> $restart;$

Load TruncatedSeries2022.zip from

http://www.ccas.ru/ca/_media/truncatedseries2022.zip

This archive includes two files: maple.ind and maple.lib.

Put these files to some directory, for example to "/usr/userlib"

> $libname := \text{"/usr/userlib"}, libname:$

>

By definition:

> $\theta(y(x), x, 1) = x \cdot diff(y(x), x)$

$$\theta(y(x), x, 1) = x \left(\frac{d}{dx} y(x)\right) \qquad (1)$$

S is the system with truncated series coefficients wich has Laurent solutions such that they are the solutions for any prolongation of S. The order of S equals 2; the number of equations and the number of unknown functions equal 2; y(x) is a vector of two unknow functions: y[1](x), y[2](x)

> $$S := \begin{bmatrix} 3 \cdot x + O(x^2) & 7 \cdot x^2 + O(x^4) \\ O(x^2) & 17 \cdot x^2 + O(x^4) \end{bmatrix} . \theta(y(x), x, 2) + \begin{bmatrix} -1 + 2 \cdot x + O(x^2) & x + 5 \cdot x^2 + O(x^4) \\ O(x^2) & 11 \cdot x^2 + O(x^4) \end{bmatrix} . \theta(y(x), x, 1) + \begin{bmatrix} O(1) & x - 3 \cdot x^2 + O(x^4) \\ 1 + O(x^2) & -6 \cdot x^2 + O(x^4) \end{bmatrix} . y(x) = 0:$$

Laurent solutions of S:

> $TruncatedSeries:\text{-}LaurentSolution(S, y(x))$

$$\left[\left[6 x^2 \_c_1 + O(x^3), \frac{\_c_1}{x} + \_c_1 + O(x)\right]\right] \qquad (1.1)$$

the name _c[1] is an arbitrary constant.
The result is a list with values of unknow functions.
Their valuations are 2 and -1,
their truncation degrees are 2 and 0, respectively:

> $y[1](x) = \mathbf{(1.1)}[1][1]; y[2](x) = \mathbf{(1.1)}[1][2];$

$$y_1(x) = 6 x^2 \_c_1 + O(x^3)$$

$$y_2(x) = \frac{\_c_1}{x} + \_c_1 + O(x) \qquad (1.2)$$

Any prolongation of S has solutions the initial terms of which coincide with known initial terms of solutions of S. For example, S1 is a prolongation of S :

> $S1 := \begin{bmatrix} 3\cdot x + O(x^2) & 7\cdot x^2 + O(x^4) \\ O(x^2) & 17\cdot x^2 + O(x^4) \end{bmatrix} . \theta(y(x), x, 2) + \begin{bmatrix} -1 + 2\cdot x + O(x^2) & x + 5\cdot x^2 + O(x^4) \\ O(x^2) & 11\cdot x^2 + O(x^4) \end{bmatrix} . \theta(y(x), x, 1) + \begin{bmatrix} 2 + O(x) & x - 3\cdot x^2 + O(x^4) \\ 1 + O(x^2) & -6\cdot x^2 + O(x^4) \end{bmatrix} . y(x) = 0 :$

Laurent solutions of S1:

> *TruncatedSeries:-LaurentSolution*(*S1*, *y*(*x*))

$$\left[\left[6\,x^2\,\_c_1 + O(x^3),\ \frac{\_c_1}{x} + \_c_1 + \frac{3}{2}\,x\,\_c_1 + O(x^2)\right]\right] \quad \textbf{(1.3)}$$

Solutions valuations are 2 and -1,
their truncation degrees are 2 and 1, respectively.

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S2 is the system wich has no Laurent solution such that it is a solution for any prolongation of S2:

> $S2 := \begin{bmatrix} O(x^5) & -1 + O(x^5) \\ 1 + O(x^5) & O(x^5) \end{bmatrix} . \theta(y(x), x, 1) + \begin{bmatrix} O(x^5) & O(1) \\ 2 + O(x^5) & O(x^5) \end{bmatrix} . y(x) :$

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> *TruncatedSeries:-LaurentSolution*(*S2*, *y*(*x*))

$$FAIL \quad \textbf{(2.1)}$$

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For example, S3 is a prolongation of S2, which has solutions with valuation 5:

> $S3 := \begin{bmatrix} O(x^5) & -1 + O(x^5) \\ 1 + O(x^5) & O(x^5) \end{bmatrix} . \theta(y(x), x, 1) + \begin{bmatrix} O(x^5) & 5 + O(x) \\ 2 + O(x^5) & O(x^5) \end{bmatrix} . y(x) = 0 :$

TruncatedSeries:-LaurentSolution(*S3*, *y*(*x*))

$$\left[\left[O(x^{10}),\ x^5\,\_c_1 + O(x^6)\right]\right] \quad \textbf{(2.2)}$$

The valuation of y[1](x) is more then 9, and the one of y[2](x) is 5,
their truncation degrees are 9 and 5, respectively.

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>

And another prolongation has solutions with valuations -2 and 0, and has solutions with the valuations -2 and more then 2, and has solutions with the valuations more then 4 and 0:

$$> S4 := \begin{bmatrix} O(x^5) & -1 + O(x^5) \\ 1 + O(x^5) & O(x^5) \end{bmatrix} . \theta(y(x), x, 1) + \begin{bmatrix} O(x^5) & O(x) \\ 2 + O(x^5) & O(x^5) \end{bmatrix} . y(x) = 0 :$$

TruncatedSeries:-*LaurentSolution*$(S4, y(x))$

$$\left[\left[\frac{\_c_1}{x^2} + O(x^3), \_c_2 + O(x)\right], \left[\frac{\_c_1}{x^2} + O(x^3), O(x^3)\right], \left[O(x^5), \_c_2 + O(x)\right]\right] \quad \textbf{(2.3)}$$

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