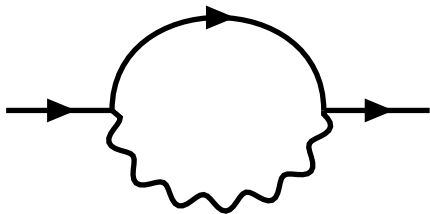
One loop example, result

```
File read correctly
1-loop diagrams are being generated
Completed100%
Diagrams created
Aligning momenta ...
Diagram generation completed
Diagram generation took 0.027 sec
Diagrams are being calculated...
0
-1*e^2*F#_1^1{-1}*F_1_2{1}*d_{2}{11}*K4{2}*I{2^-2}
1
1/8*e^2*F#_1^1{-1}*F_1_2{1}*D#_1{^2}(D_1(D#_1{_2}
(d_{2}{11})))*I{2^-4}*[K4{2}-K5^-1{2}]
2
-1/8*e^2*F#_1^1{-1}*F_2_2{1}*d_{-2}{12}
*D_2(D#_1(d_{2}{12}))*K4{2}*I{2^-2}*I{2^-2}
3
1/64*e^2*F#_1^1{-1}*F_2_2{1}*D#_1{^2}
(D_1(D#_1{_2}(d_{-2}{12})))*D_2(D#_1(d_{2}{12}))
*I{2^-4}*[K4{2}-K5^-1{2}]*I{2^-2}
Diagram calculation completed
Diagram calculation took 0.038  sec
Collecting terms...
Collecting terms completed
Result:
-1/2*e^2*F#_1^1{-1}*F_1_2{1}*I{2^-4}*K5^-1{2}
```

I.E. Shirokov   Automation of quantum corrections calculation

```
Result:
-1/2*e^2*F#_1^1{-1}*F_1_2{1}*I{2^-4}*K5^-1{2}
```

$$\Delta\Gamma = -\frac{1}{2}e_0^2 \int \frac{d^4K}{(2\pi)^4} d^4\theta\, \phi_\alpha^*(0,\theta)\phi_\alpha(0,\theta)\frac{1}{R_K K^4}.$$

$$\Gamma_\phi^{(2)} = \frac{1}{4} \int d^4\theta\, \phi_\alpha^*(0,\theta)\phi_\alpha(0,\theta)G(\alpha_0)\bigg|_{p=0}$$

Contribution to Green function

$$\Delta G^{(1)}(\alpha_0)\bigg|_{p=0} = -2e_0^2 \int \frac{d^4K}{(2\pi)^4} \frac{1}{K^4 R_K}.$$

```
Type:
F#_1^1{-1}*F_1^1{1}
Option:
SQED
Loops:
2
Order:
4
Propagators:
V_1^1{1}*V_2^1{1}
-1/4*i*e^0*[K4{1}-K5{1}]*I{1^-4}*D#_1^{^1}(D_1(D#_1(_1(d_{1}{12}))))
2*i*e^0*I{1^-2}*K4{1}*d_{1}{12}
F#_1^1{1}*F_2^1{1}
-1/4*i*e^0*I{1^-2}*D_1(D#_2(d_{1}{12}))
F1#_1^1{1}*F1_2^1{1}
-1/4*i*e^0*I{1^-2}*D_1(D#_2(d_{1}{12}))
F2#_1^1{1}*F2_2^1{1}
-1/4*i*e^0*K2{1}*D_1(D#_2(d_{1}{12}))
F3#_1^1{1}*F3_2^1{1}
-1/4*i*e^0*K2{1}*D_1(D#_2(d_{1}{12}))
F2_1^1{1}*F3_2^1{1}
-1*i*e^0*K3{1}*D#_1(d_{1}{12})
F2#_1^1{1}*F3#_2^1{1}
-1*i*e^0*K3{1}*D_1(d_{1}{12})
```

Two loop example

```
Vertexes:
1/2*i*e*F#_1^1{6}*V_1^1{-3}*F_1_1{-2}
1/2*i*e^2*F#_1^1{30}*V_1^1{-3}*V_1^1{-5}*F_1_1{-2}
1/3*i*e^3*F#_1^1{210}*V_1^1{-3}*V_1^1{-5}*V_1^1{-7}*F_1_1{-2}
1/6*i*e^4*F#_1^1{1890}*V_1^1{-3}*V_1^1{-5}*V_1^1{-7}*V_1^1{-9}*F_1_1{-2}
-1/2*i*e*F1#_1^1{6}*V_1^1{-3}*F1_1_1{-2}
1/2*i*e^2*F1#_1^1{30}*V_1^1{-3}*V_1^1{-5}*F1_1_1{-2}
-1/3*i*e^3*F1#_1^1{210}*V_1^1{-3}*V_1^1{-5}*V_1^1{-7}*F1_1_1{-2}
1/6*i*e^4*F1#_1^1{1890}*V_1^1{-3}*V_1^1{-5}*V_1^1{-7}*V_1^1{-9}*F1_1_1{-2}
1/2*i*e*F2#_1^1{6}*V_1^1{-3}*F2_1_1{-2}
1/2*i*e^2*F2#_1^1{30}*V_1^1{-3}*V_1^1{-5}*F2_1_1{-2}
1/3*i*e^3*F2#_1^1{210}*V_1^1{-3}*V_1^1{-5}*V_1^1{-7}*F2_1_1{-2}
1/6*i*e^4*F2#_1^1{1890}*V_1^1{-3}*V_1^1{-5}*V_1^1{-7}*V_1^1{-9}*F2_1_1{-2}
-1/2*i*e*F3#_1^1{6}*V_1^1{-3}*F3_1_1{-2}
1/2*i*e^2*F3#_1^1{30}*V_1^1{-3}*V_1^1{-5}*F3_1_1{-2}
-1/3*i*e^3*F3#_1^1{210}*V_1^1{-3}*V_1^1{-5}*V_1^1{-7}*F3_1_1{-2}
1/6*i*e^4*F3#_1^1{1890}*V_1^1{-3}*V_1^1{-5}*V_1^1{-7}*V_1^1{-9}*F3_1_1{-2}
```

Two loop contribution
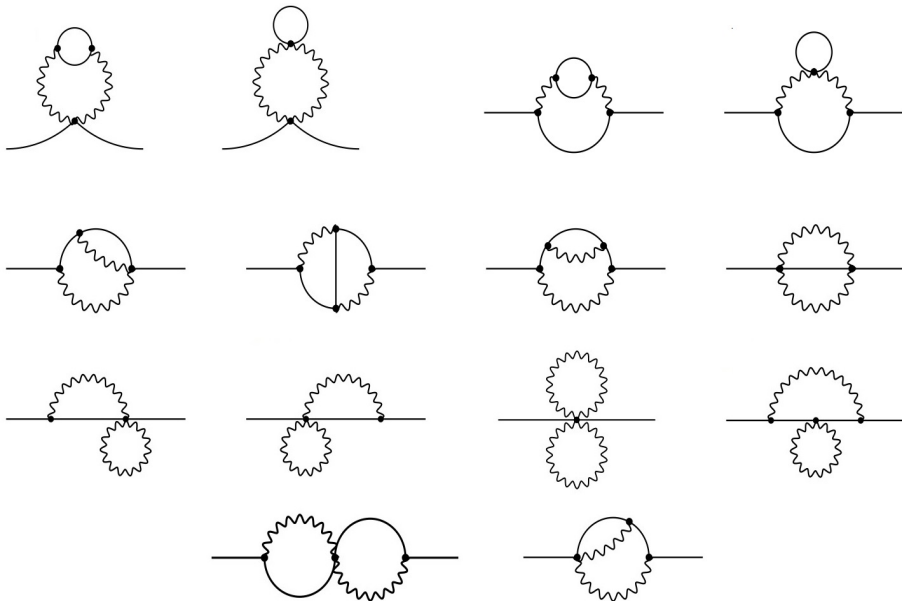
```
Result:
e^4*F#_1^1{-1}*F_1_2{1}*I{3^-2}*I{2^-4}*I{6^-2}*K5^-2{2}
-2*e^4*F#_1^1{-1}*F_1_2{1}*K3{3}*I{2^-6}*K3{6}*K5^-2{2}
-2*e^4*F#_1^1{-1}*F_1_2{1}*K2{6}*I{2^-6}*K2{3}*I{3^2}*K5^-2{2}
e^4*F#_1^1{-1}*F_1_2{1}*K2{3}*I{2^-4}*K2{6}*K5^-2{2}
2*e^4*F#_1^1{-1}*F_1_2{1}*K2{3}*I{2^-6}*K5^-2{2}
e^4*F#_1^1{-1}*F_1_2{1}*I{2^-4}*I{3^-2}*I{6^-2}*K5^-1{2}*K5^-1{3}
```

Contribution to Green function:

$$\Delta G^{(2)}\Big|_{p=0} = 4e_0^4 \int \frac{d^4K}{(2\pi)^4} \frac{d^4L}{(2\pi)^4} \frac{1}{R_K R_L L^2 K^4 (K+L)^2}$$

$$+ 4e_0^4 N_f \int \frac{d^4K}{(2\pi)^4} \frac{d^4L}{(2\pi)^4} \frac{1}{R_K^2 K^4} \left(\frac{1}{(K+L)^2 L^2} - \frac{1}{((K+L)^2 + M^2)(L^2 + M^2)} \right).$$

Result coincides with

S.S.Aleshin, *et al.* Nucl. Phys. B **956** (2020), 115020;
A.L.Kataev and K.V.Stepanyantz, Theor.Math.Phys. **181** (2014) 1531.

We give the total number of graphs for the massless theory up to 4 loops.

	1 loop	2 loops	3 loops	4 loops
	2	14	189	3702

Timing (Windows)

Operating system: Windows 10 x64
Processor: AMD Ryzen 5 1600 Six-Core Processor 3.20 GHz
RAM: 8 GB
Compiler: GNU GCC Compiler
Compilation options: -march=native, -O3

contribution $\sim N_f^0$	1 loop	2 loops	3 loops
$\xi_0 = 1$	0.052 s.	0.14 s.	2.6 s.
$\xi_0 \neq 1$	0.067 s.	0.57 s.	2 min. 27 s.

contribution $\sim N_f^1$	2 loops	3 loops
$\xi_0 = 1,\ m = 0$	0.16 s.	6.6 s.
$\xi_0 \neq 1,\ m = 0$	0.52 s.	13 min. 49 s.
$\xi_0 = 1,\ m \neq 0$	0.41 s.	41.5 s.
$\xi_0 \neq 1,\ m \neq 0$	1.23 s.	3 h. 54 min.

contribution $\sim N_f^2$	3 loops
$\xi_0 = 1,\ m = 0$	4.2 s.
$\xi_0 \neq 1,\ m = 0$	6.6 s.
$\xi_0 = 1,\ m \neq 0$	35 s.
$\xi_0 \neq 1,\ m \neq 0$	2 min. 58 s.

Timing (Ubuntu)

Operating system: Ubuntu 22.04.1 LTS
Processor: AMD Ryzen 5 1600 Six-Core Processor 3.20 GHz
RAM: 8 GB
Compiler: GNU GCC Compiler
Compilation options: -march=native, -O3

contribution $\sim N_f^0$	1 loop	2 loops	3 loops
$\xi_0 = 1$	0.0031 s.	0.0077 s.	0.16 s.
$\xi_0 \neq 1$	0.0048 s.	0.013 s.	5.43 s.

contribution $\sim N_f^1$	2 loops	3 loops
$\xi_0 = 1,\ m = 0$	0.0047 s.	0.17 s.
$\xi_0 \neq 1,\ m = 0$	0.012 s.	1 min. 57 s.
$\xi_0 = 1,\ m \neq 0$	0.011 s.	1.35 s.
$\xi_0 \neq 1,\ m \neq 0$	0.036 s.	6 min. 14 s.

contribution $\sim N_f^2$	3 loops
$\xi_0 = 1,\ m = 0$	0.12 s.
$\xi_0 \neq 1,\ m = 0$	0.34 s.
$\xi_0 = 1,\ m \neq 0$	1.24 s.
$\xi_0 \neq 1,\ m \neq 0$	12.6 s.

Here is a graph of the dependence of the operating time on the number of loops for different operating systems:
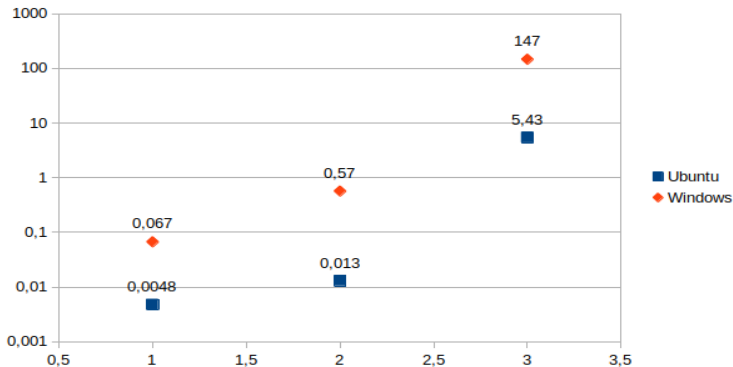
As you can see, the difference in operating time is from 40 to 13 times.

$$\Delta G^{(3)}\Big|_{p=0} = \int \frac{d^4K}{(2\pi)^4} \frac{d^4L}{(2\pi)^4} \frac{d^4Q}{(2\pi)^4} \frac{8e_0^6}{R_K R_L R_Q} \left[-\frac{1}{3K^4L^4Q^4} + \frac{1}{K^4L^2Q^4(Q+L)^2} + \frac{1}{(K+L)^2} \right.$$

$$\times \frac{1}{K^2L^4(Q+K+L)^2} \left(\frac{1}{Q^2} - \frac{2}{(Q+L)^2} \right) \bigg] + N_f \int \frac{d^4K}{(2\pi)^4} \frac{d^4L}{(2\pi)^4} \frac{d^4Q}{(2\pi)^4} \frac{16e_0^6 \, K_\mu L^\mu}{R_K^2 R_L K^4 L^4 (K+L)^2}$$

$$\times \left(\frac{1}{Q^2(Q+K)^2} - \frac{1}{(Q^2+M^2)((Q+K)^2+M^2)} \right) + N_f \int \frac{d^4K}{(2\pi)^4} \frac{d^4L}{(2\pi)^4} \frac{d^4Q}{(2\pi)^4} \frac{8e_0^6}{R_K^2 R_L K^4}$$

$$\times \frac{1}{L^2} \left(\frac{2(Q+K+L)^2 - K^2 - L^2}{Q^2(Q+K)^2(Q+L)^2(Q+K+L)^2} - \frac{2(Q+K+L)^2 - K^2 - L^2}{(Q^2+M^2)((Q+K)^2+M^2)((Q+L)^2+M^2)} \right.$$

$$\times \frac{1}{((Q+K+L)^2+M^2)} + \frac{4M^2}{(Q^2+M^2)^2((Q+K)^2+M^2)((Q+L)^2+M^2)} \right) - (N_f)^2$$

$$\times \int \frac{d^4K}{(2\pi)^4} \frac{d^4L}{(2\pi)^4} \frac{d^4Q}{(2\pi)^4} \frac{8e_0^6}{R_K^3 K^4} \left(\frac{1}{Q^2(Q+K)^2} - \frac{1}{(Q^2+M^2)((Q+K)^2+M^2)} \right) \left(\frac{1}{L^2(L+K)^2} \right.$$

$$\left. - \frac{1}{(L^2+M^2)((L+K)^2+M^2)} \right) + O(e_0^8),$$

Dependence of the ratio of the Feynman diagram generation time to the total operation time

Let us present the dependence (in percent) of the ratio of the generation time to the total operation time on the number of loops in the massless theory c ($\xi_0 = 1$).
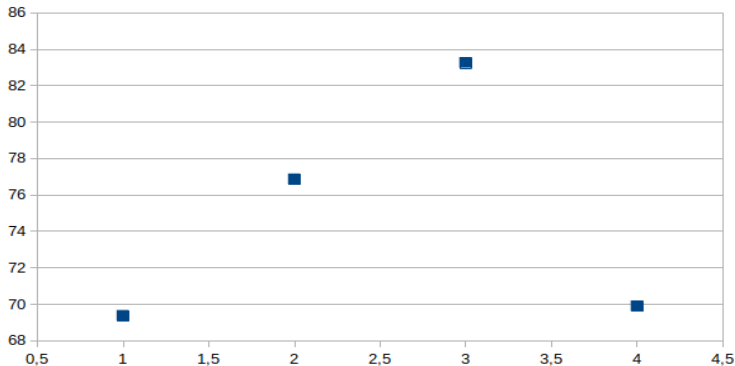
As can be seen, the generation makes a larger contribution, however, the growth of this contribution is observed only up to three loops.

Conclusion and perspectives

- A new computer-algebraic approach to the generation and calculation of Feynman diagrams in supersymmetric theories was proposed, based on the generation of diagrams from first principles.

- A program was created that implements this approach, at the moment it obtains results suitable for analysis in supersymmetric quantum electrodynamics up to three loops.

- At the moment, there is a problem of final reduction of similar ones (including the possible finding of an analogue of master integrals). This makes it impossible to use the results of calculations in four loops and higher.

- It is necessary to improve the algorithm for generating diagrams in order to calculate diagrams in higher loops in this theory, as well as in other theories with a large number of vertices.

- It is also planned to generalize the program for calculations of other types, for example, for calculations of vacuum graphs (that is, those that do not contain external lines).

Thank you for your attention!