```
> restart;
>
```

## Background

```
> eq := (Sum(x^k, k=7 .. infinity) + x^5) * diff(y(x), x$4) + (Sum((10 + 2 * k) * x^k, k=7 .. infinity) + 23
    * x^6 + 18 * x^4) * diff(y(x), x$3) + (Sum(x^k * (k^2 + 20 * k + 26), k=6 .. infinity) + 167 * x^5
    + 96 * x^3) * diff(y(x), x$2) + (Sum(2 * x^k * (5 * k^2 + 22 * k + 17), k=5 .. infinity) + 440 * x^4
    + 168 * x^2) * diff(y(x), x) + (340 * x^3 + 72 * x + Sum(x^k * (k^3 + 13 * k^2 + 32 * k + 20), k=4
    .. infinity)) * y(x) = 0;
```

$$eq := \left(\left(\sum_{k=7}^{\infty} x^k\right) + x^5\right)\left(\frac{d^4}{dx^4}\, y(x)\right) + \left(\left(\sum_{k=7}^{\infty} (10 + 2\,k)\, x^k\right) + 23\,x^6 + 18\,x^4\right)\left(\frac{d^3}{dx^3}\, y(x)\right) + \left(\left(\sum_{k=6}^{\infty} x^k\,(k^2 + 20\,k + 26)\right) + 167\,x^5 + 96\,x^3\right)\left(\frac{d^2}{dx^2}\, y(x)\right) + \left(\left(\sum_{k=5}^{\infty} 2\,x^k\,(5\,k^2 + 22\,k + 17)\right) + 440\,x^4 + 168\,x^2\right)\left(\frac{d}{dx}\, y(x)\right) + \left(340\,x^3 + 72\,x + \left(\sum_{k=4}^{\infty} x^k\,(k^3 + 13\,k^2 + 32\,k + 20)\right)\right) y(x) = 0 \quad \textbf{(1.1)}$$

```
>
```

Abramov S., Khmelnov D.
Regular Solutions of Linear Differential Systems with Power Series Coefficients.
Programming and Computer Software. — 2014. — Vol. 40, no. 2. — P. 98–106

Abramov S., Khmelnov D., Ryabenko A.
Procedures for Searching Local Solutions of Linear Differential Systems with Infinite Power Series in the Role of Coefficients.
Programming and Computer Software. — 2016. — Vol. 42, no. 2. — P. 55–64

Load from http://www.ccas.ru/ca/_media/truncatedseries2020.zip the archive with two files: maple.ind and maple.lib.
- Put these files to some directory, for example to "/usr/userlib".
- Assign libname := "/usr/userlib", libname in the Maple session.

```
> libname := "maple.lib", libname :
> with(TruncatedSeries);
```

$$[FormalSolution, LaurentSolution, RegularSolution] \quad \textbf{(1.2)}$$

```
> y(x) = LaurentSolution(eq, y(x))[1];
```

$$y(x) = \frac{\_c_1}{x^4} + \frac{\_c_2}{x^3} + \frac{\_c_3}{x^2} + O\left(\frac{1}{x}\right) \quad \textbf{(1.3)}$$

```
> y(x) = LaurentSolution(eq, y(x), degree = 0)[1]
```

$$y(x) = \frac{\_c_1}{x^4} + \frac{\_c_2}{x^3} + \frac{\_c_3}{x^2} + \frac{-\frac{\_c_2}{3} - \frac{\_c_1}{3}}{x} - \frac{5\,\_c_3}{12} - \frac{\_c_2}{6} - \frac{\_c_1}{4} + O(x) \quad \textbf{(1.4)}$$

```
>
```

Abramov S., Barkatou M., Pflügel E.
Higher-order linear differential systems with truncated coefficients
CASC'2011 Proceedings, Lecture Notes in Computer Science. — 2011. — Vol. 6885. — P. 10–24

```
> eq7 := eval(map(el→convert(series(remove(has, el, y), x, 8), polynom)·select(has, el, y), lhs(eq))) = 0;
  y(x) = LaurentSolution(eq7, y(x), degree = 1)[1];
```

$$eq7 := (x^7 + x^5)\left(\frac{d^4}{dx^4}\, y(x)\right) + (24\,x^7 + 23\,x^6 + 18\,x^4)\left(\frac{d^3}{dx^3}\, y(x)\right) + (215\,x^7 + 182\,x^6 + 167\,x^5$$

$$+96x^3\Big)\left(\frac{d^2}{dx^2}y(x)\right)+\left(832x^7+658x^6+504x^5+440x^4+168x^2\right)\left(\frac{d}{dx}y(x)\right)+\Big(1224x^7$$
$$+896x^6+630x^5+420x^4+340x^3+72x\Big)y(x)=0$$

$$y(x)=\frac{\_c_1}{x^4}+\frac{\_c_2}{x^3}+\frac{\_c_3}{x^2}+\frac{-\frac{\_c_2}{3}+\frac{209\_c_1}{3}}{x}-\frac{5\_c_3}{12}+\frac{29\_c_2}{6}-\frac{383\_c_1}{12}+x\left(-\frac{299\_c_2}{60}-\frac{247\_c_1}{10}+\frac{3\_c_3}{10}\right)+O(x^2) \quad \textbf{(1.5)}$$

> eq6 := eval(map(el→convert(series(remove(has, el, y), x, 7), polynom)·select(has, el, y), lhs(eq))) = 0;
> y(x) = LaurentSolution(eq6, y(x), degree = 0)[1];

$$eq6 := x^5\left(\frac{d^4}{dx^4}y(x)\right)+\left(23x^6+18x^4\right)\left(\frac{d^3}{dx^3}y(x)\right)+\left(182x^6+167x^5+96x^3\right)\left(\frac{d^2}{dx^2}y(x)\right)$$
$$+\left(658x^6+504x^5+440x^4+168x^2\right)\left(\frac{d}{dx}y(x)\right)+\Big(896x^6+630x^5+420x^4+340x^3$$
$$+72x\Big)y(x)=0$$

$$y(x)=\frac{\_c_1}{x^3}+\frac{\_c_2}{x^2}+\frac{89\_c_1}{3x}+\frac{5\_c_2}{4}-\frac{91\_c_1}{6}+O(x) \quad \textbf{(1.6)}$$

>

## The current problem

$$> eq := \left(\left(\sum_{k=7}^{\infty}x^k\right)+x^5\right)\frac{d^4}{dx^4}y(x)+\left(\left(\sum_{k=7}^{\infty}(10+2k)x^k\right)+23x^6+18x^4\right)\frac{d^3}{dx^3}y(x)+$$
$$\left(\left(\sum_{k=6}^{\infty}x^k\left(k^2+20k+26\right)\right)+167x^5+96x^3\right)\frac{d^2}{dx^2}y(x)+\left(\left(\sum_{k=5}^{\infty}2x^k\left(5k^2+22k+17\right)\right)+440x^4$$
$$+168x^2\right)\frac{d}{dx}y(x)+\left(340x^3+72x+\left(\sum_{k=4}^{\infty}x^k\left(k^3+13k^2+32k+20\right)\right)\right)y(x)=0:$$

$$> eqT := \left(x^5+x^7+O(x^8)\right)\frac{d^4}{dx^4}y(x)+\left(18x^4+23x^6+O(x^7)\right)\frac{d^3}{dx^3}y(x)+$$
$$\left(96x^3+167x^5+O(x^6)\right)\frac{d^2}{dx^2}y(x)+\left(168x^2+440x^4+O(x^5)\right)\frac{d}{dx}y(x)+$$
$$\left(72x+340x^3+420x^4+O(x^5)\right)y(x)=0:$$

>

> LaurentSolution(eqT, y(x));

$$\left[\frac{\_c_1}{x^4}+\frac{\_c_2}{x^3}+\frac{\_c_3}{x^2}+O\left(\frac{1}{x}\right),\ \frac{\_c_2}{x^3}+\frac{\_c_3}{x^2}-\frac{\_c_2}{3x}+O(1),\ \frac{\_c_3}{x^2}-\frac{5\_c_3}{12}+O(x)\right] \quad \textbf{(2.1)}$$

>

>

## An example

>

> (1 + O(x))·θ(y(x), x, 1) + (−1 + O(x))·y(x) = 0 :

$$> \left(1+\left(\sum_{k=1}^{\infty}\frac{x^k}{(k+1)!}\right)\right)\cdot\theta(y(x),x,1)+\left(-1-\sum_{k=1}^{\infty}\frac{x^k}{k!}\right)\cdot y(x)=0:$$
LaurentSolution(%, y(x), degree = 4)[1];

**(3.1)**

$$x\_c_1 + \frac{x^2\_c_1}{2} + \frac{x^3\_c_1}{6} + \frac{x^4\_c_1}{24} + O(x^5) \quad \textbf{(3.1)}$$

> $$\left(1 + \sum_{k=1}^{\infty} x^k \left(\begin{cases} \frac{(-1)^{\frac{k}{2}}}{(k+1)!} & k :: even \\ 0 & otherwise \end{cases}\right)\right) \cdot \theta(y(x), x, 1) + \left(-1 - \sum_{k=1}^{\infty} x^k \left(\begin{cases} \frac{(-1)^{\frac{k}{2}}}{k!} & k :: even \\ 0 & otherwise \end{cases}\right)\right) \cdot y(x) :$$

*LaurentSolution*(%, y(x), *degree* = 5)[1];

$$x\_c_1 - \frac{x^3\_c_1}{6} + \frac{x^5\_c_1}{120} + O(x^6) \quad \textbf{(3.2)}$$

>

> (1 + O(x))·θ(y(x), x, 1) + (-1 + O(x))·y(x) = 0 :
*LaurentSolution*(%, y(x));

$$\left[x\_c_1 + O(x^2)\right] \quad \textbf{(3.3)}$$

>

# Examples 1,2,3

> (-1 + O(x))·θ(y(x), x, 2) + (-2 + O(x))·θ(y(x), x, 1) + (O(x))·y(x) = 0 :
*LaurentSolution*(%, y(x));

$$\left[\_c_1 + O(x)\right] \quad \textbf{(4.1)}$$

> (-1 + x + x^2 + O(x^3))·θ(y(x), x, 2) + (-2 + O(x^3))·θ(y(x), x, 1) +
(x + 6· x^2 + O(x^4))·y(x) = 0 :
*LaurentSolution*(%, y(x));

$$\left[\frac{\_c_1}{x^2} - \frac{5\_c_1}{x} + \_c_2 + O(x), \_c_2 + \frac{x\_c_2}{3} + \frac{5x^2\_c_2}{6} + \frac{13x^3\_c_2}{30} + O(x^4)\right] \quad \textbf{(4.2)}$$

> (-1 + x + x^2 + O(x^3))·θ(y(x), x, 2) + $(-2 + x^2 + O(x^3))$·θ(y(x), x, 1) +
O(x^4)·y(x) = 0 :
*LaurentSolution*(%, y(x));

$$\left[\_c_1 + O(x^4)\right] \quad \textbf{(4.3)}$$

>

>

> (1 + O(x))·θ(y(x), x, 1) + (x^4 + O(x^5))·y(x) = 0 :
*LaurentSolution*(%, y(x));

$$\left[\_c_1 - \frac{x^4\_c_1}{4} + O(x^5)\right] \quad \textbf{(5.1)}$$

> (1 + O(x))·θ(y(x), x, 1) + O(x)·y(x) :
*LaurentSolution*(%, y(x));

$$\left[\_c_1 + O(x)\right] \quad \textbf{(5.2)}$$

>

>

> $eqT := \left(x^5 + x^7 + O\left(x^8\right)\right)\left(\frac{d^4}{dx^4}\,y(x)\right) + \left(18\,x^4 + 23\,x^6 + O\left(x^7\right)\right)\left(\frac{d^3}{dx^3}\,y(x)\right) + \left(96\,x^3 + 167\,x^5 + O\left(x^6\right)\right)\left(\frac{d^2}{dx^2}\,y(x)\right) + \left(168\,x^2 + 440\,x^4 + O\left(x^5\right)\right)\left(\frac{d}{dx}\,y(x)\right) + \left(72\,x + 340\,x^3 + 420\,x^4 + O\left(x^5\right)\right)y(x):$

> $LaurentSolution(eqT, y(x));$

$$\left[\frac{\_c_1}{x^4} + \frac{\_c_2}{x^3} + \frac{\_c_3}{x^2} + O\left(\frac{1}{x}\right), \frac{\_c_2}{x^3} + \frac{\_c_3}{x^2} - \frac{\_c_2}{3\,x} + O(1), \frac{\_c_3}{x^2} - \frac{5\,\_c_3}{12} + O(x)\right] \tag{6.1}$$

>

> $(2 + O(x))\cdot\theta(y(x), x, 1) + (1 + O(x))\cdot y(x) = 0:$
$LaurentSolution(\%, y(x));$

$$[\ ] \tag{7.1}$$

>

> $(x\hat{}2 + O(x\hat{}3))\cdot diff(y(x), x, x) + O(x)\cdot diff(y(x), x) + (1 + O(x))\cdot y(x) = 0:$
$LaurentSolution(\%, y(x));$

$$FAIL \tag{7.2}$$

>

>